

Mathematical studies
Standard level
Paper 2

Wednesday 13 May 2015 (afternoon)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

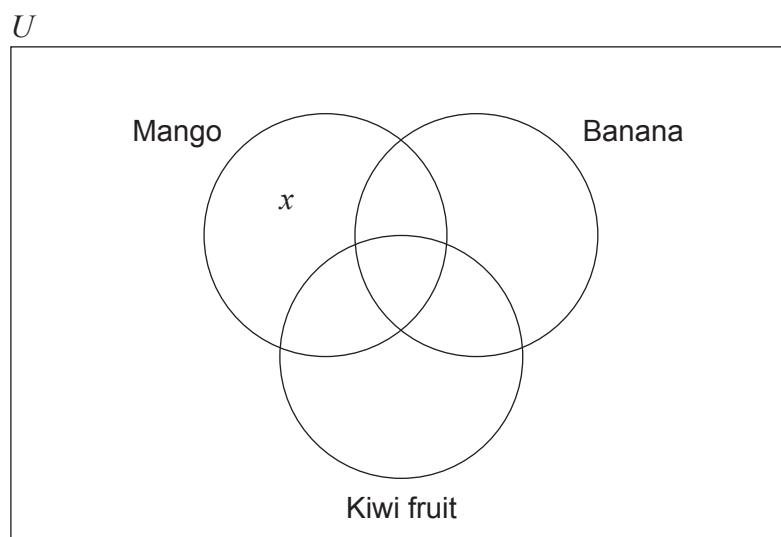
1. [Maximum mark: 16]

A group of 100 customers in a restaurant are asked which fruits they like from a choice of mangoes, bananas and kiwi fruits. The results are as follows.

- 15 like all three fruits
- 22 like mangoes and bananas
- 33 like mangoes and kiwi fruits
- 27 like bananas and kiwi fruits
- 8 like none of these three fruits
- x like **only** mangoes

(a) **Copy** the following Venn diagram and correctly insert all values from the above information.

[3]



The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. This number is half of the number of customers that like **only** bananas.

(b) Complete your Venn diagram from part (a) with this additional information **in terms of x** .

[2]

(c) Find the value of x .

[2]

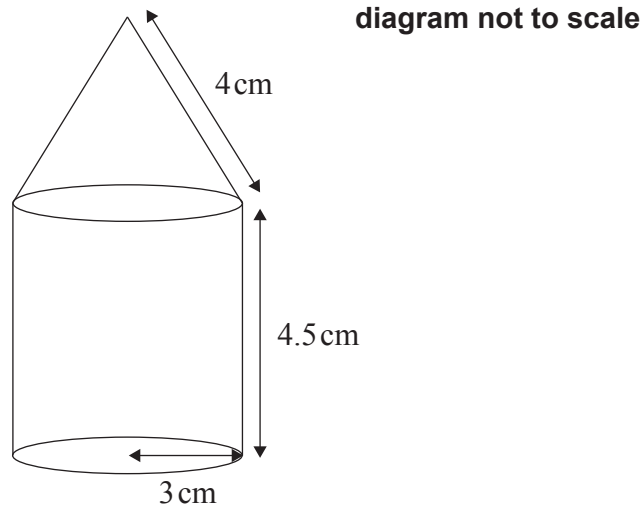
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(Question 1 continued)

- (d) Write down the number of customers who like
- (i) mangoes;
 - (ii) mangoes or bananas. [2]
- (e) A customer is chosen at random from the 100 customers. Find the probability that this customer
- (i) likes none of the three fruits;
 - (ii) likes only two of the fruits;
 - (iii) likes all three fruits given that the customer likes mangoes and bananas. [4]
- (f) Two customers are chosen at random from the 100 customers. Find the probability that the two customers like none of the three fruits. [3]

2. [Maximum mark: 17]

The following diagram shows a perfume bottle made up of a cylinder and a cone.



The radius of both the cylinder and the base of the cone is 3 cm.

The height of the cylinder is 4.5 cm.

The slant height of the cone is 4 cm.

(a) (i) Show that the vertical height of the cone is 2.65 cm correct to three significant figures.

(ii) Calculate the volume of the perfume bottle.

[6]

The bottle contains 125 cm^3 of perfume. The bottle is **not** full and all of the perfume is in the cylinder part.

(b) Find the height of the perfume in the bottle.

[2]

Temi makes some crafts with perfume bottles, like the one above, once they are empty.

Temi wants to know the surface area of one perfume bottle.

(c) Find the **total** surface area of the perfume bottle.

[4]

(This question continues on the following page)

(Question 2 continued)

Temi covers the perfume bottles with a paint that costs 3 South African rand (ZAR) per millilitre. One millilitre of this paint covers an area of 7 cm^2 .

- (d) Calculate the cost, in ZAR, of painting the perfume bottle. **Give your answer correct to two decimal places.** [3]

Temi sells her perfume bottles in a craft fair for 325 ZAR each. Dominique from France buys one and wants to know how much she has spent, in euros (EUR). The exchange rate is $1 \text{ EUR} = 13.03 \text{ ZAR}$.

- (e) Find the price, in EUR, that Dominique paid for the perfume bottle. **Give your answer correct to two decimal places.** [2]

3. [Maximum mark: 12]

The sum of the first n terms of an arithmetic sequence is given by $S_n = 6n + n^2$.

(a) Write down the value of

(i) S_1 ;

(ii) S_2 .

[2]

The n^{th} term of the arithmetic sequence is given by u_n .

(b) Show that $u_2 = 9$.

[1]

(c) Find the common difference of the sequence.

[2]

(d) Find u_{10} .

[2]

(e) Find the lowest value of n for which u_n is greater than 1000.

[3]

(f) There is a value of n for which

$$u_1 + u_2 + \dots + u_n = 1512.$$

Find the value of n .

[2]

4. [Maximum mark: 12]

Daniel grows apples and chooses at random a sample of 100 apples from his harvest. He measures the diameters of the apples to the nearest cm. The following table shows the distribution of the diameters.

Diameter (to the nearest cm)	5	6	7	8	9
Frequency	15	27	33	17	8

(a) Using your graphic display calculator, write down the value of

(i) the mean of the diameters in this sample;

(ii) the standard deviation of the diameters in this sample.

[3]

Daniel assumes that the diameters of all of the apples from his harvest are normally distributed with a mean of 7 cm and a standard deviation of 1.2 cm. He classifies the apples according to their diameters as shown in the following table.

Classification	Diameter (cm)
Small	Diameter < 6.5
Medium	$6.5 \leq \text{Diameter} < a$
Large	Diameter $\geq a$

(b) Calculate the percentage of **small** apples in Daniel's harvest.

[3]

Of the apples harvested, 5% are **large** apples.

(c) Find the value of a .

[2]

(d) Find the percentage of **medium** apples.

[2]

This year, Daniel estimates that he will grow 100 000 apples.

(e) Estimate the number of **large** apples that Daniel will grow this year.

[2]

5. [Maximum mark: 17]

Consider the function $f(x) = 0.5x^2 - \frac{8}{x}$, $x \neq 0$.

(a) Find $f(-2)$. [2]

(b) Find $f'(x)$. [3]

(c) Find the gradient of the graph of f at $x = -2$. [2]

Let T be the tangent to the graph of f at $x = -2$.

(d) Write down the equation of T . [2]

(e) Sketch the graph of f for $-5 \leq x \leq 5$ and $-20 \leq y \leq 20$. [4]

(f) Draw T on your sketch. [2]

The tangent, T , intersects the graph of f at a second point, P.

(g) Use your graphic display calculator to find the coordinates of P. [2]

6. [Maximum mark: 16]

The following table shows the number of bicycles, x , produced daily by a factory and their total production cost, y , in US dollars (USD). The table shows data recorded over seven days.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Number of bicycles, x	12	15	14	17	20	18	21
Production cost, y	3900	4600	4100	5300	6000	5400	6000

- (a) (i) Write down the Pearson's product-moment correlation coefficient, r , for these data.
- (ii) Hence comment on the result. [4]
- (b) Write down the equation of the regression line y on x for these data, in the form $y = ax + b$. [2]
- (c) Estimate the total cost, **to the nearest USD**, of producing 13 bicycles on a particular day. [3]
- All the bicycles that are produced are sold. The bicycles are sold for 304 USD **each**.
- (d) Explain why the factory does **not** make a profit when producing 13 bicycles on a particular day. [2]
- (e) (i) Write down an expression for the total selling price of x bicycles.
- (ii) Write down an expression for the **profit** the factory makes when producing x bicycles on a particular day.
- (iii) Find the least number of bicycles that the factory should produce, on a particular day, in order to make a profit. [5]
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