

**Mathematical studies**  
**Standard level**  
**Paper 2**

Thursday 12 November 2015 (afternoon)

1 hour 30 minutes

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**Instructions to candidates**

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 14]

A shop sells strawberry, coffee and orange flavoured chocolates. 50 customers were asked which flavoured chocolates they had bought.

The results are as follows:

7 bought only strawberry chocolates  
6 bought only coffee chocolates  
10 bought only orange chocolates  
3 bought coffee and strawberry, **but not** orange  
5 bought strawberry and orange, **but not** coffee  
4 bought orange and coffee, **but not** strawberry  
 $x$  bought strawberry, coffee and orange.

- (a) Represent this information on a Venn diagram. [4]
- (b) Find the value of  $x$ , given that 13 of these 50 customers did not buy any chocolates. [2]
- (c) (i) Find the probability that a customer, chosen at random from these 50 customers, bought strawberry flavoured chocolates.
- (ii) Find the probability that a customer, chosen at random from these 50 customers, bought orange flavoured chocolates.
- (iii) Determine whether the events in parts (c)(i) and (c)(ii) are independent. Give a reason for your answer. [6]
- (d) A customer is chosen at random from these 50 customers. Given that the customer bought coffee flavoured chocolates, find the probability that they also bought strawberry flavoured chocolates. [2]

2. [Maximum mark: 19]

The following table shows the internal assessment marks and examination marks for six students.

|   |    |    |    |    |    |    |
|---|----|----|----|----|----|----|
| <b>Internal assessment marks (<math>x</math>)</b> | 4  | 10 | 12 | 16 | 18 | 20 |
| <b>Examination marks (<math>y</math>)</b>         | 35 | 45 | 52 | 55 | 65 | 70 |

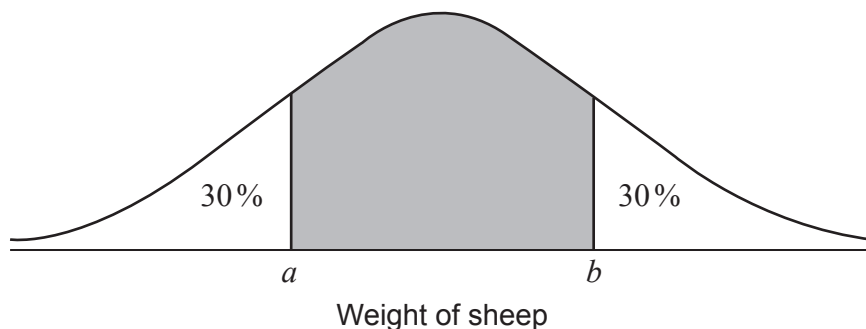
- (a) **On graph paper**, draw a scatter diagram for the above data. Use a scale of 1 cm to represent 2 marks on the  $x$ -axis and 1 cm to represent 10 marks on the  $y$ -axis. [3]
- (b) (i) Write down the Pearson's product-moment correlation coefficient,  $r$ , for the above data.
- (ii) Describe the correlation between internal assessment marks and examination marks. [4]
- (c) Find the
- (i) mean internal assessment mark,  $\bar{x}$  ;
- (ii) mean examination mark,  $\bar{y}$ . [2]
- (d) Plot and label the point  $M(\bar{x}, \bar{y})$  on your scatter diagram from part (a). [2]
- (e) Write down the equation of the regression line for  $y$  on  $x$ . [2]
- (f) Use your **equation** from part (e) to estimate the examination mark of a student who scored 8 marks on their internal assessment. [2]
- (g) Draw the regression line  $y$  on  $x$  on your scatter diagram from part (a). [2]
- A **new student** scores 30 marks on her internal assessment and uses the above data to estimate that she will score 89 marks on the examination.
- (h) State whether this estimate is reliable and give a reason for your answer. [2]

## 3. [Maximum mark: 13]

The weights of sheep on a farm are normally distributed with a mean of 110 kg and a standard deviation of 8 kg.

- (a) Sketch a diagram of the distribution of the weights of these sheep. On your diagram, label the mean and label one standard deviation above and below the mean. [2]
- (b) (i) A sheep has a weight of 94 kg. Write down the number of standard deviations that this weight is below the mean. [2]
- (ii) Find the probability that a sheep, chosen at random, weighs more than 94 kg. [3]
- (c) (i) Find the probability that a sheep, chosen at random, weighs between 88 kg and 116 kg. [2]
- (ii) The farmer weighs 160 sheep. Find the number of sheep that he would expect to weigh between 88 kg and 116 kg. [4]
- (d) Given that 75% of the sheep weigh **less than**  $w$  kg, find the value of  $w$ . [2]

A sheep is chosen at random. Its weight is within the central shaded region of the following diagram.



- (e) Find the value of  $a$  and of  $b$ . [2]

4. [Maximum mark: 15]

A lake is in the shape of a triangle,  $ABC$ , where  $AB$ ,  $BC$  and  $CA$  are paths around the lake. The paths have the following lengths:  $AB = 140\text{ m}$ ,  $BC = 110\text{ m}$  and  $CA = 120\text{ m}$ .

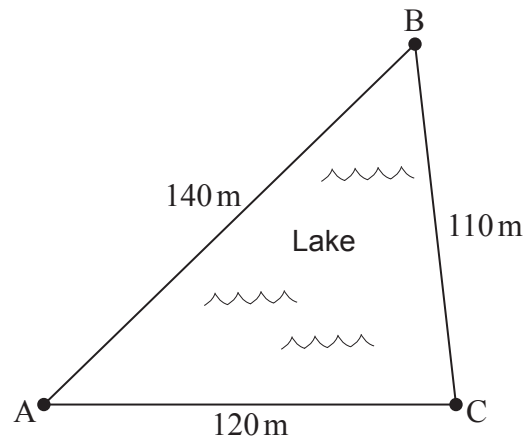


diagram not to scale

D  
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(a) Find the size of angle  $\hat{BAC}$ . [3]

(b) Find the surface area of the lake. [3]

A farmhouse is located at point  $D$ , away from the lake, such that angle  $\hat{DBC}$  is  $80^\circ$  and angle  $\hat{BCD}$  is  $40^\circ$ . The farmer has built straight paths from her farmhouse,  $D$ , to points  $B$  and  $C$ .

(c) Show that angle  $\hat{BDC} = 60^\circ$ . [1]

(d) Find the distance between  $C$  and  $D$ . [3]

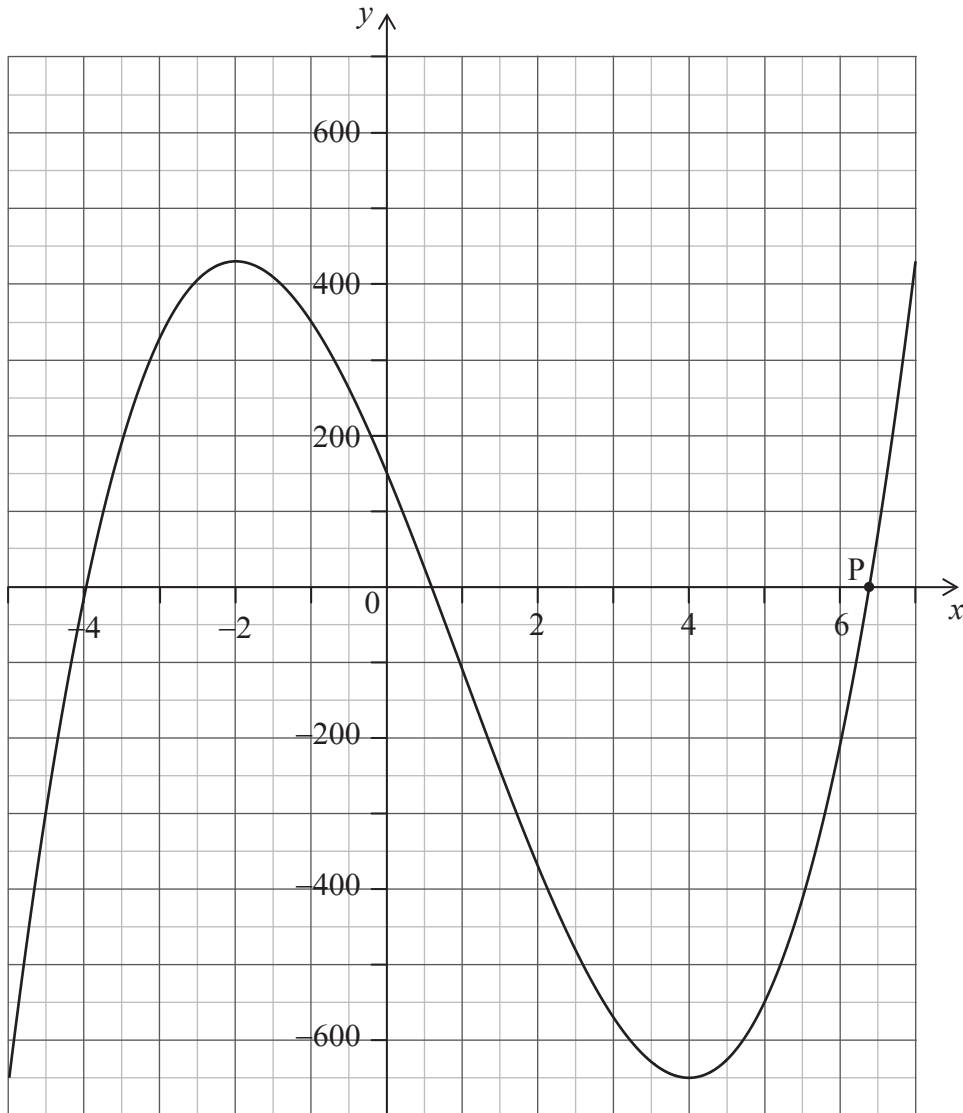
The farmer's tractor is located at point  $A$ , on the other side of the lake. The farmer walks along the straight paths from her farmhouse,  $D$ , and around the lake, to the tractor at  $A$ .

(e) Show that the **shortest** possible route that the farmer can walk passes through point  $B$ . [5]

5. [Maximum mark: 17]

The following diagram shows the graph of the function

$$f(x) = nx^3 + px^2 + qx + r, \quad n \neq 0, \quad \text{for } -5 \leq x \leq 7.$$



(a) State whether the function is increasing or decreasing at  $x = -3$ . Give a reason for your answer. [2]

(b) Write down the value of  $r$ . [1]

The values of  $p$  and  $q$  are such that  $f(x) = nx^3 - 30x^2 - 240x + r$ .

(c) Find  $f'(x)$ . [3]

(This question continues on the following page)

**(Question 5 continued)**

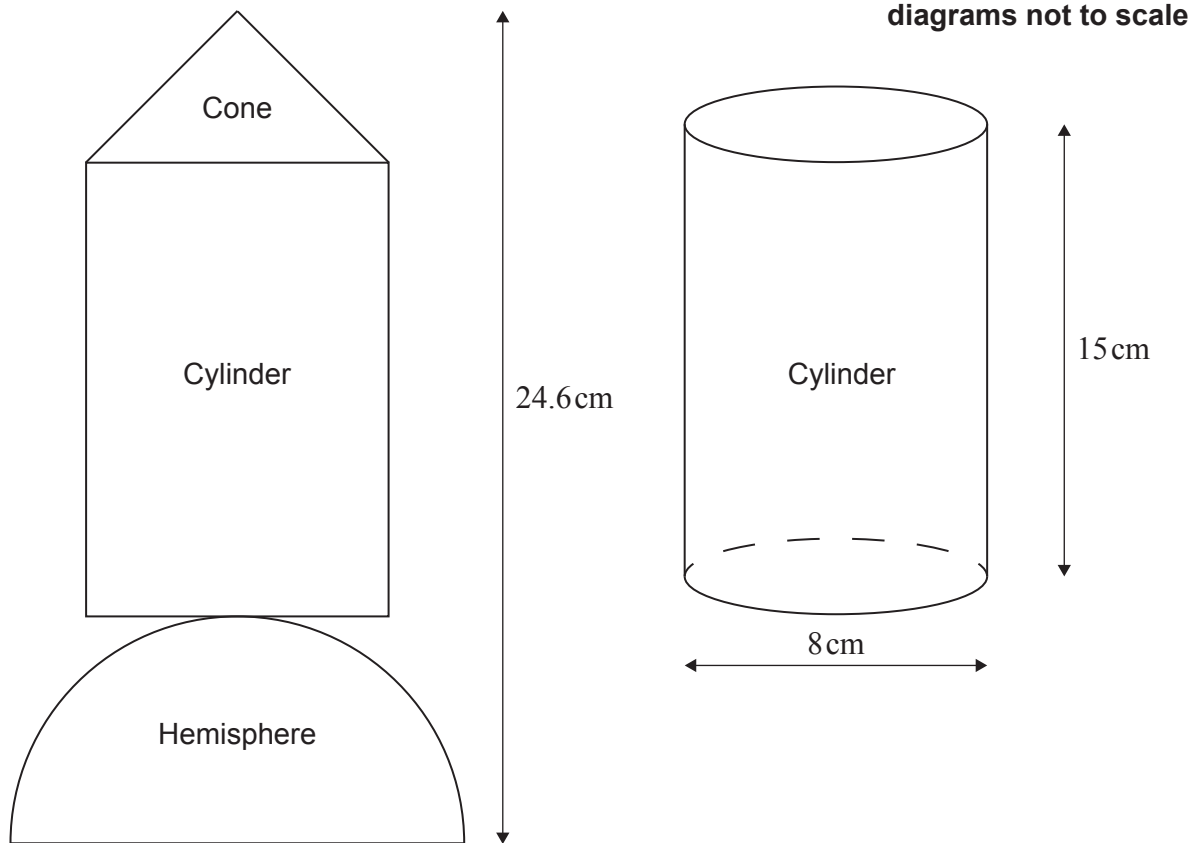
- (d) Write down the coordinates of the local minimum point. [1]
- (e) Show that the value of  $n$  is 10. [2]
- (f) (i) Calculate  $f'(-1)$ .
- (ii) Find the equation of the tangent to the graph at the point  $(-1, 350)$ . Give your answer in the form  $ax + by + d = 0$ .
- (iii) Write down the gradient of the normal to the graph at  $x = -1$ . [5]

The graph of the function intersects the  $x$ -axis at point P, as shown in the diagram.

- (g) Use your graphic display calculator to find the  $x$ -coordinate of P. [1]
- (h) Let  $g(x) = 100x + 400$ , for  $-5 \leq x \leq 7$ . Use your graphic display calculator to find the values of  $x$  where  $f(x) = g(x)$ , for the given domain. [2]

## 6. [Maximum mark: 12]

Yutaka makes a toy spaceship. It has three separate solid parts: a cone, a cylinder and a hemisphere. This toy is represented by the following two-dimensional diagram. The cylinder is also shown in a separate diagram.



The cylinder has height 15 cm and diameter 8 cm.

- (a) Find the volume of the cylinder. [2]

The cone has diameter 8 cm and volume  $85 \text{ cm}^3$ .

- (b) Find the height of the cone. [2]

The toy spaceship's total height is 24.6 cm.

- (c) Find the volume of the hemisphere. [4]

Yutaka decides to paint the cone of the toy spaceship.

- (d) Calculate the curved surface area of the cone. Give your answer to the nearest  $\text{cm}^2$ . [4]