Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2017

Further Mathematics

Unit 2 **(With calculator)**
Mechanics and Statistics

[GMF21]

GMF21

THURSDAY 22 JUNE, MORNING

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page, on blank pages or tracing paper.
Complete in black ink only. **Do not write with a gel pen.**
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all fifteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on pages 2 and 3.

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Formula Sheet

PURE MATHEMATICS

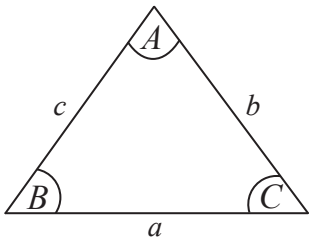
Quadratic equations: If $ax^2 + bx + c = 0$ $(a \neq 0)$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometry: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$



Differentiation: If $y = ax^n$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = nax^{n-1}$

Integration: $\int ax^n \, \mathrm{d}x = \dfrac{ax^{n+1}}{n+1} + c$ $(n \neq -1)$

Logarithms: If $a^x = n$ then $x = \log_a n$

$\log (ab) = \log a + \log b$

$\log \left(\dfrac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Matrices: If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\det \mathbf{A} = ad - bc$

and $\mathbf{A}^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $(ad - bc \neq 0)$

MECHANICS

Vectors: Magnitude of $x\mathbf{i} + y\mathbf{j}$ is given by $\sqrt{x^2 + y^2}$

Angle between $x\mathbf{i} + y\mathbf{j}$ and $\mathbf{i}$ is given by $\tan^{-1}\left(\frac{y}{x}\right)$

Uniform Acceleration: $v = u + at$ $\quad s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$ $\quad s = ut + \frac{1}{2}at^2$

where u is initial velocity, v is final velocity, a is acceleration, t is time, s is change in displacement

Newton's Second Law: $F = ma$

where F is resultant force, a is acceleration, m is mass

STATISTICS

Statistical measures: $\text{Mean} = \frac{\Sigma fx}{\Sigma f}$ $\quad \text{Median} = L_1 + \frac{\left\{\frac{N}{2} - (\Sigma f)_1\right\}c}{f_{median}}$

where L_1 is lower class boundary of the median class
N is total frequency
$(\Sigma f)_1$ is the sum of the frequencies up to but not including the median class
f_{median} is the frequency of the median class
c is the width of the median class

$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$ where $\bar{x}$ is the mean

Probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

Bivariate Analysis: Spearman's coefficient of rank correlation is given by

$$r = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

[Turn over

Section A

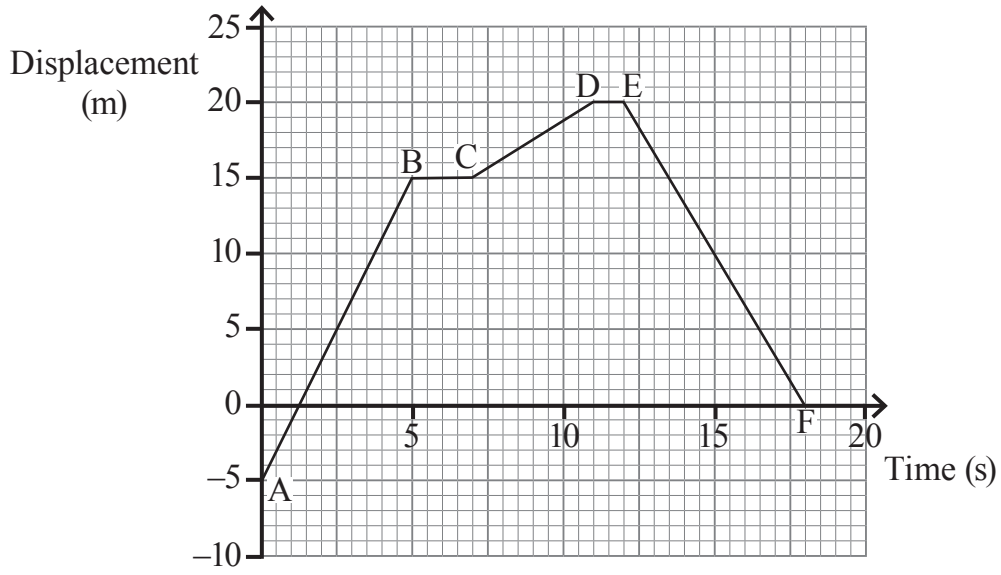
Mechanics

You should spend approximately **one hour** on this section.

Take $\mathbf{g = 10\,m/s^2}$ when required.

1 A particle starts at a point A and travels along a straight line.

The displacement/time graph for its motion is shown below.



(i) In which part of the motion did the particle have its maximum speed? Give a reason for your answer.

Answer _____ [1]

Reason _____ [1]

(ii) Calculate the average speed of the particle for the motion from A to F.

Answer _____ m/s [2]

[Turn over

2 (Throughout this question **i** and **j** denote unit vectors parallel to a set of standard x-y axes.)

Two forces **a** and **b** are defined by

$$\mathbf{a} = (2\mathbf{i} - 3\mathbf{j}) \text{ N} \quad \text{and} \quad \mathbf{b} = (-7\mathbf{i} + 4\mathbf{j}) \text{ N}$$

(i) Find the resultant **R** of the forces **a** and **b**.

Answer _____ N [2]

(ii) Calculate the magnitude of the resultant **R**.

Answer _____ N [2]

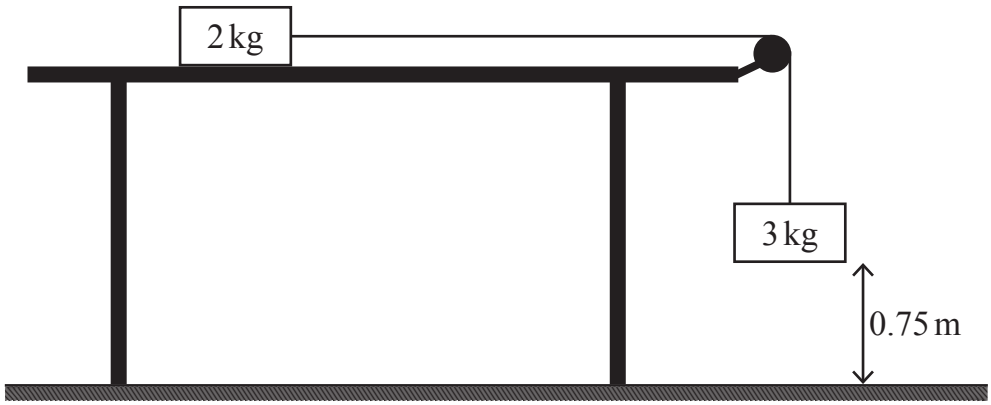
A force **c** is defined by $\mathbf{c} = (x\mathbf{i} + y\mathbf{j})$ N.

(iii) Given that $2\mathbf{a} + 5\mathbf{c} = 4\mathbf{b}$, find the values of x and y.

Answer $x =$ _____ , $y =$ _____ [3]

[Turn over

3 A block of mass 2 kg is held at rest on a smooth table. It is connected to a block of mass 3 kg by a light inextensible string which passes over a smooth pulley attached to the end of the table, as shown in the diagram below. The 3 kg block hangs freely and is 0.75 m above the floor.



The blocks are released from rest.

(i) Mark clearly on the diagram above all the forces acting on the blocks. [2]

Calculate

(ii) the acceleration of the blocks,

Answer _____ m/s^2 [3]

(iii) the tension in the string,

Answer _____ N [1]

(iv) the time taken for the 3 kg block to hit the floor, assuming that the 2 kg block does not reach the pulley first.

Answer _____ s [2]

[Turn over

4 A uniform plank AB, of length 8 m and mass 3 kg, rests horizontally on two supports at C and D, where AC = 1 m and BD = 2 m.

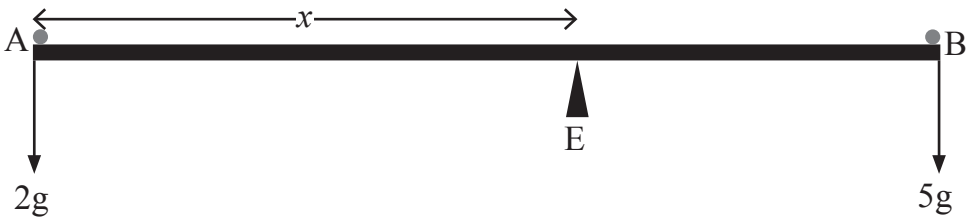
A mass of 2 kg is placed at A and a mass of 5 kg is placed at B, as shown below.

The plank remains horizontal and in equilibrium.

(i) Calculate the reactions at C and D.

Answer Reaction at C = _____ N, Reaction at D = _____ N [5]

The supports at C and D are removed and replaced by a single support at the point E, which is x metres from A, as shown below.



The plank remains horizontal and in equilibrium.

(ii) Calculate the value of x.

Answer _____ [3]

[Turn over

5 A ball is thrown vertically upwards with a speed of 12 m/s from a point which is 1.2 m above the ground.

The ball moves freely under gravity, rising to a maximum height and then falling to the ground.

Calculate

(i) the maximum height above **the ground** reached by the ball,

Answer _____ m [3]

(ii) the speed of the ball as it hits the ground,

Answer _____ m/s [2]

(iii) the time that elapses between the ball being thrown upwards and hitting the ground.

Answer _____ s [4]

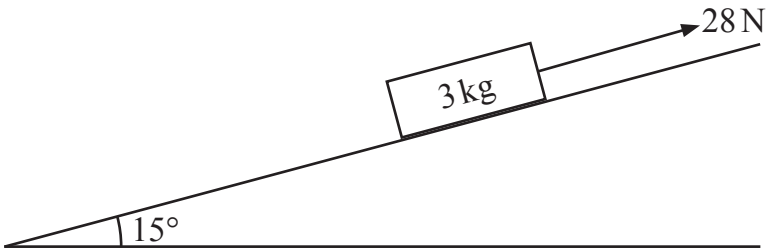
[Turn over

6 A parcel of mass 3 kg is initially at rest on a rough ramp which is inclined at 15° to the horizontal.

The parcel is pulled up the ramp by a force of 28 N acting parallel to the ramp.

The coefficient of friction between the parcel and the ramp is 0.2

(i) Mark on the diagram below all the forces acting on the parcel. [2]



(ii) Calculate the acceleration of the parcel up the ramp.

Answer _____ m/s^2 [5]

[Turn over

After 5 seconds the force of 28 N ceases to act.

Calculate

(iii) the speed of the parcel when the force of 28 N ceases,

Answer _____ m/s [2]

(iv) the additional time it takes for the parcel to stop moving up the ramp.

Answer _____ s [5]

Section B

Statistics

You should spend approximately **one hour** on this section.

7 Simon and Louis are two judges on the singing competition, the Y Factor.

They each gave points to 5 singers and the rank orders of these points were noted.

The table below shows the rank order of the points given by Simon.

Singer	Monica	Colin	Michaela	Sam	Maurice
Rank Order (Simon)	1	2	3	4	5
Rank Order (Louis)					

The Spearman's coefficient of rank correlation was $r = -1$

(i) State briefly what $r = -1$ implies.

Answer _____ [1]

(ii) Complete the table above to show the rank order of the points given by Louis. [1]

[Turn over

8 The table below shows the distances travelled, **to the nearest 5 km**, by 30 people going to work.

Distance (km)	Number of people
5 – 15	12
20 – 40	7
45 – 80	4
85 – 100	2
105 – 110	5

(i) Calculate an estimate for the **mean** distance travelled.

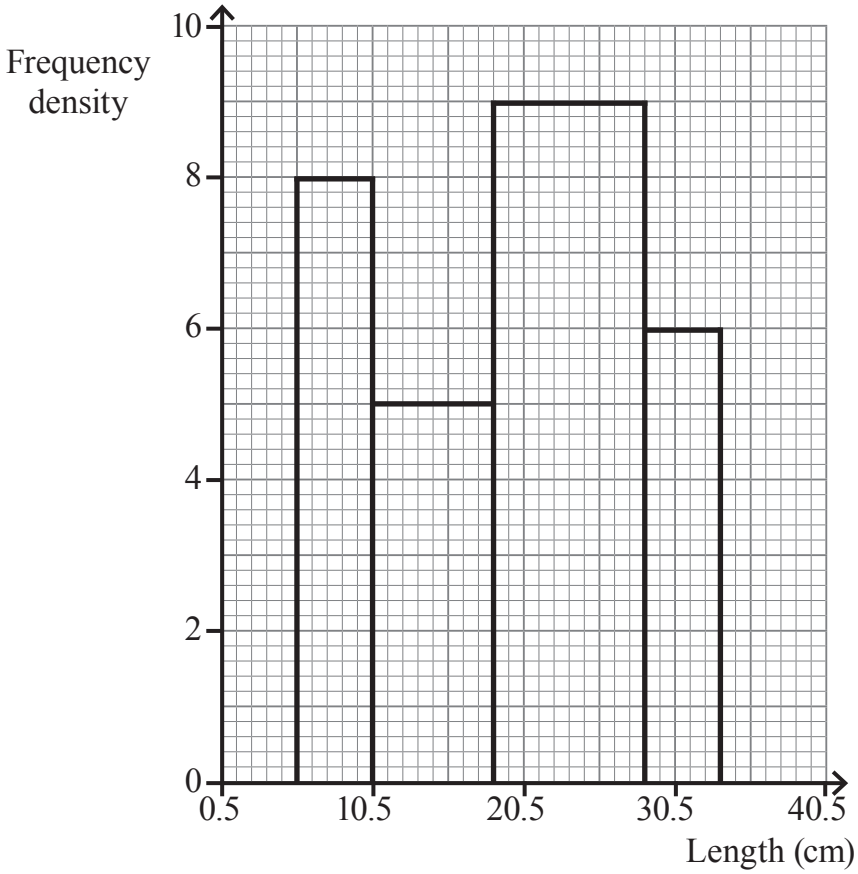
Answer _____ km [3]

(ii) Calculate an estimate for the **median** distance travelled.

Answer _____ km [4]

9 The lengths of 200 tiles, each measured to the nearest centimetre, were recorded.

The histogram below summarises the results.



(i) Complete the following table for these lengths.

Length (cm)	Frequency
6 – 10	

[4]

A tile is chosen at random.

(ii) Calculate the probability that it is at least 28.5 cm long, given that it is at least 10.5 cm long.

Answer _____ [2]

10 A company employs 40 graduate and 60 non-graduate workers.

The mean and standard deviation of the monthly salaries of **all** the workers are £1852 and £96 respectively.

The mean and standard deviation of the monthly salaries of the **graduate** workers are £1960 and £32 respectively.

For the **non-graduate** workers, calculate

(i) the mean of their monthly salaries,

Answer £ _____ [3]

(ii) the standard deviation of their monthly salaries.

Answer £ _____ [4]

[Turn over

11 Quinn recorded in the table below the number of points scored by eight football teams in their home and away matches.

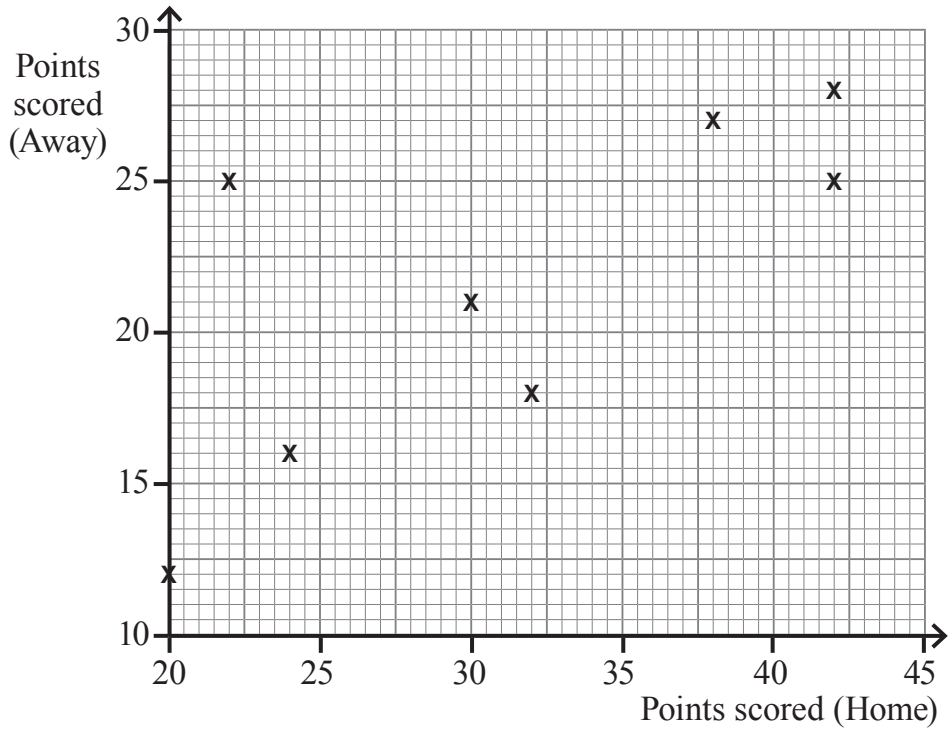
Points scored (Home)	38	42	32	24	42	30	22	20
Points scored (Away)	27	25	18	16	28	21	25	12

(i) Calculate the mean number of points scored in both the home and the away matches.

Answer Home _____ , Away _____ [1]

The data from the above table are plotted on the graph below.

(ii) Draw your line of best fit on the graph.



[2]

(iii) Determine the equation of the line of best fit which you have drawn.

Answer _____ [3]

12 There are 1215 pupils in a school, of which 540 are girls.

The probability that a **pupil** chosen at random wears glasses is $\frac{1}{3}$

The number of **girls** who wear glasses is 189

(i) Calculate the probability that a **boy** chosen at random wears glasses.

Answer _____ [3]

A pupil is chosen at random from the school.

(ii) Calculate the probability that the pupil is a boy, given that the pupil chosen does not wear glasses.

Answer _____ [3]

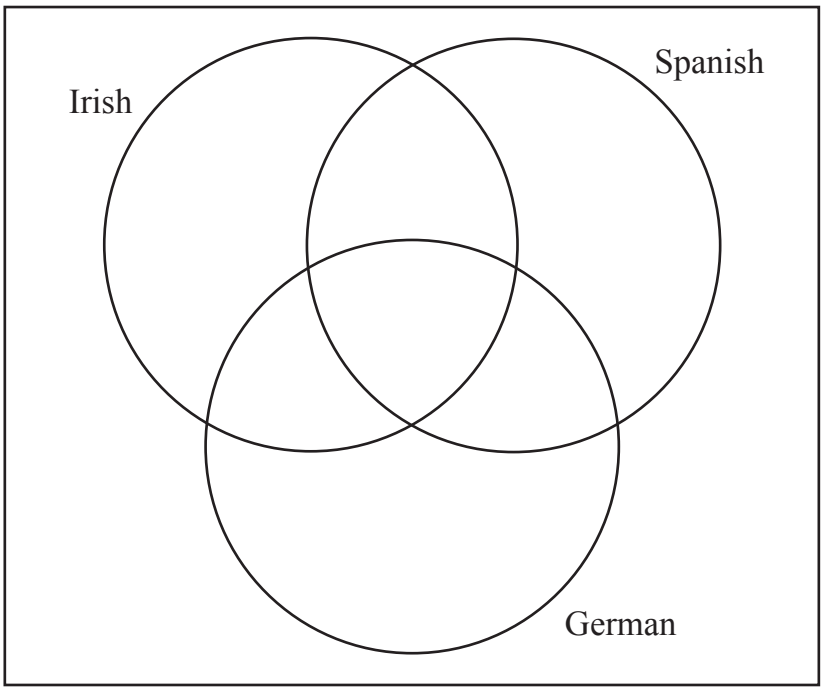
[Turn over

13 There are 125 tourists in a hotel.

Of the tourists,

x	can speak Irish, Spanish and German
12	can speak Irish and Spanish,
7	can speak Irish and German,
10	can speak Spanish and German,
14	can speak Irish, but not Spanish or German,
11	can speak Spanish, but not Irish or German,
8	can speak German, but not Irish or Spanish,
73	cannot speak Irish or Spanish or German.

(i) Illustrate this information on the Venn diagram below.



[3]

(ii) Hence find the value of x.

Answer _____ [2]

Sarah was chosen at random from the tourists.

(iii) Find the probability that she speaks Irish.

Answer _____ [2]

[Turn over

14 Cadence has two bags, A and B, containing equal sized beads.

In bag A there are 20 beads, of which 6 are blue, 12 are yellow and 2 are white.

In bag B there are 24 beads, of which 5 are blue, 15 are yellow and 4 are white.

Cadence takes a bead at random from bag A and **places it in bag B**. She then takes a bead at random from bag B and places it in bag A.

Calculate the probability that

(i) a blue bead was chosen each time,

Answer _____ [2]

(ii) each bag has the same number of beads of each colour as it had at the start.

Answer _____ [2]

[Turn over

15 Mairead has a box containing 2 gold and x diamond earrings. The earrings are all the same size.

She takes 2 earrings, without replacement, from the box.

The probability that the first earring is gold is $\frac{2}{2+x}$

(i) Given that the first earring is gold, write down an expression for the probability that the second earring is also gold.

Answer _____ [1]

The probability that both earrings are gold is $\frac{1}{15}$

(ii) Derive a quadratic equation for x and **hence** determine the number of diamond earrings in the box at the start.

Answer _____ [4]

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For Examiner's use only	
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