



Rewarding Learning

General Certificate of Secondary Education
2018

Centre Number

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Candidate Number

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Further Mathematics

Unit 2 (With calculator)
Mechanics and Statistics



[GMF21]

GMF21

TUESDAY 19 JUNE, MORNING

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.

You must answer the questions in the spaces provided.

Do not write outside the boxed area on each page, on blank pages or tracing paper.

Complete in black ink only. **Do not write with a gel pen.**

All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.

Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.

Answer **all thirteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

You may use a calculator.

The Formula Sheet is on pages 2 and 3.



Formula Sheet

PURE MATHEMATICS

Quadratic equations: If $ax^2 + bx + c = 0$ ($a \neq 0$)

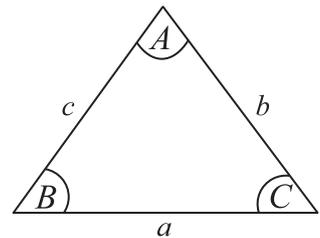
$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometry:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of triangle} = \frac{1}{2} ab \sin C$$



Differentiation:

If $y = ax^n$ then $\frac{dy}{dx} = nax^{n-1}$

Integration:

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c \quad (n \neq -1)$$

Logarithms:

If $a^x = n$ then $x = \log_a n$

$$\log(ab) = \log a + \log b$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

$$\log a^n = n \log a$$

Matrices:

If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\det \mathbf{A} = ad - bc$

and $\mathbf{A}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad (ad - bc \neq 0)$



MECHANICS

Vectors: Magnitude of $x\mathbf{i} + y\mathbf{j}$ is given by $\sqrt{x^2 + y^2}$

Angle between $x\mathbf{i} + y\mathbf{j}$ and \mathbf{i} is given by $\tan^{-1}\left(\frac{y}{x}\right)$

Uniform Acceleration: $v = u + at$ $s = \frac{1}{2}(u + v)t$
 $v^2 = u^2 + 2as$ $s = ut + \frac{1}{2}at^2$

where u is initial velocity t is time
 v is final velocity s is change in displacement
 a is acceleration

Newton's Second Law: $F = ma$

where F is resultant force m is mass
 a is acceleration

STATISTICS

Statistical measures: Mean = $\frac{\sum fx}{\sum f}$ Median = $L_1 + \frac{\left\{\frac{N}{2} - (\sum f)_1\right\}c}{f_{median}}$

where L_1 is lower class boundary of the median class
 N is total frequency
 $(\sum f)_1$ is the sum of the frequencies up to but not including the median class
 f_{median} is the frequency of the median class
 c is the width of the median class

Standard deviation = $\sqrt{\frac{\sum fx^2}{\sum f} - (\bar{x})^2}$ where \bar{x} is the mean

Probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Bivariate Analysis: Spearman's coefficient of rank correlation is given by

$$r = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

[Turn over



Section A

Mechanics

You should spend approximately **one hour** on this section.

Take $g = 10 \text{ m/s}^2$ when required.

- 1 (Throughout this question \mathbf{i} and \mathbf{j} denote unit vectors parallel to a set of standard x - y axes.)

A body is acted upon by three forces \mathbf{a} , \mathbf{b} and \mathbf{c} where

$$\mathbf{a} = (p\mathbf{i} - 5\mathbf{j}) \text{ N} \quad \mathbf{b} = (3\mathbf{i} + q\mathbf{j}) \text{ N} \quad \mathbf{c} = (-5\mathbf{i} - 3\mathbf{j}) \text{ N}$$

- (i) Given that the body is in equilibrium, calculate the values of p and q .

Answer $p = \underline{\hspace{2cm}}$ $q = \underline{\hspace{2cm}}$ [2]



The three forces are now removed from the body and replaced with a single force \mathbf{F} . The body, which is of mass 6 kg, now moves with a constant acceleration of $(3\mathbf{i} - 2\mathbf{j}) \text{ m/s}^2$.

(ii) Calculate the **magnitude** of \mathbf{F} .

Answer _____ N [3]

(iii) Calculate the acute angle the acceleration makes with the positive x -axis.

Answer _____ ° [2]

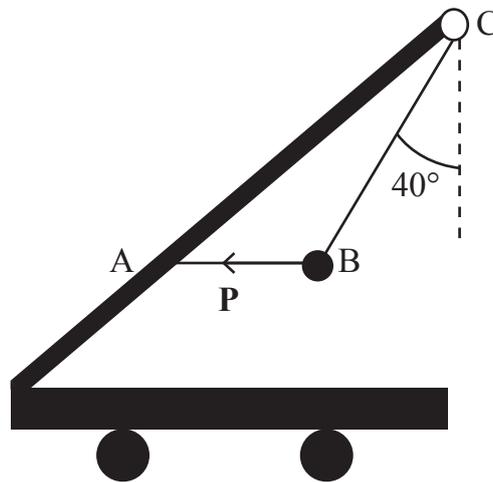
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- 2 A lorry with a wrecking ball is called to a demolition job.

The wrecking ball B has mass 100 kg. It is connected to the top of the lorry by a light, inextensible wire BC.

When the lorry is in position, the rope AB is pulled by a **horizontal** force **P** N to hold the wrecking ball at an angle of 40° to the vertical, as shown in the diagram below.



- (i) Calculate the tension in the wire BC.

Answer _____ N [2]



(ii) Calculate the value of P.

Answer _____ [2]



3 Sarah is running at a constant speed of 4 m/s when she passes a shop.

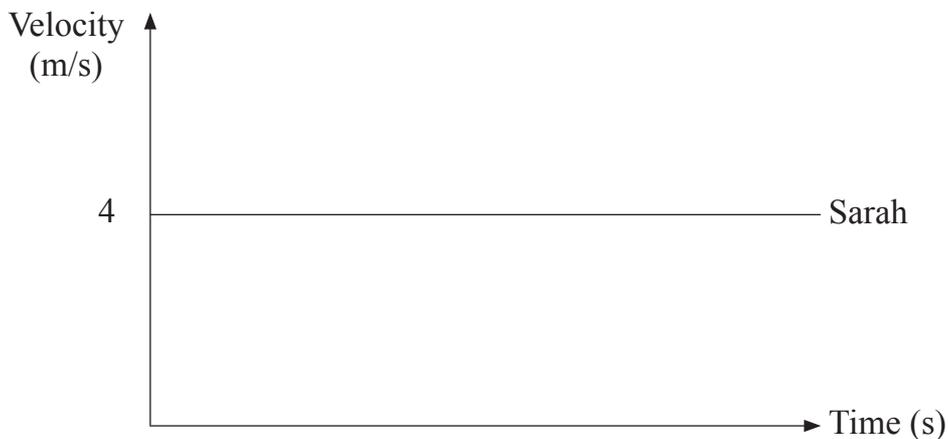
At the moment Sarah passes the shop, a cyclist, Matthew, starts from rest at the shop and follows Sarah.

Matthew accelerates for 12 seconds to reach a maximum speed of 6 m/s. He then continues at this maximum speed until he catches up with Sarah.

(i) Calculate Matthew's acceleration.

Answer _____ m/s² [2]

(ii) Sketch the velocity–time graph of Matthew's journey on the diagram below.



[2]



(iii) Calculate the time it takes Matthew to catch up with Sarah.

Answer _____ s [4]



Immediately after Matthew catches up with Sarah, he decelerates uniformly to rest in 10 seconds.

Sarah continues to travel at the constant speed of 4 m/s during this time.

(iv) After the 10 seconds, who is in the lead and by what distance?

Answer _____ is in the lead by _____ m [3]



- 4 A uniform rod AB, of length 10 m and mass 8 kg, rests on a smooth pivot at the point C where the distance BC is 4 m.

When a mass of 20 kg is placed at end A and a mass of M kg is placed at end B, the rod is horizontal and in equilibrium.

- (i) Mark on the diagram below all the forces acting on the rod.



[2]

- (ii) Calculate the value of M .

Answer _____ [3]

[Turn over



The pivot is now moved to a position D, where the distance AD is x metres.

The mass of M kg at B is removed and replaced with a mass of 12 kg as shown below.



The rod remains horizontal and in equilibrium.

(iii) Calculate the reaction at the pivot.

Answer _____ N [1]



(iv) Calculate the value of x .

Answer _____ [3]

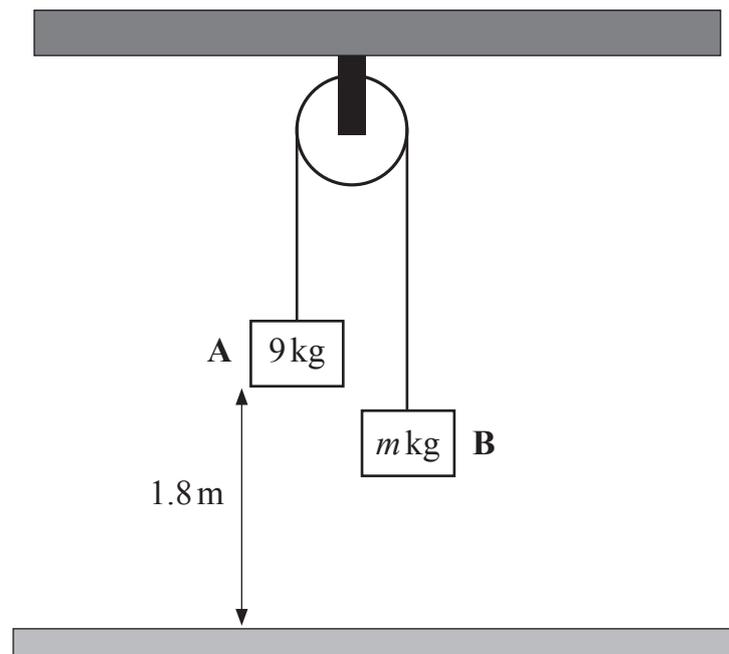
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- 5 Two blocks, **A** and **B**, of masses 9 kg and $m\text{ kg}$ respectively, are connected by a light inextensible string that passes over a smooth fixed pulley which is attached to a ceiling.

The mass of block **A** is greater than the mass of block **B**.

Block **A** is held at a distance of 1.8 m above horizontal ground, as shown in the diagram below.



The blocks are released from rest.

- (i) Mark on the diagram above all the forces acting on the blocks.

[2]



Block **A** hits the ground after 1.5 seconds.

When the blocks are in motion, and assuming Block **B** does not hit the pulley,

(ii) show that the acceleration of the blocks is 1.6 m/s^2 ,

[2]

(iii) calculate the tension in the string,

Answer _____ N [2]

[Turn over



(iv) calculate the value of m ,

Answer _____ [2]

(v) calculate the force exerted by the string on the pulley.

Answer _____ N [1]





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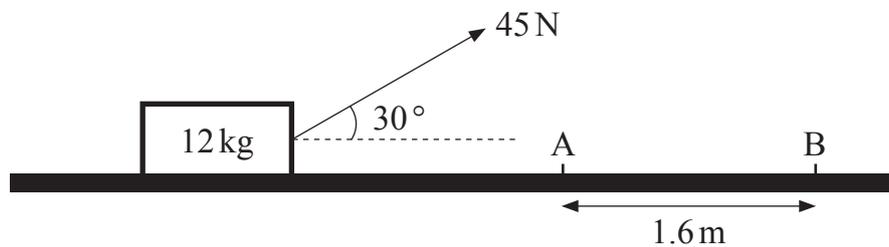
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- 6 A block of mass 12 kg is pulled along a rough horizontal surface by a light, inextensible rope held at an angle of 30° to the horizontal, as shown in the diagram below.

The tension in the rope is 45 N.



The block passes through the points A and B, 1.6 m apart, with speeds 1.2 m/s and 2.8 m/s respectively.

- (i) Calculate the acceleration of the block.

Answer _____ m/s² [2]



(ii) Hence calculate the magnitude of the frictional force.

Answer _____ N [3]

(iii) Hence calculate the value of the coefficient of friction.

Answer _____ [5]

[Turn over



Section B**Statistics**

You should spend approximately **one hour** on this section.

- 7 Jason recorded, to the nearest minute, how long he took to complete his English homeworks. His results are summarised in the table below.

Time (minutes)	10–15	16–25	26–30	31–40	41–45	46–50
Number of homeworks	3	9	6	16	11	5

Calculate an estimate for the median time.

Answer _____ minutes [4]

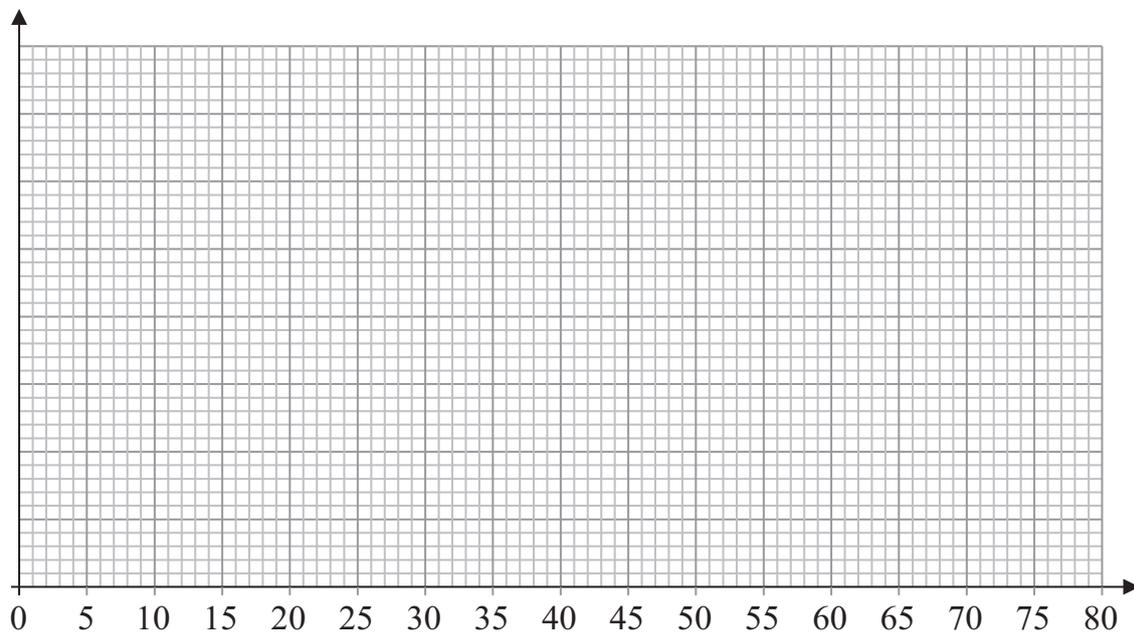


- 8 On Thursday, a doctor recorded the ages of people visiting his health centre. The results are summarised in the table below.

Age (years)	0–29	30–49	50–59	60–69	70–79	80–
Number of people	18	32	22	36	24	0

Draw a histogram on the axes below to represent this information.

Label each axis clearly.



[5]

[Turn over



- 9 The table below shows the lengths and weights of newborn babies in a hospital one day.

Length (cm)	45	55	49	50	56	52	53	48	51
Weight (kg)	2.8	4.2	3.2	3.5	4.1	3.5	3.5	3.3	3.4
Ranks (Length)									
Ranks (Weight)									

- (i) Write down the rank orders for the lengths and weights in the table. [2]
- (ii) Calculate Spearman's coefficient of rank correlation.

Answer _____ [4]



(iii) Interpret your answer to part (ii).

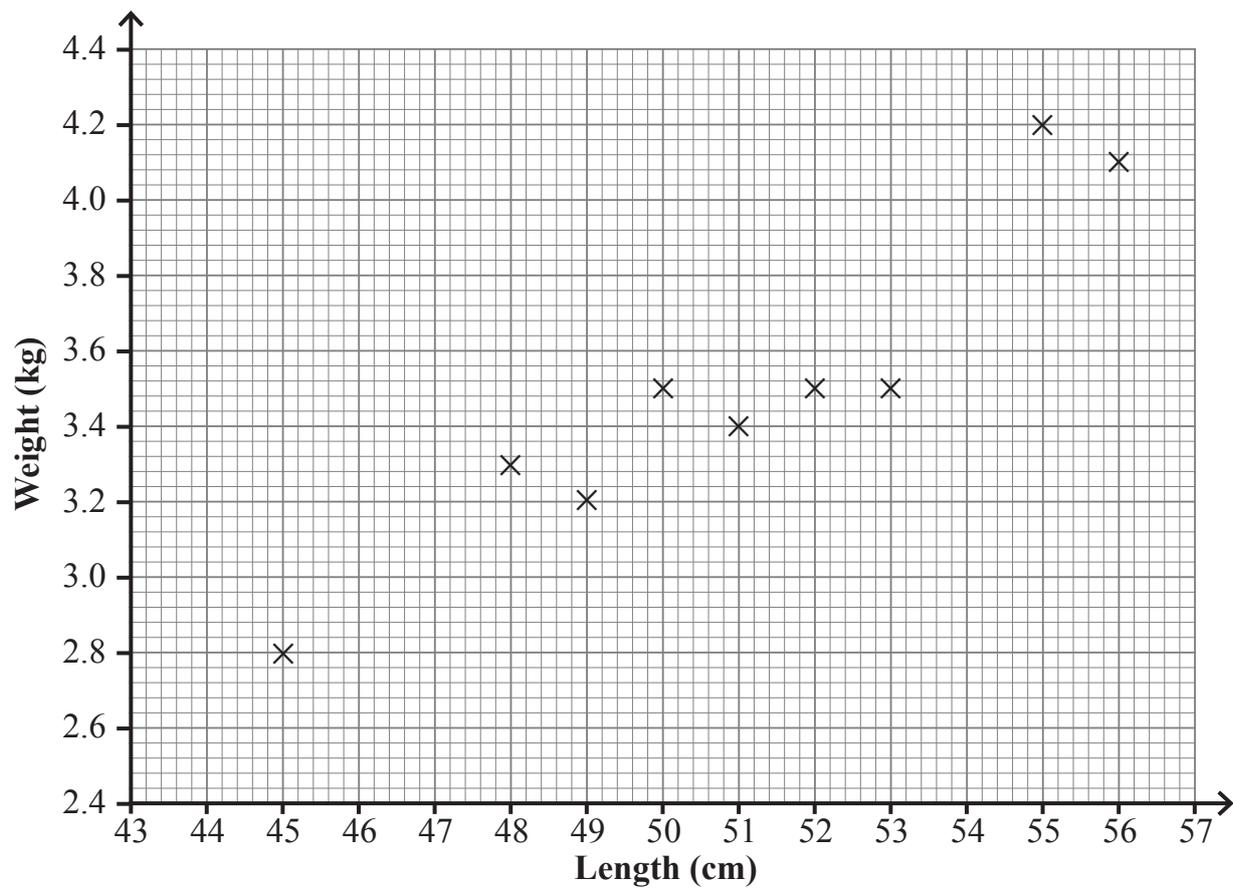
Answer _____ [1]

(iv) Calculate the mean length and the mean weight.

Answer mean length _____ cm, mean weight _____ kg [1]



The data from the table are plotted on the graph below.



(v) Draw your line of best fit on the graph.

[2]



(vi) Determine the equation of the line of best fit which you have drawn.

Answer _____ [3]



10 A class of 8 girls and 12 boys sat a Mathematics examination.

For the girls, the mean mark was 78 and the standard deviation was 6

For the boys, the mean mark was 82 and the standard deviation was 12

(i) Calculate the mean mark for the whole class.

Answer _____ [2]



(ii) Calculate the standard deviation for the whole class.

Answer _____ [4]

[Turn over



11 At Newtown College 180 pupils chose their subjects for GCSE.

86 pupils chose Geography

78 pupils chose History

48 pupils did not choose Geography or History

Using a Venn diagram, or otherwise, answer the following questions.

- (i) What is the probability that a pupil, selected at random, chose Geography **and** History?

Answer _____ [2]



(ii) Given that a pupil, selected at random, chose Geography, what is the probability that this pupil also chose History?

Answer _____ [2]

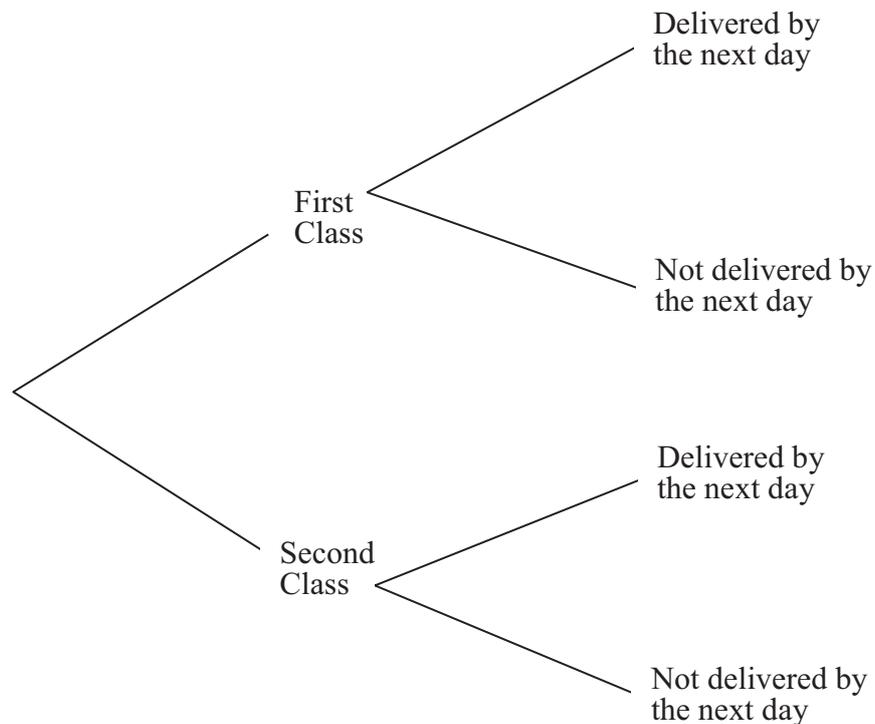
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- 12 Brian is a solicitor who posts many letters using First and Second Class post. He uses First Class post 60% of the time.

If he sends a letter by First Class post, the probability that it is delivered by the next day is 0.9

If he sends a letter by Second Class post, the probability that it is delivered by the next day is 0.4



By completing the tree diagram above, or otherwise, answer the following questions.

- (i) Calculate the probability that a letter, selected at random, is delivered by the next day.

Answer _____ [2]



(ii) If 20 letters are selected at random, how many of them would you expect to be delivered by the next day?

Answer _____ [2]

(iii) Given that a letter, selected at random, was delivered by the next day, what is the probability that it was sent by First Class post?

Answer _____ [3]

[Turn over



(iv) If 3 letters are selected at random, what is the probability that only one of them is delivered by the next day?

Answer _____ [2]





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- 13 (a) In a study of a club's sporting facilities, the data collected gave a mean of 33.4 and a standard deviation of 2.6

To compare the results with a study of similar data collected in a second club, each data item was adjusted by adding 5 and then multiplying by 1.5

Find the mean and standard deviation of the adjusted data.

Answer mean = _____, standard deviation = _____ [2]

- (b) For the last two seasons Alan recorded the number of goals scored in each match by his local football club. The results are given in the following incomplete tables.

Season 1	
Number of goals	Frequency Season 1
0	3
1	3
2	4
3	7
4	
More than 4	0

Season 2	
Number of goals	Frequency Season 2
0	3
1	0
2	6
3	3
4	
More than 4	0

For both seasons Alan forgot to fill in the number of matches in which the club scored 4 goals, but he remembered that this number was the **same** for both seasons.

He also knew that the mean number of goals scored for all matches in Season 1 was the **same** as that for Season 2.



In how many matches in each season had his local football club scored 4 goals?

You **must** show all your working and a **solution by trial and improvement will not be accepted.**

Answer _____ [7]

THIS IS THE END OF THE QUESTION PAPER



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For Examiner's use only	
Question Number	Marks
1	
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Total Marks	
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Examiner Number

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