



General Certificate of Secondary Education  
2017

Centre Number

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Candidate Number

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## Further Mathematics

Unit 2 (With calculator)  
Mechanics and Statistics

[GMF21]



MV18

THURSDAY 22 JUNE, MORNING

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### Time

2 hours, plus your additional time allowance.

### Instructions to Candidates

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.

**You must answer the questions in the spaces provided.**

Complete in black ink only.

All working should be clearly shown in the spaces provided.

Marks may be awarded for partially correct solutions.

Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.

Answer **all fifteen** questions.

## **Information for Candidates**

The total mark for this paper is 100.

Figures in brackets printed at the end of each question indicate the marks awarded to each question or part question.

You may use a calculator.

The Formula Sheet is on pages 4–7.

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**(Questions start on page 8)**

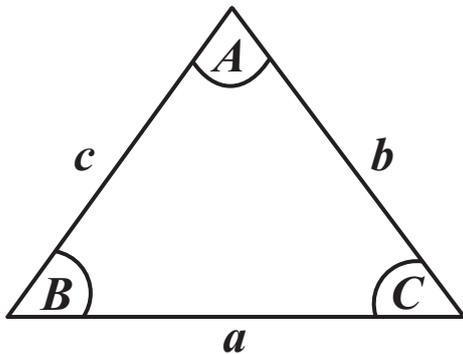
# Formula Sheet for use with GCSE Further Mathematics

## Pure Mathematics

Quadratic equations: If  $ax^2 + bx + c = 0$  ( $a \neq 0$ )

$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometry:



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of triangle} = \frac{1}{2}ab \sin C$$

Differentiation: If  $y = ax^n$  then  $\frac{dy}{dx} = nax^{n-1}$

Integration:  $\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$  ( $n \neq -1$ )

Logarithms: If  $a^x = n$  then  $x = \log_a n$

$$\log(ab) = \log a + \log b$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

$$\log a^n = n \log a$$

Matrices:

$$\text{If } \mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{then } \det \mathbf{A} = ad - bc$$

$$\text{and } \mathbf{A}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$(ad - bc \neq 0)$$

## Mechanics

Vectors: Magnitude of  $x\mathbf{i} + y\mathbf{j}$  is given by  $\sqrt{x^2 + y^2}$   
 Angle between  $x\mathbf{i} + y\mathbf{j}$  and  $\mathbf{i}$  is given by  $\tan^{-1}\left(\frac{y}{x}\right)$

Uniform Acceleration:  $v = u + at$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

$$s = ut + \frac{1}{2}at^2$$

where  $u$  is initial velocity  
 $v$  is final velocity  
 $t$  is time  
 $s$  is change in displacement  
 $a$  is acceleration

Newton's Second Law:  $F = ma$

where  $F$  is resultant force  
 $m$  is mass  
 $a$  is acceleration

## Statistics

Statistical measures: Mean =  $\frac{\sum fx}{\sum f}$

$$\text{Median} = L_1 + \frac{\left\{ \frac{N}{2} - (\sum f)_1 \right\} c}{f_{\text{median}}}$$

where  $L_1$  is lower class boundary of the median class  
 $N$  is total frequency  
 $(\sum f)_1$  is the sum of the frequencies up to but not including the median class  
 $f_{\text{median}}$  is the frequency of the median class  
 $c$  is the width of the median class

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - (\bar{x})^2}$$

where  $\bar{x}$  is the mean

Probability:  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Bivariate Analysis: Spearman's coefficient of rank correlation is given by

$$r = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

## Section A

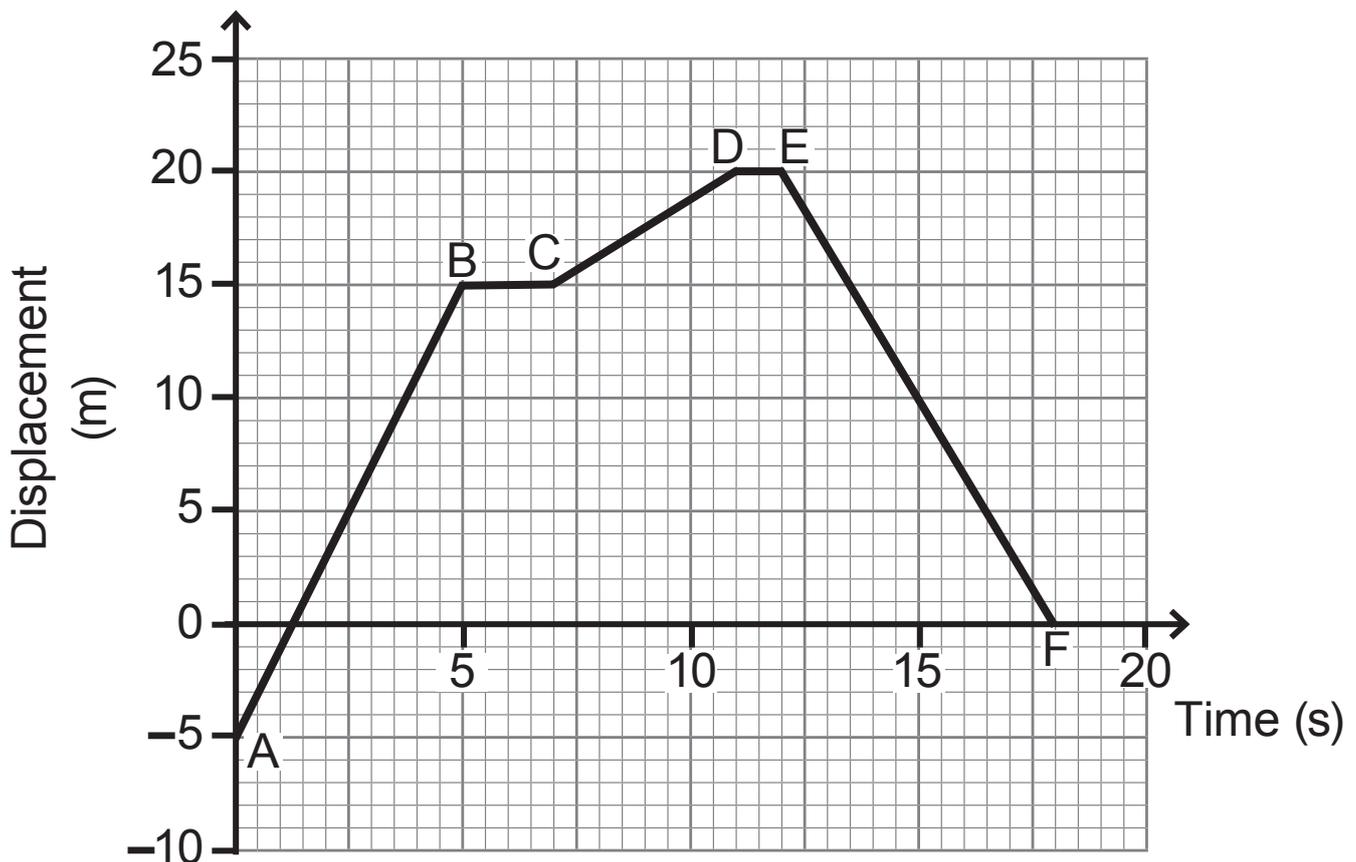
### Mechanics

You should spend approximately **one hour** on this section.

Take  $g = 10 \text{ m/s}^2$  when required.

1 A particle starts at a point A and travels along a straight line.

The displacement/time graph for its motion is shown below.



- (i) In which part of the motion did the particle have its maximum speed? Give a reason for your answer.  
[2 marks, 1 mark for the answer, 1 mark for the reason]

Answer \_\_\_\_\_

Reason \_\_\_\_\_

- (ii) Calculate the average speed of the particle for the motion from A to F. [2 marks]

Answer \_\_\_\_\_ m/s

- 2** (Throughout this question  $\mathbf{i}$  and  $\mathbf{j}$  denote unit vectors parallel to a set of standard  $x$ - $y$  axes.)

Two forces  $\mathbf{a}$  and  $\mathbf{b}$  are defined by

$$\mathbf{a} = (2\mathbf{i} - 3\mathbf{j}) \text{ N} \quad \text{and} \quad \mathbf{b} = (-7\mathbf{i} + 4\mathbf{j}) \text{ N}$$

- (i)** Find the resultant  $\mathbf{R}$  of the forces  $\mathbf{a}$  and  $\mathbf{b}$ . [2 marks]

Answer \_\_\_\_\_ N

- (ii)** Calculate the magnitude of the resultant  $\mathbf{R}$ . [2 marks]

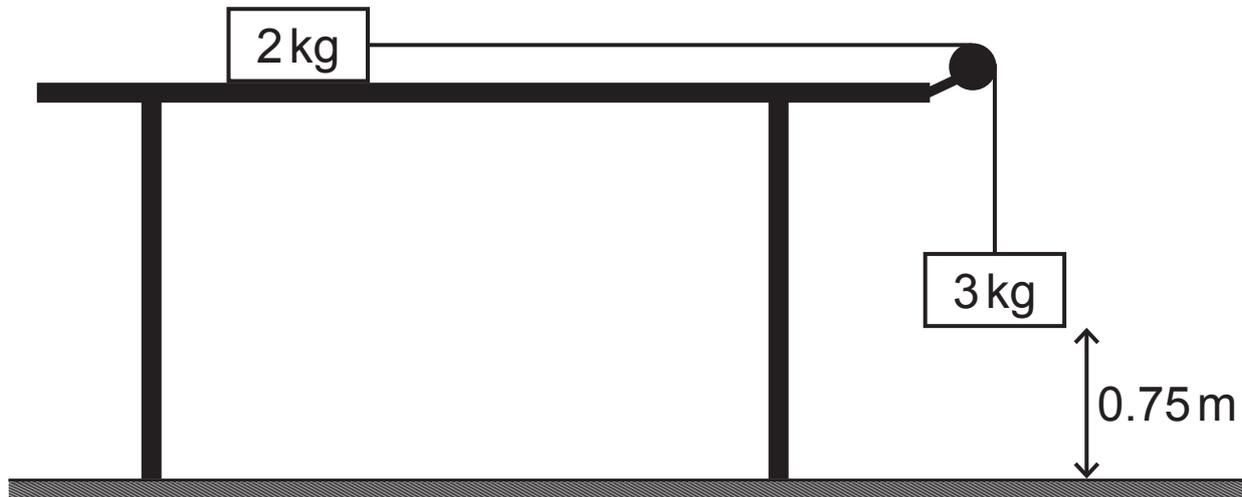
Answer \_\_\_\_\_ N

A force  $\mathbf{c}$  is defined by  $\mathbf{c} = (x\mathbf{i} + y\mathbf{j}) \text{ N}$ .

(iii) Given that  $2\mathbf{a} + 5\mathbf{c} = 4\mathbf{b}$ , find the values of  $x$  and  $y$ .  
[3 marks]

Answer  $x = \underline{\hspace{2cm}}$  ,  $y = \underline{\hspace{2cm}}$

- 3 A block of mass 2 kg is held at rest on a smooth table. It is connected to a block of mass 3 kg by a light inextensible string which passes over a smooth pulley attached to the end of the table, as shown in the diagram below. The 3 kg block hangs freely and is 0.75 m above the floor.



The blocks are released from rest.

- (i) Mark clearly on the diagram above all the forces acting on the blocks. [2 marks]

Calculate

- (ii) the acceleration of the blocks, [3 marks]

Answer \_\_\_\_\_ m/s<sup>2</sup>

(iii) the tension in the string, [1 mark]

Answer \_\_\_\_\_ N

(iv) the time taken for the 3 kg block to hit the floor, assuming that the 2 kg block does not reach the pulley first. [2 marks]

Answer \_\_\_\_\_ s

- 4 A uniform plank AB, of length 8 m and mass 3 kg, rests horizontally on two supports at C and D, where  $AC = 1$  m and  $BD = 2$  m.

A mass of 2 kg is placed at A and a mass of 5 kg is placed at B, as shown below.

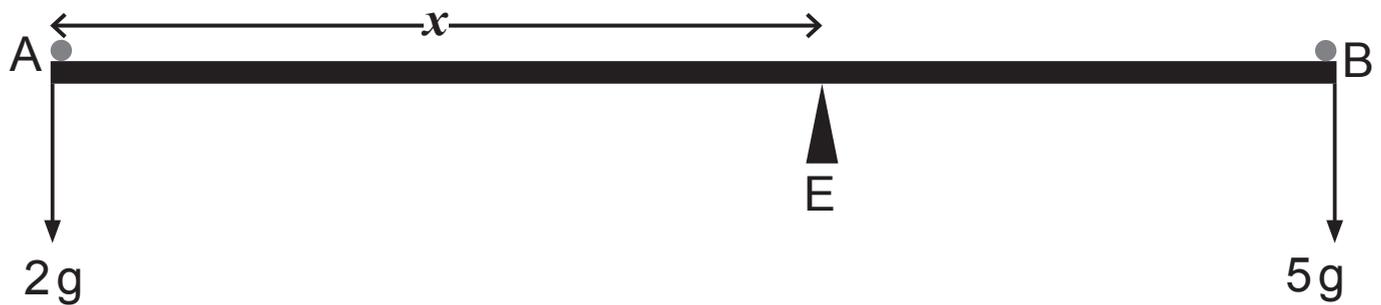


The plank remains horizontal and in equilibrium.

- (i) Calculate the reactions at C and D. [5 marks]

Answer      Reaction at C = \_\_\_\_\_ N,  
                   Reaction at D = \_\_\_\_\_ N

The supports at C and D are removed and replaced by a single support at the point E, which is  $x$  metres from A, as shown below.



The plank remains horizontal and in equilibrium.

(ii) Calculate the value of  $x$ . [3 marks]

Answer \_\_\_\_\_

- 5 A ball is thrown vertically upwards with a speed of 12 m/s from a point which is 1.2 m above the ground.

The ball moves freely under gravity, rising to a maximum height and then falling to the ground.

Calculate

- (i) the maximum height above **the ground** reached by the ball, [3 marks]

Answer \_\_\_\_\_ m

(ii) the speed of the ball as it hits the ground, [2 marks]

Answer \_\_\_\_\_ m/s

(iii) the time that elapses between the ball being thrown upwards and hitting the ground. [4 marks]

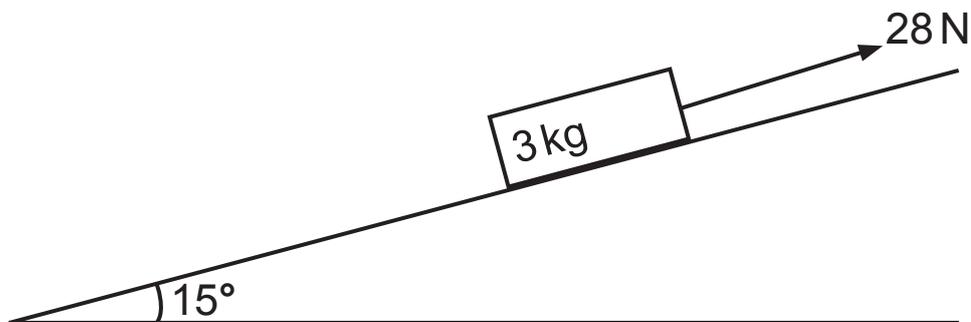
Answer \_\_\_\_\_ s

- 6 A parcel of mass 3 kg is initially at rest on a rough ramp which is inclined at  $15^\circ$  to the horizontal.

The parcel is pulled up the ramp by a force of 28 N acting parallel to the ramp.

The coefficient of friction between the parcel and the ramp is 0.2

- (i) Mark on the diagram below all the forces acting on the parcel. [2 marks]



(ii) Calculate the acceleration of the parcel up the ramp.  
[5 marks]

Answer \_\_\_\_\_ m/s<sup>2</sup>

After 5 seconds the force of 28 N ceases to act.

Calculate

(iii) the speed of the parcel when the force of 28 N ceases,  
[2 marks]

Answer \_\_\_\_\_ m/s

(iv) the additional time it takes for the parcel to stop moving  
up the ramp. [5 marks]

Answer \_\_\_\_\_ s

## Section B

### Statistics

You should spend approximately **one hour** on this section.

- 7 Simon and Louis are two judges on the singing competition, the Y Factor.

They each gave points to 5 singers and the rank orders of these points were noted.

The table below shows the rank order of the points given by Simon.

Singer	Monica	Colin	Michaela	Sam	Maurice
Rank Order (Simon)	1	2	3	4	5
Rank Order (Louis)					

The Spearman's coefficient of rank correlation was  $r = -1$

- (i) State briefly what  $r = -1$  implies. [1 mark]

Answer \_\_\_\_\_

- (ii) Complete the table above to show the rank order of the points given by Louis. [1 mark]

- 8 The table below shows the distances travelled, to the nearest 5 km, by 30 people going to work.

Distance (km)	Number of people
5 – 15	12
20 – 40	7
45 – 80	4
85 – 100	2
105 – 110	5

- (i) Calculate an estimate for the **mean** distance travelled.  
[3 marks]

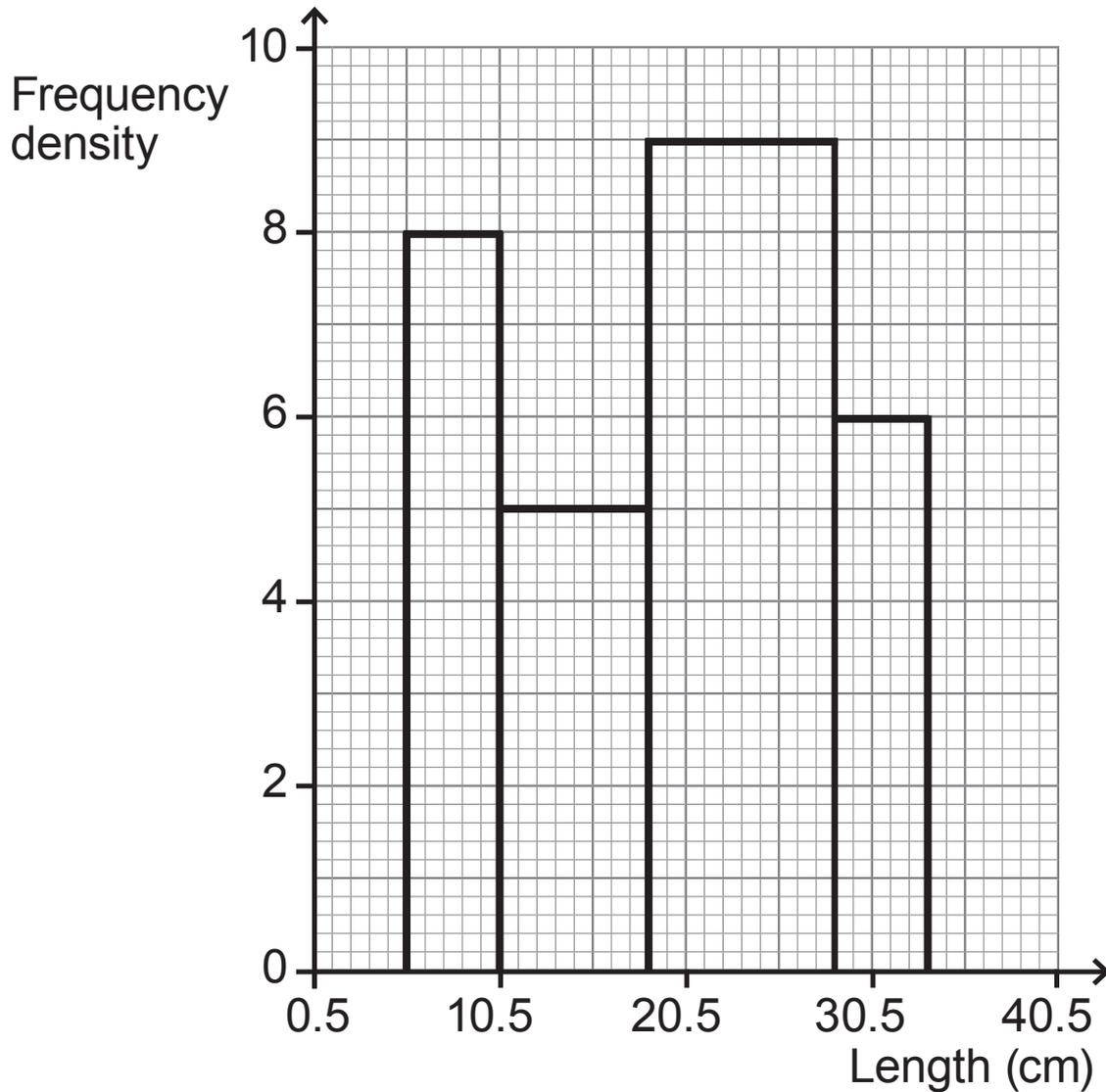
Answer \_\_\_\_\_ km

(ii) Calculate an estimate for the **median** distance travelled.  
[4 marks]

Answer \_\_\_\_\_ km

- 9 The lengths of 200 tiles, each measured to the nearest centimetre, were recorded.

The histogram below summarises the results.



- (i) Complete the following table for these lengths.  
[4 marks]

Length (cm)	Frequency
6 – 10	

A tile is chosen at random.

(ii) Calculate the probability that it is at least 28.5 cm long, given that it is at least 10.5 cm long. [2 marks]

Answer \_\_\_\_\_

**10** A company employs 40 graduate and 60 non-graduate workers.

The mean and standard deviation of the monthly salaries of **all** the workers are £1852 and £96 respectively.

The mean and standard deviation of the monthly salaries of the **graduate** workers are £1960 and £32 respectively.

For the **non-graduate** workers, calculate

**(i)** the mean of their monthly salaries, [3 marks]

Answer £ \_\_\_\_\_

(ii) the standard deviation of their monthly salaries.  
[4 marks]

Answer £ \_\_\_\_\_

- 11 Quinn recorded in the table below the number of points scored by eight football teams in their home and away matches.

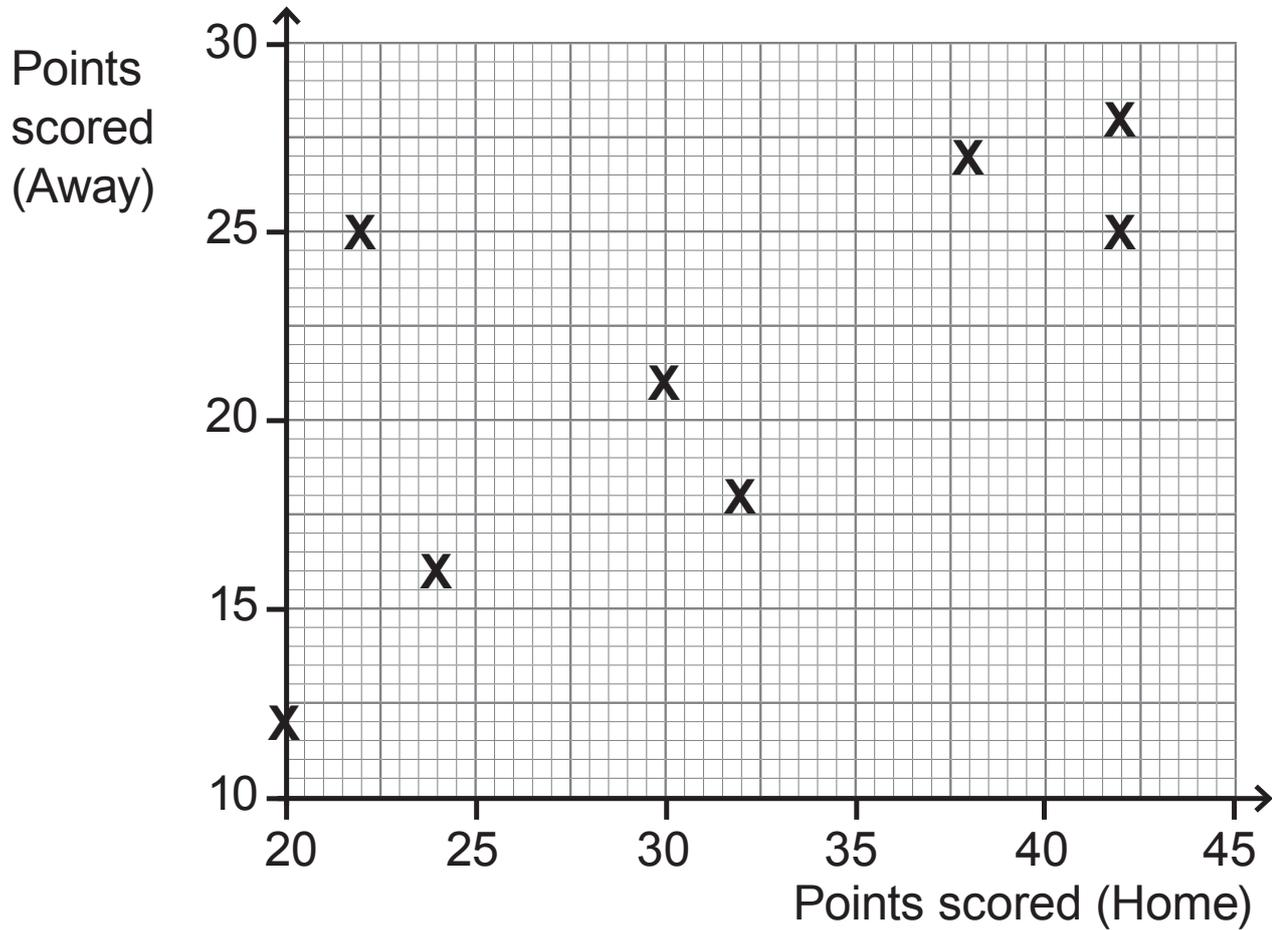
<b>Points scored (Home)</b>	38	42	32	24	42	30	22	20
<b>Points scored (Away)</b>	27	25	18	16	28	21	25	12

- (i) Calculate the mean number of points scored in both the home and the away matches. [1 mark]

Answer Home \_\_\_\_\_ , Away \_\_\_\_\_

The data from the table on page 28 are plotted on the graph below.

(ii) Draw your line of best fit on the graph. [2 marks]



(iii) Determine the equation of the line of best fit which you have drawn. [3 marks]

Answer \_\_\_\_\_

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**(Questions continue overleaf)**

**12** There are 1215 pupils in a school, of which 540 are girls.

The probability that a **pupil** chosen at random wears glasses is  $\frac{1}{3}$

The number of **girls** who wear glasses is 189

**(i)** Calculate the probability that a **boy** chosen at random wears glasses. [3 marks]

Answer \_\_\_\_\_

A pupil is chosen at random from the school.

(ii) Calculate the probability that the pupil is a boy, given that the pupil chosen does not wear glasses.

[3 marks]

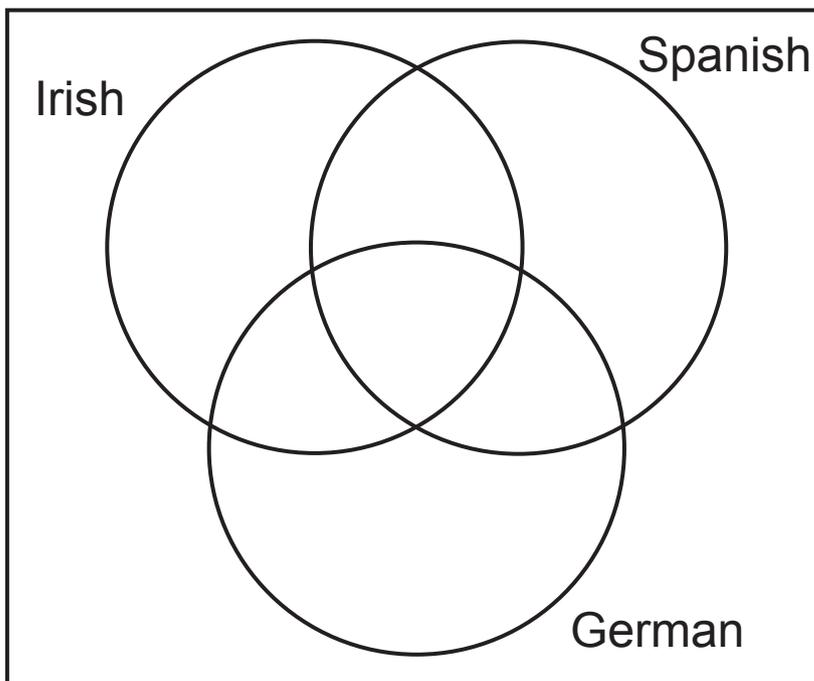
Answer \_\_\_\_\_

**13** There are 125 tourists in a hotel.

Of the tourists,

- $x$  can speak Irish, Spanish and German
- 12 can speak Irish and Spanish,
- 7 can speak Irish and German,
- 10 can speak Spanish and German,
- 14 can speak Irish, but not Spanish or German,
- 11 can speak Spanish, but not Irish or German,
- 8 can speak German, but not Irish or Spanish,
- 73 cannot speak Irish or Spanish or German.

**(i)** Illustrate this information on the Venn diagram below.  
[3 marks]



**(ii) Hence** find the value of  $x$ . [2 marks]

Answer \_\_\_\_\_

Sarah was chosen at random from the tourists.

**(iii)** Find the probability that she speaks Irish. [2 marks]

Answer \_\_\_\_\_

**14** Cadence has two bags, A and B, containing equal sized beads.

In bag A there are 20 beads, of which 6 are blue, 12 are yellow and 2 are white.

In bag B there are 24 beads, of which 5 are blue, 15 are yellow and 4 are white.

Cadence takes a bead at random from bag A and **places it in bag B**. She then takes a bead at random from bag B and places it in bag A.

Calculate the probability that

(i) a blue bead was chosen each time, [2 marks]

Answer \_\_\_\_\_

(ii) each bag has the same number of beads of each colour as it had at the start. [2 marks]

Answer \_\_\_\_\_

**15** Mairead has a box containing 2 gold and  $x$  diamond earrings. The earrings are all the same size.

She takes 2 earrings, without replacement, from the box.

The probability that the first earring is gold is  $\frac{2}{2+x}$

- (i) Given that the first earring is gold, write down an expression for the probability that the second earring is also gold. [1 mark]

Answer \_\_\_\_\_

The probability that both earrings are gold is  $\frac{1}{15}$

- (ii) Derive a quadratic equation for  $x$  and **hence** determine the number of diamond earrings in the box at the start.  
[4 marks]

Answer \_\_\_\_\_

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**THIS IS THE END OF THE QUESTION PAPER**

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**DO NOT WRITE ON THIS PAGE**

<b>For Examiner's use only</b>	
<b>Question Number</b>	<b>Marks</b>
1	
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<b>Total Marks</b>	

Examiner Number

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