M15/5/MATSD/SP2/ENG/TZ1/XX

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 2

Wednesday 13 May 2015 (afternoon)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **Mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 10]

In a debate on voting, a survey was conducted. The survey asked people's opinion on whether or not the minimum voting age should be reduced to 16 years of age. The results are shown as follows.

	Age 18–25	**Age 26–40**	**Age 41+**	**Total**
Oppose the reduction	12	20	48	80
Favour the reduction	18	15	17	50
Total	30	35	65	130

A χ^2 test at the 1% significance level was conducted. The χ^2 critical value of the test is 9.21.

(a) State

(i) H_0, the null hypothesis for the test;

(ii) H_1, the alternative hypothesis for the test. [2]

(b) Write down the number of degrees of freedom. [1]

(c) Show that the expected frequency of those between the ages of 26 and 40 who oppose the reduction in the voting age is 21.5, correct to three significant figures. [2]

(d) Find

(i) the χ^2 statistic;

(ii) the associated p-value for the test. [3]

(e) Determine, giving a reason, whether H_0 should be accepted. [2]

2. [Maximum mark: 11]

Consider the following statements.

p: the land has been purchased
q: the building permit has been obtained
r: the land can be used for residential purposes

(a) Write the following argument in symbolic form.

"If the land has been purchased and the building permit has been obtained, then the land can be used for residential purposes." [3]

(b) **In your answer booklet**, copy and complete a truth table for the argument in part (a). Begin your truth table as follows.

p	q	r	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

[2]

(c) Use your truth table to determine whether the argument in part (a) is valid. Give a reason for your decision. [2]

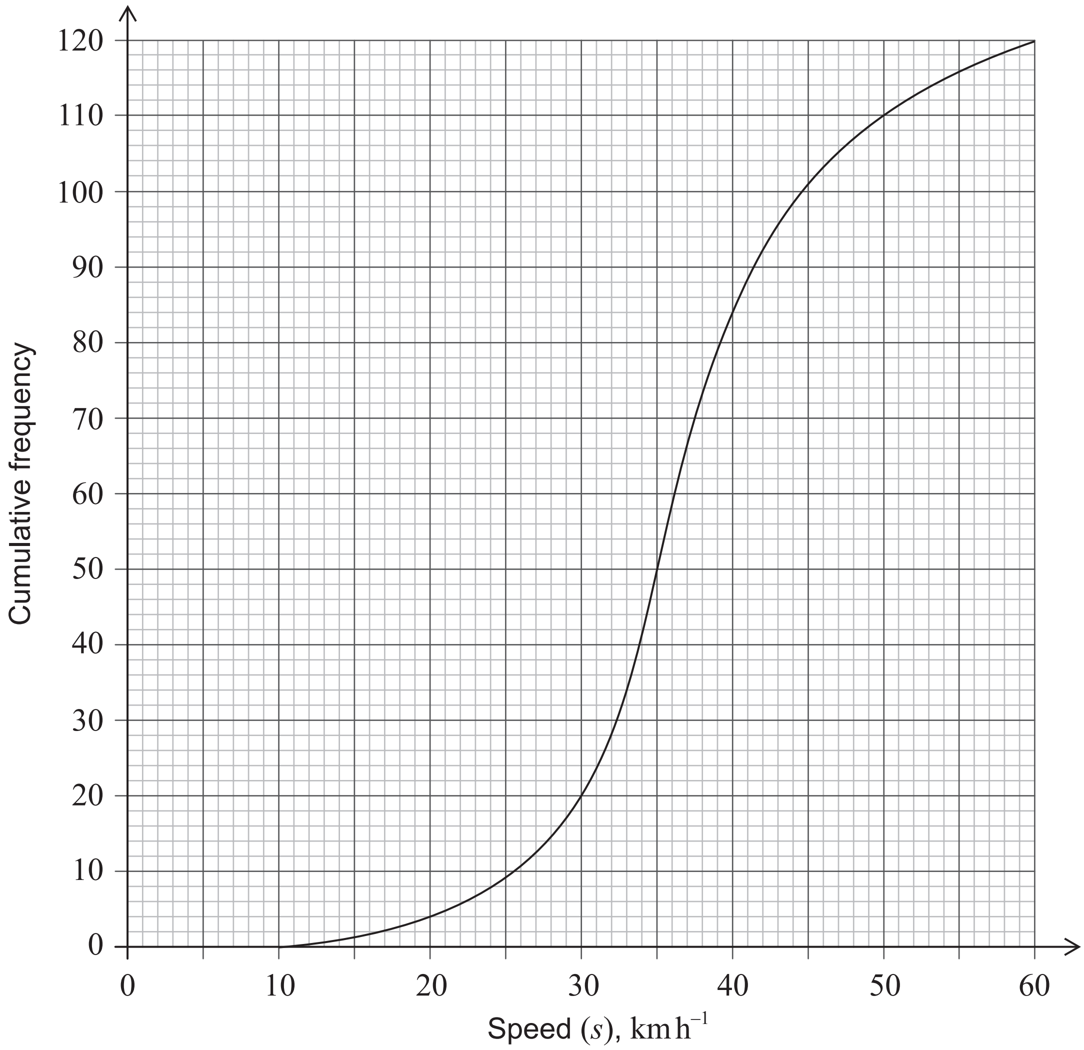
(d) Write down the inverse of the argument in part (a)

(i) in symbolic form;

(ii) in words. [4]

3. [Maximum mark: 17]

The cumulative frequency graph shows the speed, s, in km h^{-1}, of 120 vehicles passing a hospital gate.



(a) Estimate the minimum possible speed of one of these vehicles passing the hospital gate. [1]

(b) Find the median speed of the vehicles. [2]

(c) Write down the 75th percentile. [1]

(d) Calculate the interquartile range. [2]

This question continues on the following page

Question 3 continued

The speed limit past the hospital gate is $50\,\text{km}\,\text{h}^{-1}$.

(e) Find the number of these vehicles that exceed the speed limit. [2]

The table shows the speeds of these vehicles travelling past the hospital gate.

Speed of Vehicles	Number of Vehicles
$0 < s \le 10$	0
$10 < s \le 20$	p
$20 < s \le 30$	16
$30 < s \le 40$	64
$40 < s \le 50$	26
$50 < s \le 60$	q

(f) Find the value of p and of q. [2]

(g) (i) Write down the modal class.

(ii) Write down the mid-interval value for this class. [2]

(h) Use your graphic display calculator to calculate an estimate of

(i) the mean speed of these vehicles;

(ii) the standard deviation. [3]

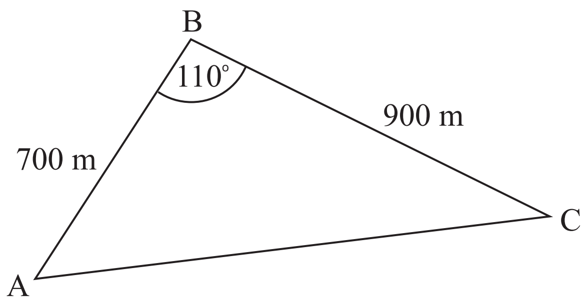
It is proposed that the speed limit past the hospital gate is reduced to $40\,\text{km}\,\text{h}^{-1}$ from the current $50\,\text{km}\,\text{h}^{-1}$.

(i) Find the percentage of these vehicles passing the hospital gate that **do not** exceed the current speed limit but **would** exceed the new speed limit. [2]

4. **[Maximum mark: 21]**

A boat race takes place around a triangular course, ABC, with $AB = 700\text{ m}$, $BC = 900\text{ m}$ and angle $ABC = 110^\circ$. The race starts and finishes at point A.

diagram not to scale



(a) Calculate the total length of the course. [4]

It is estimated that the fastest boat in the race can travel at an average speed of 1.5 m s^{-1}.

(b) Calculate an estimate of the winning time of the race. Give your answer to the nearest minute. [3]

(c) Find the size of angle ACB. [3]

To comply with safety regulations, the area inside the triangular course must be kept clear of other boats, and the shortest distance from B to AC must be greater than 375 metres.

(d) Calculate the area that must be kept clear of boats. [3]

(e) Determine, giving a reason, whether the course complies with the safety regulations. [3]

The race is filmed from a helicopter, H, which is flying vertically above point A. The angle of elevation of H from B is 15°.

(f) Calculate the vertical height, AH, of the helicopter above A. [2]

(g) Calculate the maximum possible distance from the helicopter to a boat on the course. [3]

5. **[Maximum mark: 20]**

Consider the function $f(x) = \frac{96}{x^2} + kx$, where k is a constant and $x \neq 0$.

(a) Write down $f'(x)$. [3]

The graph of $y = f(x)$ has a local minimum point at $x = 4$.

(b) Show that $k = 3$. [2]

(c) Find $f(2)$. [2]

(d) Find $f'(2)$. [2]

(e) Find the equation of the normal to the graph of $y = f(x)$ at the point where $x = 2$. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$. [3]

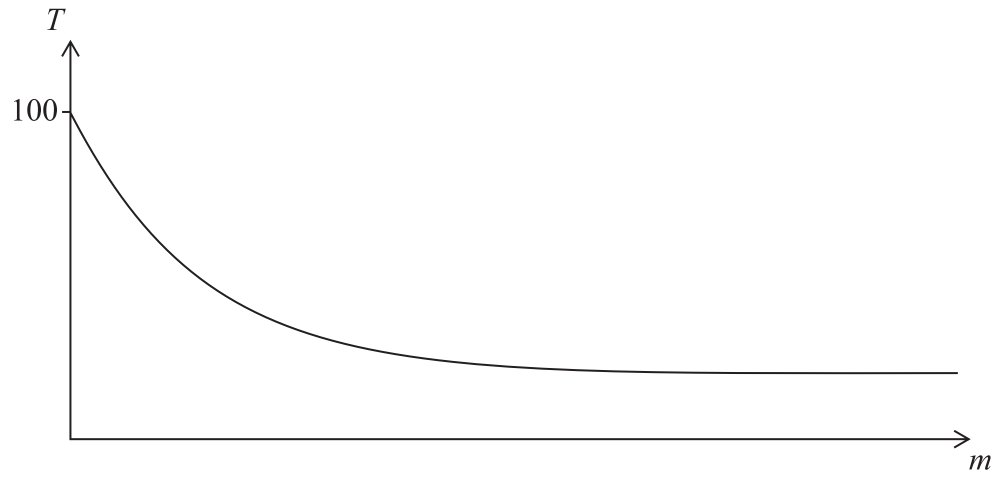
(f) Sketch the graph of $y = f(x)$, for $-5 \leq x \leq 10$ and $-10 \leq y \leq 100$. [4]

(g) Write down the coordinates of the point where the graph of $y = f(x)$ intersects the x-axis. [2]

(h) State the values of x for which $f(x)$ is decreasing. [2]

6. [Maximum mark: 11]

A cup of boiling water is placed in a room to cool. The temperature of the room is $20\,^{\circ}\mathrm{C}$. This situation can be modelled by the exponential function $T = a + b(k^{-m})$, where T is the temperature of the water, in $^{\circ}\mathrm{C}$, and m is the number of minutes for which the cup has been placed in the room. A sketch of the situation is given as follows.



(a) Explain why $a = 20$. [2]

Initially, at $m = 0$, the temperature of the water is $100\,^{\circ}\mathrm{C}$.

(b) Find the value of b. [2]

After being placed in the room for one minute, the temperature of the water is $84\,^{\circ}\mathrm{C}$.

(c) Show that $k = 1.25$. [2]

(d) Find the temperature of the water three minutes after it has been placed in the room. [2]

(e) Find the total time needed for the water to reach a temperature of $35\,^{\circ}\mathrm{C}$. Give your answer in minutes and seconds, correct to the nearest second. [3]