Rewarding Learning

General Certificate of Secondary Education
2017

Further Mathematics

Unit 2
Mechanics and Statistics

[GMF21]

THURSDAY 22 JUNE, MORNNING

MARK SCHEME

GCSE Further Mathematics

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for accurate working, whether in calculation, reading from tables, graphs or answers.

MW indicates marks for combined method and accurate working.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be **followed through** from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

It should be noted that where an error trivialises a question, or changes the nature of the skills being tested, then as a general rule, it would be the case that not more than half the marks for that question or part of that question would be awarded; in some cases the error may be such that no marks would be awarded.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

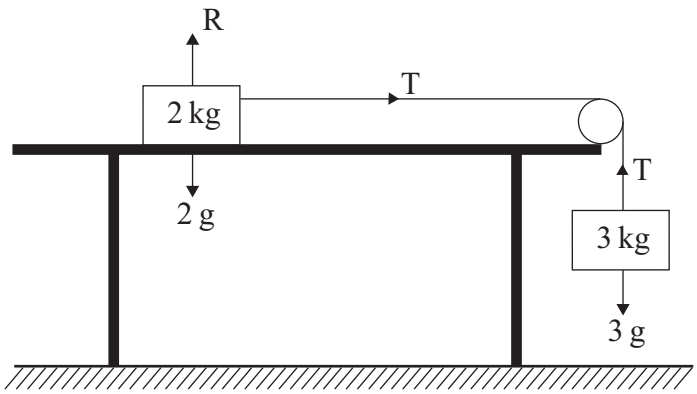
(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier, only a proportion of the marks will be available (based on the professional judgement of the examiner).

				AVAILABLE MARKS
1	(i)	Between A and B	W1	
		Graph is steepest in this region	M1	
		Alternative explanation		
		Stationary between BC and DE		
		Speed AB $= \frac{20}{5} = 4$ m/s	M1	
		Speed CD $= \frac{5}{4} = 1.25$ m/s		
		Speed EF $= \frac{20}{6} = 3.33$ m/s		
	(ii)	Total distance $= 20 + 5 + 20 = 45$ m	MW1	
		Total time $= 18$ s		
		Average speed $= \frac{45}{18} = 2.5$ m/s	MW1	4
2	(i)	$\mathbf{R} = \mathbf{a} + \mathbf{b}$		
		$= (2\mathbf{i} - 3\mathbf{j}) + (-7\mathbf{i} + 4\mathbf{j})$	M1	
		$= (-5\mathbf{i} + \mathbf{j})$N	W1	
	(ii)	$\lvert\mathbf{R}\rvert = \sqrt{(-5)^2 + 1^2}$	M1	
		$= \sqrt{26} = 5.10$ N	W1	
	(iii)	$2(2\mathbf{i} - 3\mathbf{j}) + 5(x\mathbf{i} + y\mathbf{j}) = 4(-7\mathbf{i} + 4\mathbf{j})$	MW1	
		$(4 + 5x)\mathbf{i} + (-6 + 5y)\mathbf{j} = -28\mathbf{i} + 16\mathbf{j}$		
		$4 + 5x = -28$		
		$x = -6.4$	MW1	
		$-6 + 5y = 16$		
		$y = 4.4$	MW1	7
		Alternative solution		
		$\mathbf{c} = \frac{1}{5}(4\mathbf{b} - 2\mathbf{a})$		
		$x\mathbf{i} + y\mathbf{j} = \frac{1}{5}[4(-7\mathbf{i} + 4\mathbf{j}) - 2(2\mathbf{i} - 3\mathbf{j})]$	MW1	
		$= \frac{1}{5}(-32\mathbf{i} + 22\mathbf{j})$		
		$x = -\frac{32}{5} = -6.4$	MW1	
		$y = \frac{22}{5} = 4.4$	MW1	

	AVAILABLE MARKS

3 **(i)**



W1

W1

(ii) At 2 kg mass, $T = 2a$ MW1

At 3 kg mass, $3g - T = 3a$ MW1

So $5a = 3g$

$a = \frac{30}{5} = 6 \text{ m/s}^2$ W1

(iii) $T = 2a = 2 \times 6 = 12 \text{ N}$ MW1

(iv) $u = 0, a = 6, s = 0.75$

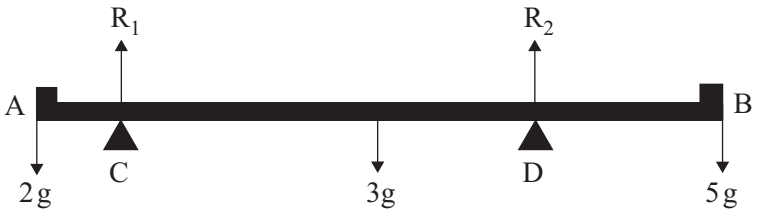
$s = ut + \frac{1}{2}at^2$

$0.75 = 3t^2$ MW1

$t = 0.5 \text{ s}$ W1 8

AVAILABLE MARKS

4 (i)



Take moments about D

$R_1 \times 5 + 5g \times 2 = 3g \times 2 + 2g \times 6$ M1, W1

$5R_1 = 8g$

$R_1 = 16$ N W1

Resolve vertically

$R_1 + R_2 = 10g$

$R_2 = 100 - 16$ MW1

$= 84$ N W1

Alternative solution

Take moments about C:

$R_2 \times 5 + 2g \times 1 = 3g \times 3 + 5g \times 7$ M1, W1

$5R_2 = 42g$

$R_2 = 84$ N W1

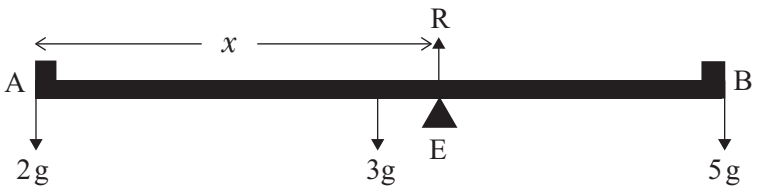
Resolve vertically

$R_1 + R_2 = 10\,g$ M1

$R_1 = 100 - 84$

$= 16$ N W1

(ii)



Resolve vertically

$R = 2g + 3g + 5g = 10g$

Take moments about A:

$10g \times x = 3g \times 4 + 5g \times 8$ MW1, MW1

$10g\,x = 52g$

$x = 5.2$ W1

Alternative solution

Take moments about E:

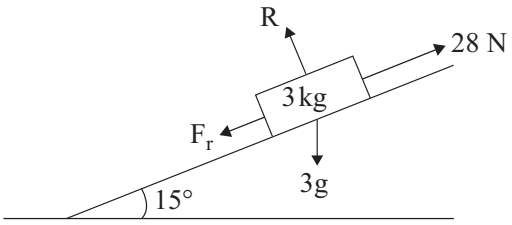
$5g \times (8 - x) = (2g \times x) + 3g \times (x - 4)$ MW1, MW1

$40g - 5gx = 2gx + 3gx - 12g$

$52g = 10gx$

$x = 5.2$ W1 8

			AVAILABLE MARKS
5	**(i)** $u = 12, v = 0, a = -10$		
	$v^2 = u^2 + 2as$		
	$0 = 12^2 + 2(-10)\,s$	M1	
	$s = 7.2$ m	W1	
	Maximum height above ground $= 7.2 + 1.2 = 8.4$ m	W1	
	(ii) $u = 0,\ a = 10, s = 8.4$		
	$v^2 = u^2 + 2as$		
	$v^2 = 0 + 2(10)(8.4)$	M1	
	$v = \sqrt{168} = 12.961$ m/s $\rightarrow$ 12.96 m/s	W1	
	Alternative solution		
	$v^2 = u^2 + 2as$		
	$v^2 = 12^2 + 2(-10)(-1.2)$	M1	
	$v = \sqrt{168} \rightarrow 12.96$ m/s	W1	
	(iii) $v = u + at$		
	On way up		
	$0 = 12 + (-10)t$	M1	
	$t = 1.2$ s	W1	
	On way down		
	$12.961 = 0 + 10t$		
	$t = 1.296$ s	MW1	
	Total time $= 1.2 + 1.296 = 2.50$ s	W1	
	Alternative solution		
	$v = u + at$		
	$-12.961 = 12 + (-10)t$	M2	
	$t = 2.50$ s	W2	
	Alternative solution		
	$s = ut + \frac{1}{2}at^2$		
	$-1.2 = 12t + (-5)t^2$	M2	
	$5t^2 - 12t - 1.2 = 0$	W1	
	$t = \dfrac{12 \pm \sqrt{168}}{10}$		
	$t = 2.50s$	W1	9

			AVAILABLE MARKS
6	**(i)**		
		MW1 (2 forces) MW1 (3rd force)	
	(ii)	Resolve perpendicular to ramp	
		$R = 3g \cos 15° = 28.978$ N — MW1	
		$F_r = \mu R$	
		$= 0.2 \times 28.978$	
		$= 5.796$ N — MW1	
		Resolve parallel to ramp and use $F = ma$	
		$28 - F_r - 3g \sin 15° = 3a$ — MW1, MW1	
		$28 - 5.796 - 7.765 = 3a$	
		$3a = 14.439$	
		$a = 4.813$ m/s^2 $\rightarrow 4.81$ m/s^2 — W1	
	(iii)	$u = 0, t = 5, a = 4.813$	
		$v = u + at$	
		$v = 0 + 4.813 \times 5$ — MW1	
		$= 24.065$ m/s $\rightarrow 24.07$ m/s — W1	
	(iv)	Resolve parallel to ramp	
		$-F_r - 3g \sin 15° = 3a$	
		$3a = -5.796 - 7.765$ — MW1, MW1	
		$a = -4.520$ m/s^2 — W1	
		$v = u + at$	
		$0 = 24.065 + (-4.520)t$ — MW1	
		$t = 5.32$ s — W1	14
7	**(i)**	$r = -1$ implies e.g. perfect negative correlation, rankings exactly reversed — M1	
	(ii)		

Singer	Monica	Colin	Michaela	Sam	Maurice
Rank Order (Simon)	1	2	3	4	5
Rank Order (Louis)	5	4	3	2	1

MW1 — 2

AVAILABLE MARKS

8 **(i)**

Midpoint of interval x	Number of people f	fx
10	12	120
30	7	210
62.5	4	250
92.5	2	185
107.5	5	537.5

$\sum fx = 1302.5$ M1, W1

$\text{mean} = \dfrac{\sum fx}{\sum f} = \dfrac{1302.5}{30} = 43.42 \text{ km}$ W1

(ii) median class is 20–40

$\text{median} = 17.5 + \dfrac{\left(\frac{30}{2} - 12\right) \times 25}{7}$ M1 (17.5+) MW1 (15 – 12) MW1 $\left(\frac{25}{7}\right)$

$= 28.21 \text{ km}$ W1 7

9 **(i)**

Length (cm)	Frequency
6–10	$5 \times 8 = 40$
11–18	$8 \times 5 = 40$
19–28	$10 \times 9 = 90$
29–33	$5 \times 6 = 30$

M1, W1 (group limits)

M1, W1 (frequency)

(ii) $P(> 28.5 \mid > 10.5) = \dfrac{P(> 28.5)}{P(> 10.5)} = \dfrac{30/200}{(40 + 90 + 30)/200}$ MW1

$= \dfrac{30}{40 + 90 + 30}$

$= \dfrac{3}{16}$ or 0.1875 W1 6

AVAILABLE MARKS

10 **(i)** Total for all workers = $1852 \times 100 = 185\,200$

Total for graduates = $1960 \times 40 = 78\,400$

Total for all non-graduates = $185\,200 - 78\,400$ M1

$= 106\,800$ W1

$$\text{Mean for non-graduates} = \frac{106\,800}{60}$$

$= £1780$ MW1

(ii) For all workers

$$\text{S.D.} = 96 = \sqrt{\frac{\sum x^2}{100} - 1852^2}$$

$$\sum x^2 = (96^2 + 1852^2) \times 100 = 343\,912\,000$$ MW1

For graduates

$$\text{S.D.} = 32 = \sqrt{\frac{\sum x^2}{40} - 1960^2}$$

$$\sum x^2 = (32^2 + 1960^2) \times 40 = 153\,704\,960$$ MW1

For non-graduates

$$\sum x^2 = 343\,912\,000 - 153\,704\,960$$
$$= 190\,207\,040$$

$$\text{S.D.} = \sqrt{\frac{190\,207\,040}{60} - 1780^2}$$ M1

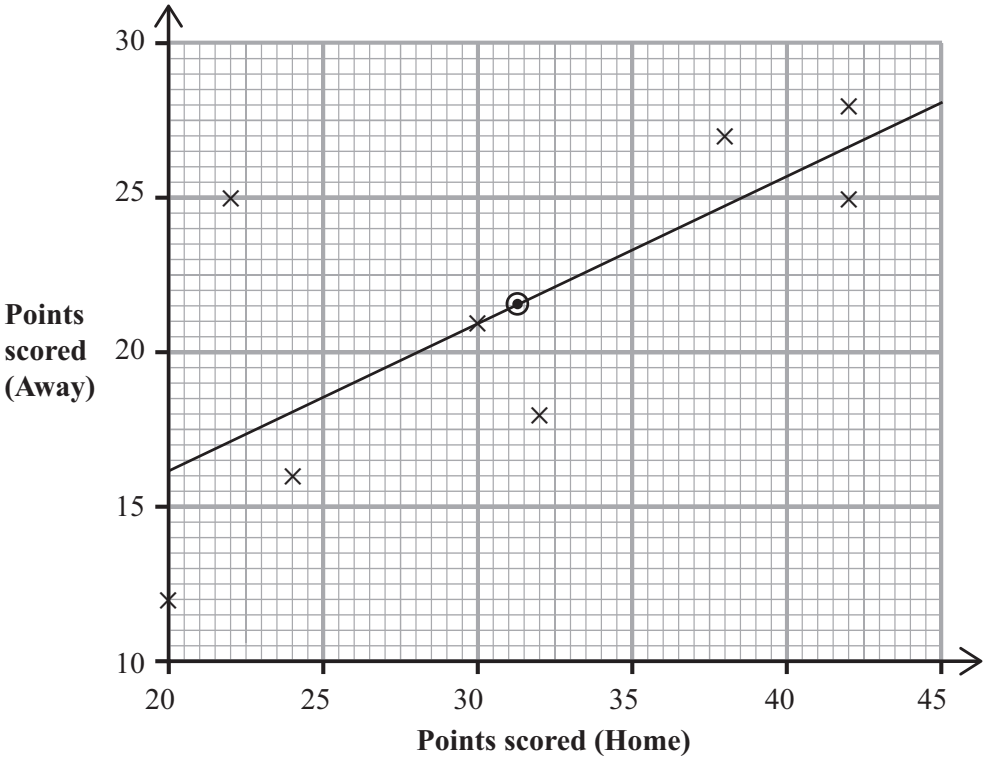
$= £41.44$ W1 7

AVAILABLE MARKS

11 **(i)** Mean for home matches $= \frac{250}{8} = 31.25$

Mean for away matches $= \frac{172}{8} = 21.5$ MW1

(ii)



M1
(through means)
W1
(slope)

(iii) Gradient $= \frac{28 - 21.5}{45 - 31.25}$

$= 0.473$ M1

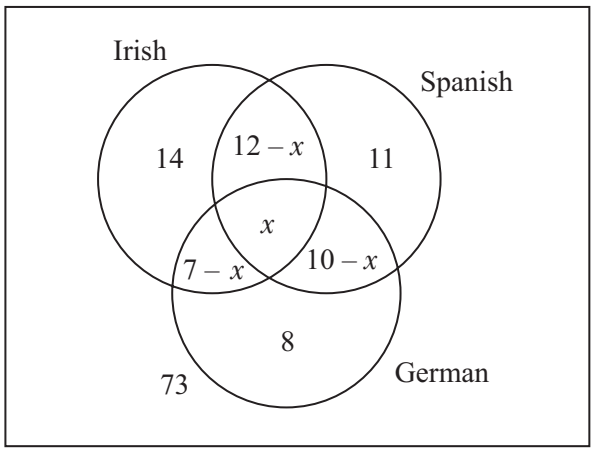
Using means for equation of line

$21.5 = 0.473\ (31.25) + c$

$c = 6.718$ M1

So equation is

$y = 0.47x + 6.72$ W1 6

			AVAILABLE MARKS
12	**(i)** Number of pupils wearing glasses $= 1215 \times \frac{1}{3} = 405$	MW1	
	Number of boys wearing glasses $= 405 - 189 = 216$	MW1	
	P(boy wears glasses) $= \frac{216}{1215 - 540} = \frac{216}{675} = \frac{8}{25}$ or 0.32	MW1	
	Alternative solution		
	Boy / Girl: Glasses 216, 189, 405; No glasses 459, 351, 810; 675, 540	M1, W1	
	P(boy wears glasses) $= \frac{216}{675} = \frac{8}{25} = 0.32$	MW1	
	(ii) Number of boys not wearing glasses $= 675 - 216 = 459$	MW1	
	Number of pupils not wearing glasses $= 1215 - 405 = 810$	MW1	
	So P(boy | no glasses) $= \frac{459}{810} = \frac{17}{30}$ or 0.57	MW1	6
13	**(i)** 	MW1 (x) MW1 ($12 - x$, $7 - x$, $10 - x$) MW1 (14, 11, 8, 73)	
	(ii) $73 + 14 + 11 + 8 + 12 - x + 7 - x + 10 - x + x = 125$	M1	
	$135 - 2x = 125$		
	$x = 5$	W1	
	(iii) Number speaking Irish $= 14 + 7 + 5 + 2 = 28$	MW1	
	So P(speaks Irish) $\frac{28}{125}$ or 0.224	MW1	7

				AVAILABLE MARKS
14	**(i)**	$P(\text{B and B}) = \frac{6}{20} \times \frac{6}{25}$	MW1	
		$= \frac{9}{125} = 0.072$	W1	
	(ii)	P(B and B) + P(Y and Y) + P (W and W)		
		$= \frac{9}{125} + \frac{12}{20} \times \frac{16}{25} + \frac{2}{20} \times \frac{5}{25}$	M1	
		$= \frac{9}{125} + \frac{48}{125} + \frac{1}{50}$		
		$= \frac{119}{250}$ or 0.476	W1	4
15	**(i)**	$P(\text{2nd earring gold}) = \frac{1}{1+x}$	MW1	
	(ii)	$\frac{2}{2+x} \times \frac{1}{1+x} = \frac{1}{15}$	M1	
		$(2+x)(1+x) = 30$		
		$x^2 + 3x + 2 = 30$		
		$x^2 + 3x - 28 = 0$	MW1	
		$(x+7)(x-4) = 0$	MW1	
		$x = -7$ (impossible)		
		or $x = 4$	W1	5
			Total	**100**