Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 2

Thursday 12 November 2015 (afternoon)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

8815 – 7402

Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 14]

A shop sells strawberry, coffee and orange flavoured chocolates. 50 customers were asked which flavoured chocolates they had bought.

The results are as follows:

7 bought only strawberry chocolates
6 bought only coffee chocolates
10 bought only orange chocolates
3 bought coffee and strawberry, **but not** orange
5 bought strawberry and orange, **but not** coffee
4 bought orange and coffee, **but not** strawberry
x bought strawberry, coffee and orange.

(a) Represent this information on a Venn diagram. [4]

(b) Find the value of x, given that 13 of these 50 customers did not buy any chocolates. [2]

(c) (i) Find the probability that a customer, chosen at random from these 50 customers, bought strawberry flavoured chocolates.

(ii) Find the probability that a customer, chosen at random from these 50 customers, bought orange flavoured chocolates.

(iii) Determine whether the events in parts (c)(i) and (c)(ii) are independent. Give a reason for your answer. [6]

(d) A customer is chosen at random from these 50 customers. Given that the customer bought coffee flavoured chocolates, find the probability that they also bought strawberry flavoured chocolates. [2]

2. [Maximum mark: 19]

The following table shows the internal assessment marks and examination marks for six students.

Internal assessment marks (x)	4	10	12	16	18	20
Examination marks (y)	35	45	52	55	65	70

(a) **On graph paper**, draw a scatter diagram for the above data. Use a scale of $1\,\text{cm}$ to represent 2 marks on the x-axis and $1\,\text{cm}$ to represent 10 marks on the y-axis. [3]

(b) (i) Write down the Pearson's product–moment correlation coefficient, r, for the above data.

(ii) Describe the correlation between internal assessment marks and examination marks. [4]

(c) Find the

(i) mean internal assessment mark, $\overline{x}$;

(ii) mean examination mark, $\overline{y}$. [2]

(d) Plot and label the point $\text{M}(\overline{x}, \overline{y})$ on your scatter diagram from part (a). [2]

(e) Write down the equation of the regression line for y on x. [2]

(f) Use your **equation** from part (e) to estimate the examination mark of a student who scored 8 marks on their internal assessment. [2]

(g) Draw the regression line y on x on your scatter diagram from part (a). [2]

A **new student** scores 30 marks on her internal assessment and uses the above data to estimate that she will score 89 marks on the examination.

(h) State whether this estimate is reliable and give a reason for your answer. [2]

3. [Maximum mark: 13]

The weights of sheep on a farm are normally distributed with a mean of $110\,\text{kg}$ and a standard deviation of $8\,\text{kg}$.

(a) Sketch a diagram of the distribution of the weights of these sheep. On your diagram, label the mean and label one standard deviation above and below the mean. [2]

(b) (i) A sheep has a weight of $94\,\text{kg}$. Write down the number of standard deviations that this weight is below the mean.

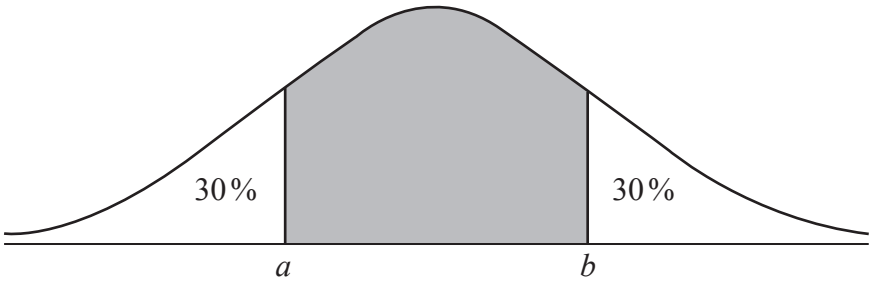
(ii) Find the probability that a sheep, chosen at random, weighs more than $94\,\text{kg}$. [3]

(c) (i) Find the probability that a sheep, chosen at random, weighs between $88\,\text{kg}$ and $116\,\text{kg}$.

(ii) The farmer weighs 160 sheep. Find the number of sheep that he would expect to weigh between $88\,\text{kg}$ and $116\,\text{kg}$. [4]

(d) Given that $75\,\%$ of the sheep weigh **less than** $w\,\text{kg}$, find the value of w. [2]

A sheep is chosen at random. Its weight is within the central shaded region of the following diagram.



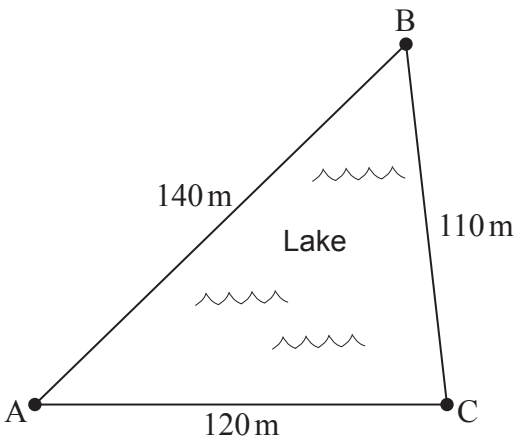
Weight of sheep

(e) Find the value of a and of b. [2]

4. [Maximum mark: 15]

A lake is in the shape of a triangle, ABC, where AB, BC and CA are paths around the lake. The paths have the following lengths: $AB = 140\,\text{m}$, $BC = 110\,\text{m}$ and $CA = 120\,\text{m}$.

diagram not to scale



(a) Find the size of angle $B\hat{A}C$. [3]

(b) Find the surface area of the lake. [3]

A farmhouse is located at point D, away from the lake, such that angle $D\hat{B}C$ is 80° and angle $B\hat{C}D$ is 40°. The farmer has built straight paths from her farmhouse, D, to points B and C.

(c) Show that angle $B\hat{D}C = 60^\circ$. [1]

(d) Find the distance between C and D. [3]

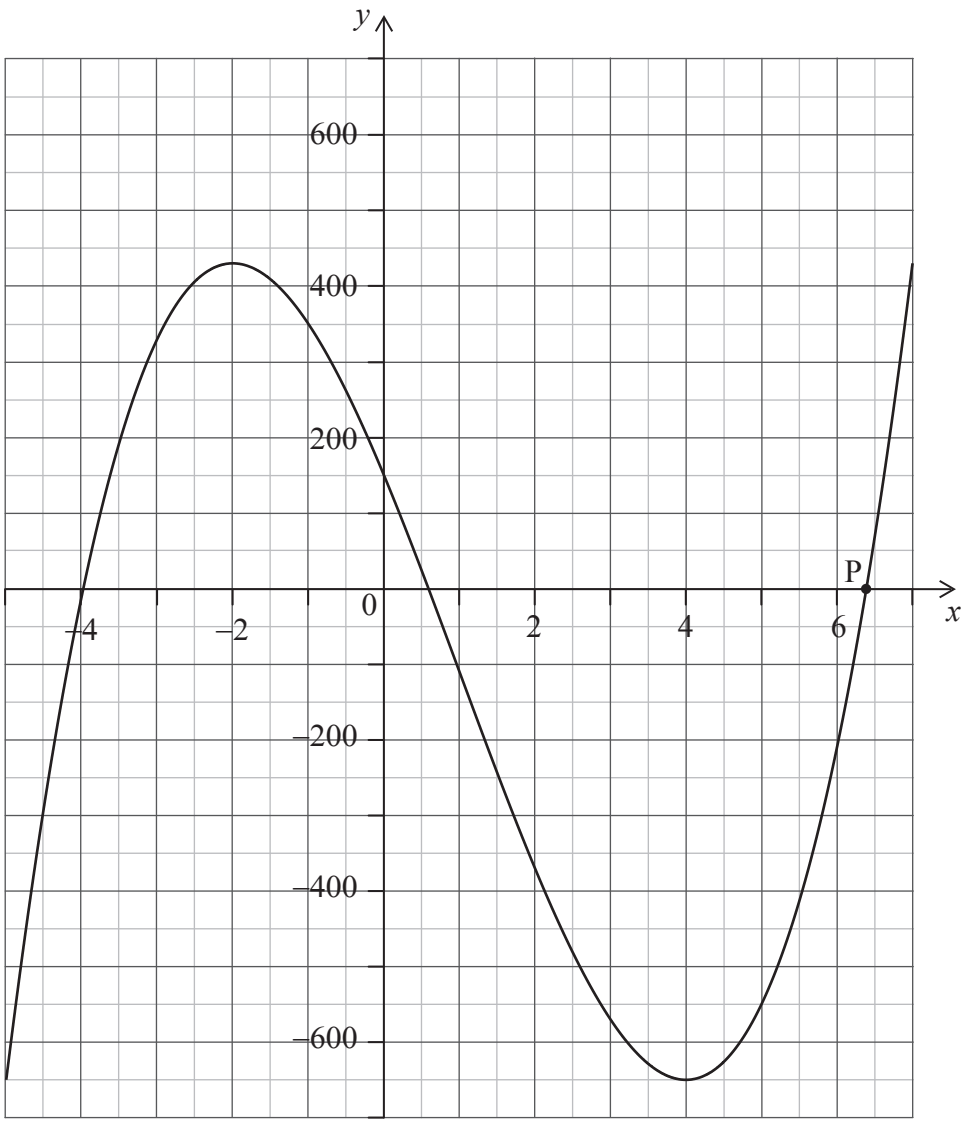
The farmer's tractor is located at point A, on the other side of the lake. The farmer walks along the straight paths from her farmhouse, D, and around the lake, to the tractor at A.

(e) Show that the **shortest** possible route that the farmer can walk passes through point B. [5]

5. [Maximum mark: 17]

The following diagram shows the graph of the function

$$f(x) = nx^3 + px^2 + qx + r,\ n \neq 0,\ \text{for } -5 \leq x \leq 7.$$



(a) State whether the function is increasing or decreasing at $x = -3$. Give a reason for your answer. [2]

(b) Write down the value of r. [1]

The values of p and q are such that $f(x) = nx^3 - 30x^2 - 240x + r$.

(c) Find $f'(x)$. [3]

(This question continues on the following page)

(Question 5 continued)

(d) Write down the coordinates of the local minimum point. [1]

(e) Show that the value of n is 10. [2]

(f) (i) Calculate $f'(-1)$.

(ii) Find the equation of the tangent to the graph at the point $(-1, 350)$. Give your answer in the form $ax + by + d = 0$.

(iii) Write down the gradient of the normal to the graph at $x = -1$. [5]

The graph of the function intersects the x-axis at point P, as shown in the diagram.

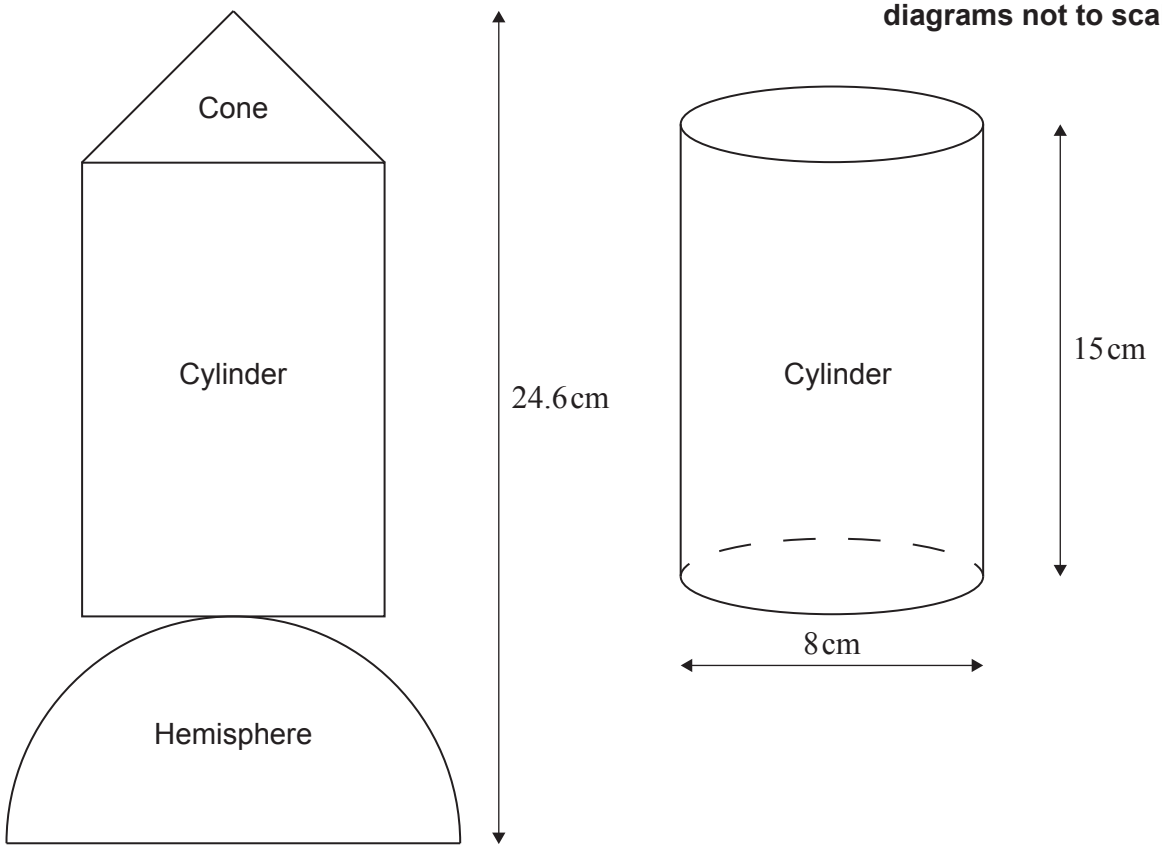
(g) Use your graphic display calculator to find the x-coordinate of P. [1]

(h) Let $g(x) = 100x + 400$, for $-5 \leq x \leq 7$. Use your graphic display calculator to find the values of x where $f(x) = g(x)$, for the given domain. [2]

6. [Maximum mark: 12]

Yutaka makes a toy spaceship. It has three separate solid parts: a cone, a cylinder and a hemisphere. This toy is represented by the following two-dimensional diagram. The cylinder is also shown in a separate diagram.

diagrams not to scale



The cylinder has height $15\,\text{cm}$ and diameter $8\,\text{cm}$.

(a) Find the volume of the cylinder. [2]

The cone has diameter $8\,\text{cm}$ and volume $85\,\text{cm}^3$.

(b) Find the height of the cone. [2]

The toy spaceship's total height is $24.6\,\text{cm}$.

(c) Find the volume of the hemisphere. [4]

Yutaka decides to paint the cone of the toy spaceship.

(d) Calculate the curved surface area of the cone. Give your answer to the nearest cm^2. [4]