Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2018

Further Mathematics

Unit 1 **(With calculator)**
Pure Mathematics

[GMF11]

GMF11

TUESDAY 12 JUNE, MORNING

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page, on blank pages or tracing paper.
Complete in black ink only. **Do not write with a gel pen.**
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all fourteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on pages 2 and 3.

11123

Formula Sheet

PURE MATHEMATICS

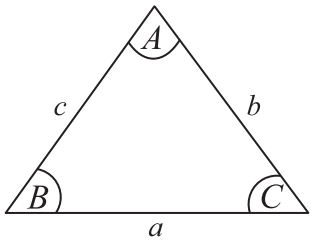
Quadratic equations: If $ax^2 + bx + c = 0$ $(a \neq 0)$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometry: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$



Differentiation: If $y = ax^n$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = nax^{n-1}$

Integration: $\displaystyle\int ax^n \,\mathrm{d}x = \frac{ax^{n+1}}{n+1} + c$ $(n \neq -1)$

Logarithms: If $a^x = n$ then $x = \log_a n$

$\log (ab) = \log a + \log b$

$\log \left(\dfrac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Matrices: If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\det \mathbf{A} = ad - bc$

and $\mathbf{A}^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $(ad - bc \neq 0)$

MECHANICS

Vectors: Magnitude of $x\mathbf{i} + y\mathbf{j}$ is given by $\sqrt{x^2 + y^2}$

Angle between $x\mathbf{i} + y\mathbf{j}$ and $\mathbf{i}$ is given by $\tan^{-1}\left(\frac{y}{x}\right)$

Uniform Acceleration: $v = u + at$ $\quad s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$ $\quad s = ut + \frac{1}{2}at^2$

where u is initial velocity, v is final velocity, a is acceleration, t is time, s is change in displacement

Newton's Second Law: $F = ma$

where F is resultant force, a is acceleration, m is mass

STATISTICS

Statistical measures: Mean $= \dfrac{\Sigma fx}{\Sigma f}$ $\quad$ Median $= L_1 + \dfrac{\left\{\frac{N}{2} - (\Sigma f)_1\right\}c}{f_{median}}$

where L_1 is lower class boundary of the median class
N is total frequency
$(\Sigma f)_1$ is the sum of the frequencies up to but not including the median class
f_{median} is the frequency of the median class
c is the width of the median class

Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$ where $\bar{x}$ is the mean

Probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Bivariate Analysis: Spearman's coefficient of rank correlation is given by

$$r = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

[Turn over

1 If $y = \frac{1}{4}x^5 - \frac{6}{x^2} + 10$

(i) find $\frac{dy}{dx}$

Answer _____ [3]

(ii) **Hence** find $\frac{d^2y}{dx^2}$

Answer _____ [2]

2 Find $\int\left(\frac{2}{7}x^2 - 4 + \frac{3}{4x^2}\right) \mathrm{d}x$

Answer _____ [4]

[Turn over

3 **(a)** Matrices **A**, **B** and **C** are defined by

$$\mathbf{A} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -2 & 3 \end{bmatrix} \quad \text{and} \quad \mathbf{C} = \begin{bmatrix} 2 & -5 \\ -4 & 3 \end{bmatrix}$$

State which two of the matrix products

AB, **AC**, **BA**, **BC**, **CA**, **CB**

cannot be formed.

Answer _____ and _____ [2]

(b) Matrices **P**, **Q** and **R** are defined by

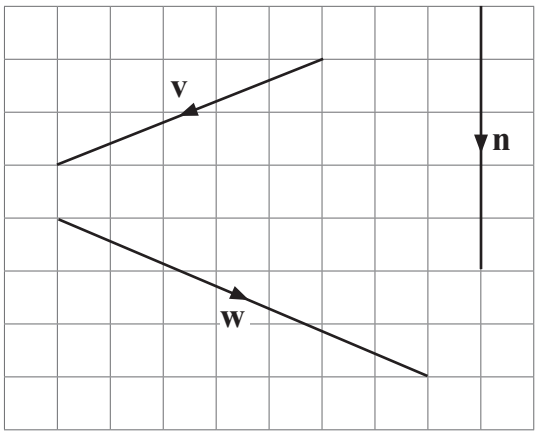
$$\mathbf{P} = \begin{bmatrix} 2 & 7 \\ -3 & 4 \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix} \text{ and } \mathbf{R} = \begin{bmatrix} 3 & -1 \\ 2 & -4 \end{bmatrix}$$

Calculate $\mathbf{P}(\mathbf{Q} - \mathbf{R})$

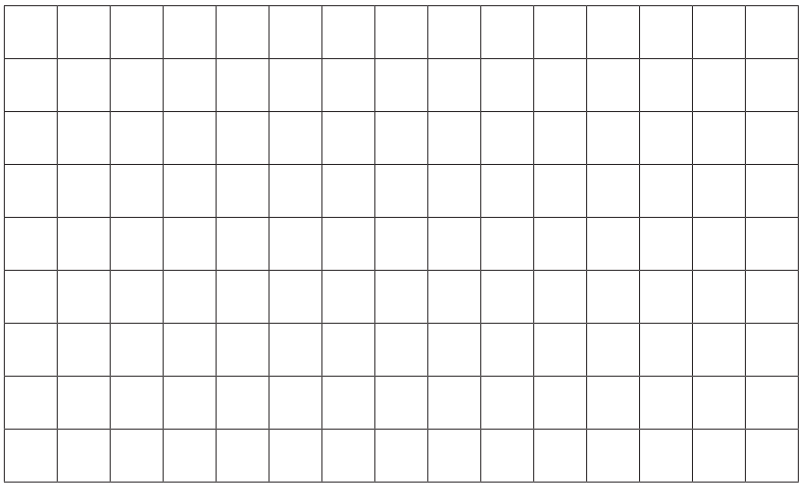
Answer _____ [4]

[Turn over

4 The vectors **v**, **w** and **n** are shown below.



(i) On the grid below draw a diagram to show the vector **v** − **w**.



[1]

(ii) Write down the exact value of the fraction k for which $\mathbf{v} + k\mathbf{w}$ is parallel to the vector $\mathbf{n}$.

Answer _____ [1]

[Turn over

5 Solve the equation

$$\cos\left(\frac{4}{5}x + 30°\right) = -0.5$$

for $0° \leq x \leq 360°$

Answer _____ [5]

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(Questions continue overleaf)

6 Matrices **Y** and **Z** are defined by

$$\mathbf{Y} = \begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix} \quad \text{and} \quad \mathbf{Z} = \begin{bmatrix} 5 & -2 \\ 1 & -5 \end{bmatrix}$$

(i) Find the matrix **A** if

$$\mathbf{A} + 2\mathbf{Y} = \mathbf{Z}$$

Answer _____ [2]

(ii) Find the matrix **B** if

$$\mathbf{YB} = \mathbf{Z}$$

Answer _____ [5]

[Turn over

7 **(a)** Solve the equation

$$3^{2x-1} = 7^{x+2}$$

Answer _____ [5]

(b) Express $\log \frac{pq^2}{r}$ in terms of $\log p$, $\log q$ and $\log r$.

Answer _____ [3]

8 Ali and Hassan are at positions A and H on the beach at Dubai. They are flying a drone over the beach.

Ali is on horizontal ground 500 m from the base of the vertical side BR of the Burj-al-Arab Hotel.

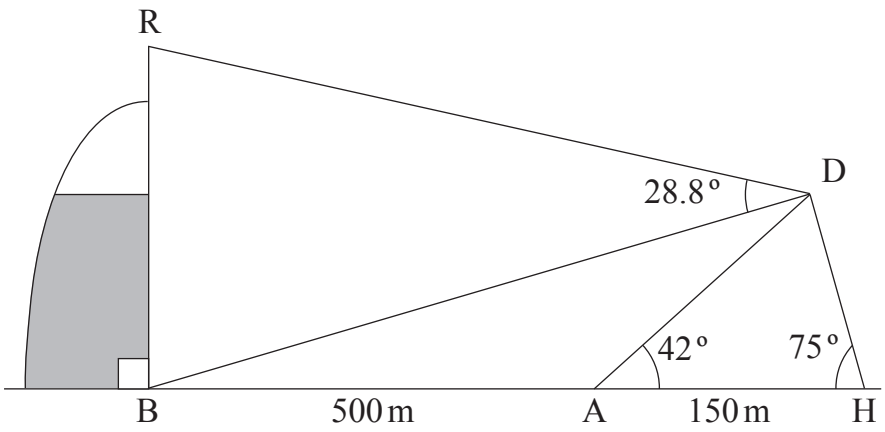
Hassan is 150 m from Ali.

BAH is a straight line.

At a given instant the drone is hovering at a point D.

Ali measures the angle of elevation of the drone as 42° and Hassan measures it as 75°, as shown in the diagram below.

All points are on the same vertical plane.



Calculate

(i) the size of the angle $A\hat{D}H$,

Answer _____ ° [1]

(ii) the distance AD,

Answer _____ m [2]

(iii) the size of the angle $B\hat{A}D$,

Answer _____ ° [1]

(iv) the distance BD,

Answer _____ m [2]

[Turn over

(v) the size of the angle $A\hat{B}D$.

Answer _____ ° [2]

From a photograph taken by the drone, the size of the angle $B\hat{D}R$ was calculated as 28.8°

(vi) Calculate the height BR of the Burj-al-Arab Hotel, giving your answer to the nearest metre.

Answer _____ m [3]

[Turn over

9 A curve is defined by the equation $y = x^3 - 6x^2 + 9x$

(i) Find the coordinates of the points where the curve meets the x-axis.

Answer _____ [3]

(ii) Find the coordinates of the turning points of the curve.

Answer _____ [5]

(iii) Using calculus, identify each turning point as either a maximum or a minimum point. You **must** show working to justify your answer.

Answer _____ [2]

[Turn over

10 Find the **coordinates** of the point at which the tangent to the curve

$$y = \frac{5}{2}x^2 + 6 - \frac{40}{x}$$

is horizontal.

Answer _____ [6]

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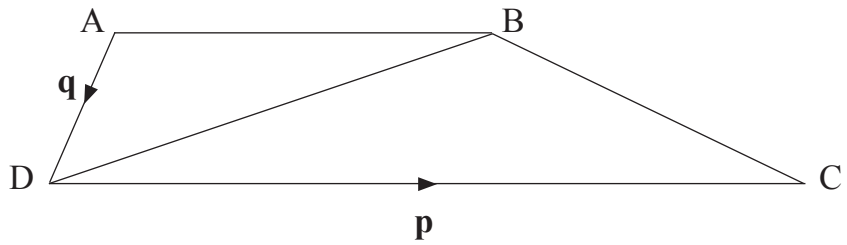
11 In the trapezium ABCD shown below, AB is parallel to DC and DC = 2 AB.



If $\overrightarrow{DC}$ = **p** and $\overrightarrow{AD}$ = **q**, express in terms of **p** and **q**

(i) $\overrightarrow{DB}$

Answer _____ [1]

(ii) $\overrightarrow{BC}$

Answer _____ [1]

E is the midpoint of DC.

The line BE is extended to a point F, where $\overrightarrow{BF} = \overrightarrow{2BE}$.

(iii) Express each of the following vectors in terms of **p** and **q**, simplifying your answers as far as possible.

(a) $\overrightarrow{DF}$

Answer _____ [2]

(b) $\overrightarrow{FC}$

Answer _____ [1]

(iv) Hence state what type of quadrilateral DFCB is, giving a reason for your answer.

Answer _____ [2]

[Turn over

12 **(a)** Simplify **fully** the algebraic expression

$$\frac{x^2 - 2x - 15}{2x + 8} \times \frac{x^2 - 16}{x^2 + 3x}$$

Answer _____ [4]

(b) (i) Express

$$\frac{x - 1}{x} + \frac{2x}{x + 1}$$

as a single fraction in its simplest form.

Answer _____ [3]

(ii) Hence, show that the equation

$$\frac{x - 1}{x} + \frac{2x}{x + 1} = 2$$

can be written as

$$x^2 - 2x - 1 = 0$$

[2]

(iii) Solve the equation $x^2 - 2x - 1 = 0$ using the method of **completing the square**.

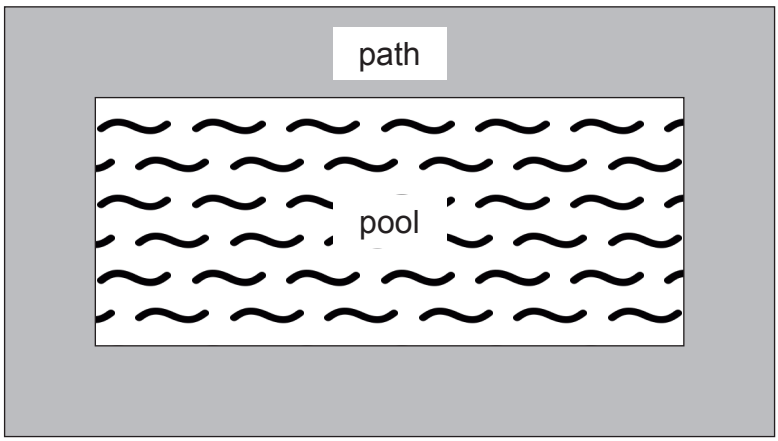
Give your answer in the form $a \pm \sqrt{b}$, where a and b are whole numbers.

Answer _____ [3]

[Turn over

13 A rectangular swimming pool is surrounded by a tiled path 1.5 m wide.

The pool is of length x m and width y m.



The area of the swimming pool is 54.6 m^2

The area of the tiled path is 56.1 m^2

Calculate the dimensions of the swimming pool.

(A solution by trial and improvement will not be accepted)

Answer _____ m, _____ m [8]

[Turn over

14 A supermarket chain wishes to build a store M somewhere on the straight road between two towns A and B.

If the store is built x km from A then, according to a survey, the number of people, N_A, from town A who will shop regularly each week at the store can be modelled by

$$N_A = 2000\left(1 - \frac{x^2}{100}\right)$$

Similarly, if the store is y km from B, then the number, N_B, from town B can be modelled by

$$N_B = 1000\left(1 - \frac{y^2}{200}\right)$$

(i) Show that the total number of regular weekly shoppers, N, from both towns is given by

$$N = 3000 - 20x^2 - 5y^2$$

[2]

The distance between A and B is 8 km.

(ii) Show that the total number of regular weekly shoppers from both towns is given by

$$N = 2680 + 80x - 25x^2$$

[3]

[Turn over

The supermarket chain wishes to position the store so that the total number of regular weekly shoppers is a maximum.

(iii) Find how far from town A the store should be built to maximise the total number of regular weekly shoppers, showing that this number is a maximum.

Answer _____ km [4]

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For Examiner's use only	
Question Number	**Marks**
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