Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 1

Wednesday 11 November 2015 (morning)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all questions.
- Write your answers in the boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

8815–7401

Please **do not** write on this page.

Answers written on this page will not be marked.

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Write your answers in the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. Complete the table below placing a tick (✓) to show which of the number sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{R}$ these numbers belong to. The first row has been completed as an example.

	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
0.8			✓	✓
$\sqrt{14}$				
$\sin 30^\circ$				
4				
-3				
4.12×10^1				
$3\frac{1}{3}$				

[6]

2. A class of 15 students were asked how many pencils they bring to class. The following results were recorded:

$$5, 7, 4, 5, 6, 7, 7, 4, 6, 5, 4, 6, 7, 2, 11$$

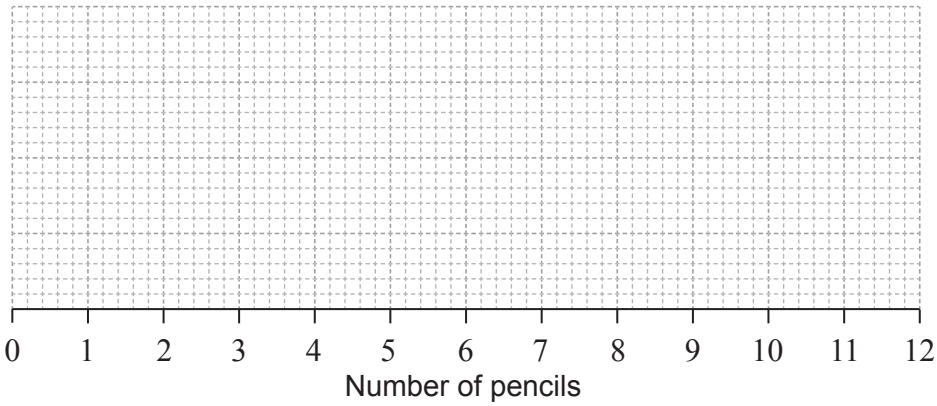
(a) For these results, write down

(i) the median;

(ii) the mode. [3]

The lower and upper quartiles of these results are 4 and 7, respectively.

(b) Draw a box-and-whisker diagram to represent these results. [3]



Working:

Answers:

(a) (i) . . . . . .

(ii) . . . . . .

3. Sebastian is travelling from Fiji to Brazil. He changes 5000 Fijian Dollars (FJD) to Brazilian Reals (BRL), at an exchange rate of 1 FJD = 1.1824 BRL.

(a) Calculate the **exact** amount of BRL that Sebastian receives. [2]

Sebastian estimated that he would receive 6000 BRL.

(b) Find the percentage error made in his estimation. [2]

While in Brazil, Sebastian spends 4000 BRL. On the way back to Fiji, he changes his remaining BRL back to FJD at the same exchange rate, 1 FJD = 1.1824 BRL.

(c) Calculate the amount of FJD he receives. Give your answer correct to two decimal places. [2]

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

4. The seventh term, u_7, of an arithmetic sequence is 28. The tenth term, u_{10}, of the same sequence is 37.

(a) Find d, the common difference of the sequence. [2]

(b) Find u_1, the first term of the sequence. [2]

(c) Find S_{200}, the sum of the first 200 terms of the sequence. [2]

Working:

Answers:

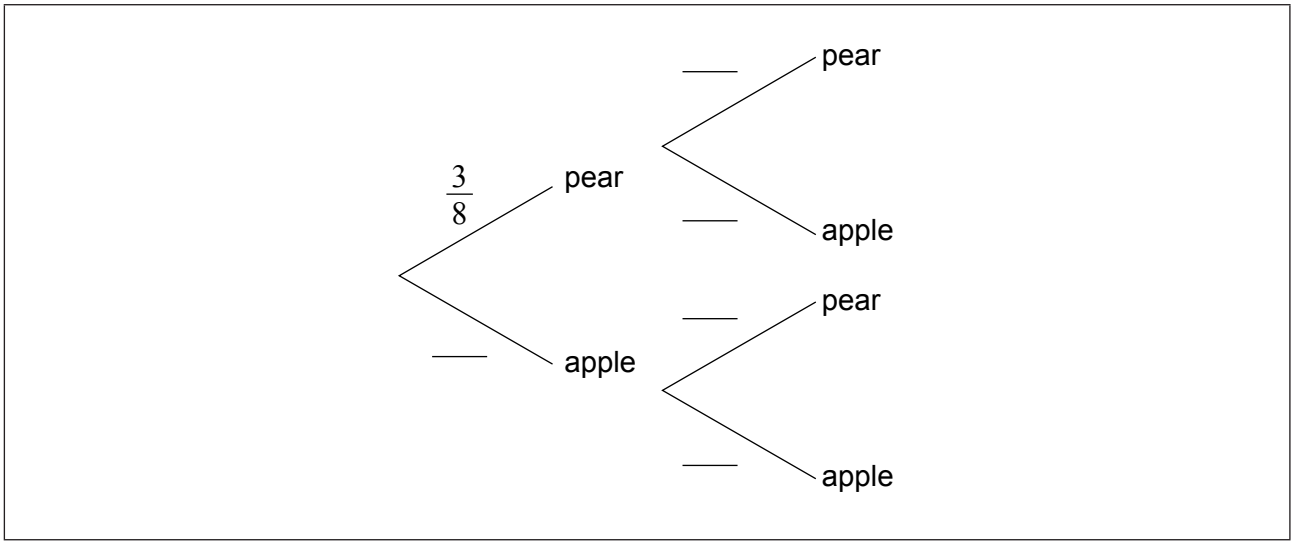
(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

5. A fruit bowl contains 3 pears and 5 apples. Hori chooses a fruit from the bowl at random and eats it. He then chooses another fruit from the same bowl at random.

(a) Complete the following tree diagram. [3]



(b) Calculate the probability that both fruits chosen by Hori are the same. [3]

Working:

Answer:

(b) . . . . . .

6. Students in a school cafeteria had their gender and drink preference recorded from a choice of tea, coffee or hot chocolate. This is shown in the following table.

	Tea	**Coffee**	**Hot Chocolate**
Boys	15	15	10
Girls	10	10	30

(a) Write down the total number of students surveyed. [1]

A χ^2 test was conducted at the 5% significance level.

(b) Write down the null hypothesis for this test. [1]

(c) Using your graphic display calculator, find the χ^2 statistic. [2]

The critical value at the 5% significance level is 5.99.

(d) State the conclusion of this test. Give a reason for your answer. [2]

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

. . . . . .

. . . . . .

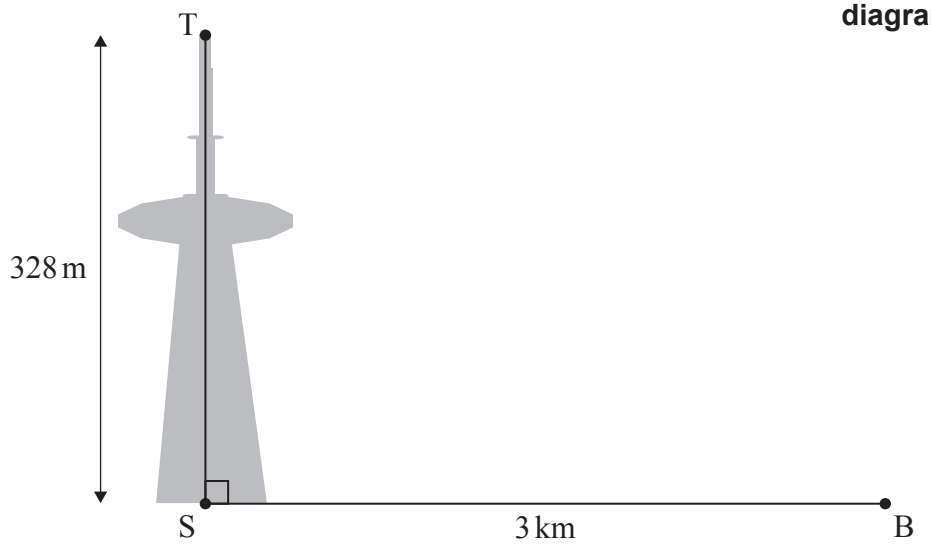
(c) . . . . . .

(d) . . . . . .

. . . . . .

. . . . . .

7. From point B, a vertical tower can be seen. Point B is 3 kilometres, horizontally, from point S at the base of the tower. The height of the tower, TS, is 328 metres.



(a) Write down the height of the tower, TS, in kilometres. [1]

(b) On the diagram, label the angle of elevation from B to T with an x. [1]

(c) Find the size of the angle of elevation from B to T. [2]

(d) Find TB, the distance **in kilometres** from the top of the tower to point B. [2]

Working:

Answers:

(a) . . . . . .

(c) . . . . . .

(d) . . . . . .

8. At the end of 2008, the population of the city of Nouméa was $97\,579$.

A year later, at the end of 2009, the population of Nouméa was $99\,921$.

(a) Find the percentage change in the population from the end of 2008 to the end of 2009. [2]

Assume that the population of Nouméa continues to grow at the same percentage rate.

(b) Use your answer to part (a) to estimate the population at the end of 2016. [2]

(c) Write your answer **to part (b)** in the form $a \times 10^k$, where $1 \leq a < 10$, $k \in \mathbb{Z}$. [2]

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

9. Consider the following logic propositions:

p: Emma plays tennis
q: Emma goes to the beach

(a) Complete the following truth table. [1]

p	q	$\neg p$	$\neg p \Rightarrow q$
T	T	F	
T	F	F	
F	T	T	
F	F	T	

(b) Write in words $\neg p \Rightarrow q$. [2]

(c) Write the inverse of $\neg p \Rightarrow q$ in symbolic form. [2]

(d) State whether $\neg p \Rightarrow q$ is a tautology, logical contradiction or neither. [1]

Working:

Answers:

(b) . . . . . .

. . . . . .

(c) . . . . . .

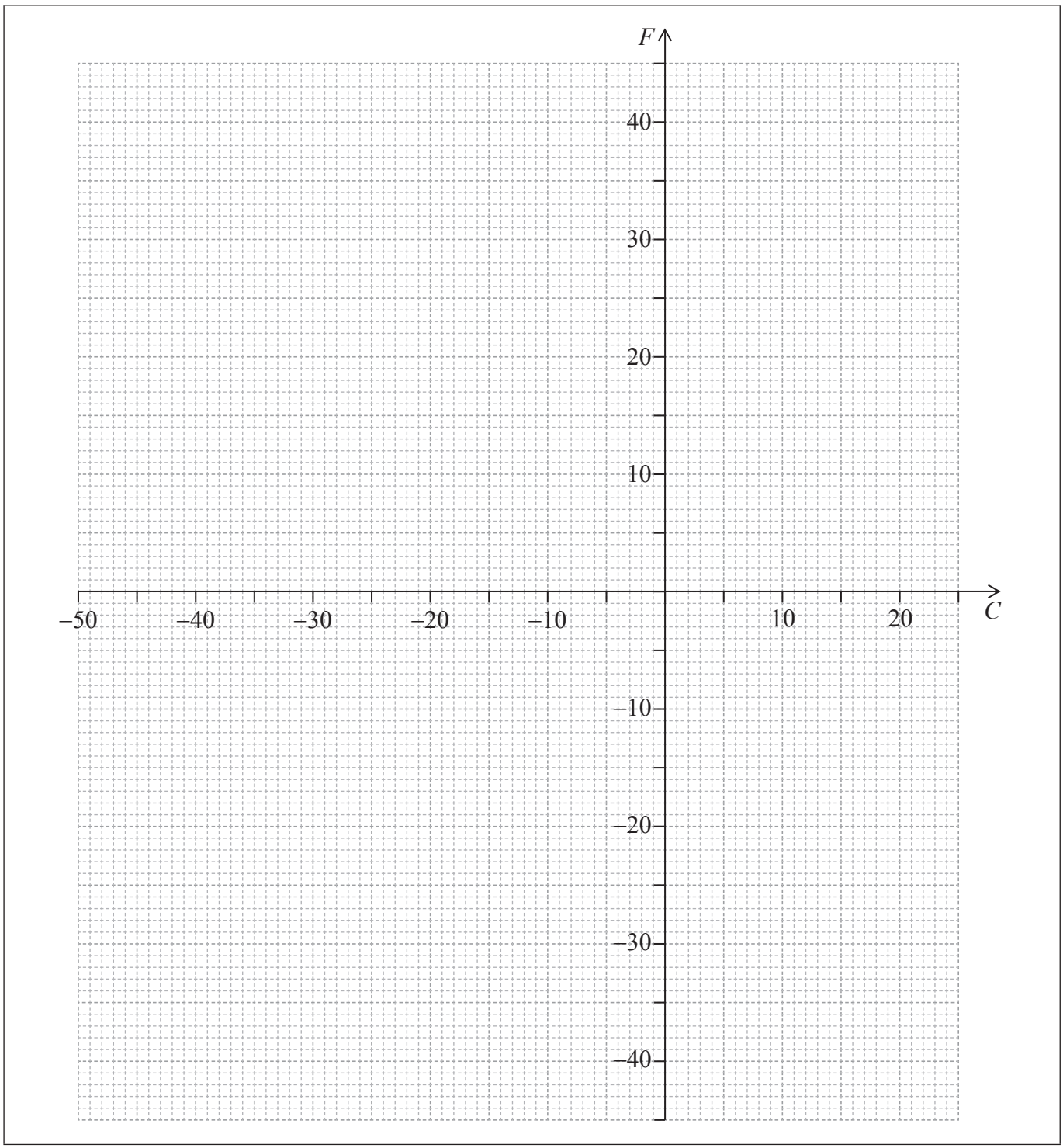
(d) . . . . . .

10. Temperature can be converted from degrees Celsius (C) to degrees Fahrenheit (F) using the equation

$$F = \frac{9}{5}C + 32.$$

(a) Calculate $100\,^{\circ}\mathrm{C}$ in degrees Fahrenheit. [1]

(b) Using the axes provided, draw the graph of $F = \frac{9}{5}C + 32$. [2]



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(Question 10 continued)

(c) Find the value of $14\,^{\circ}\mathrm{F}$ in degrees Celsius. [1]

(d) Find the value of x, such that $x\,^{\circ}\mathrm{C}$ is the same temperature as $x\,^{\circ}\mathrm{F}$. [2]

Working:

Answers:

(a) . . . . . .

(c) . . . . . .

(d) . . . . . .

11. Sophie and Kylie both received 1200 Australian dollars (AUD).

Sophie invested her 1200 AUD in an account which pays a nominal annual interest rate of 3.4%, **compounded quarterly**.

(a) Find the future value of her investment after 7 years. Give your answer correct to two decimal places. [3]

Kylie bought a car for 1200 AUD which depreciated at a rate of $r\%$ per year. The value of the car after 7 years is 669 AUD.

(b) Find the rate of depreciation. [3]

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

12. Consider the geometric sequence $2048, 1536, 1152, 864 \ldots$

(a) Find the common ratio, r. [1]

(b) Write down the next term of the sequence, u_5. [1]

(c) Find the largest term in the sequence that is **not** an integer. [2]

(d) Find the sum of the first 20 terms. [2]

Working:

Answers:

(a) . . . . . .

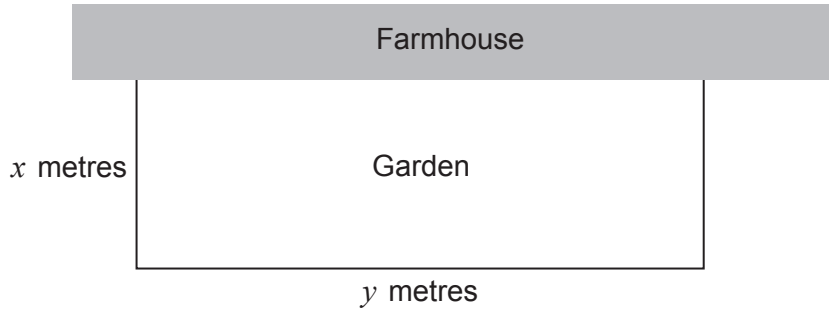
(b) . . . . . .

(c) . . . . . .

(d) . . . . . .

13. Wahanui is building a **rectangular** vegetable garden against one wall of his farmhouse. He is going to build a fence around the garden.

The total length of the fence is $60\,\text{m}$ and it will enclose the vegetable garden on three sides, as shown in the following diagram.



(a) Write down an equation for the total length of the fence, $60\,\text{m}$, **in terms of x and y**. [1]

(b) Write down the area of the garden, **in terms of x**. [2]

(c) Find the maximum area, in m^2, of the garden. [3]

Working:

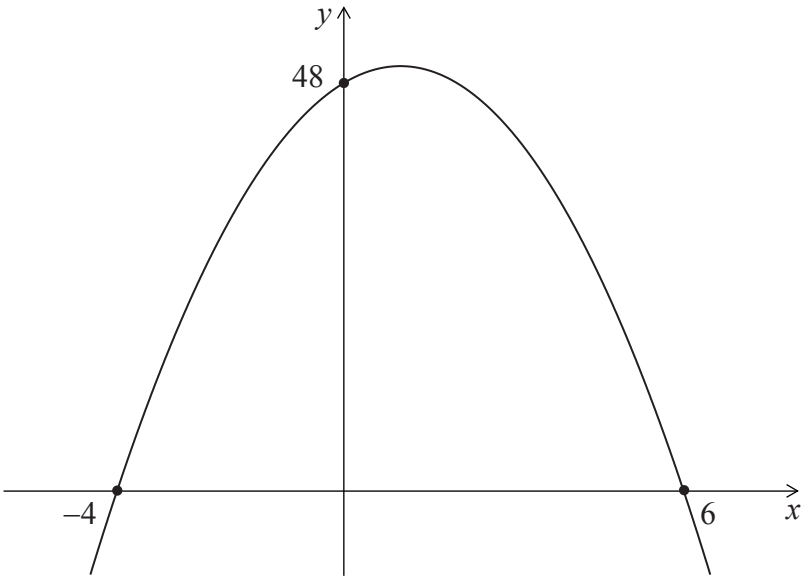
Answers:

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

14. The graph shows the curve of a quadratic function of the form $f(x) = ax^2 + bx + 48$.



(a) Write down the equation for the axis of symmetry of the curve. [2]

(b) Hence, or otherwise, find the value of a and of b. [3]

(c) Find the y-coordinate of the vertex of the curve. [1]

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

15. In an experiment, the number of bacteria, $N(t)$, is modelled by the function,

$$N(t) = 25 \times (8)^t$$

where t is the time, in hours, since the start of the experiment and $t \geq 0$.

(a) Write down the number of bacteria at the start of the experiment. [1]

(b) Find the number of bacteria after 1 hour. [1]

(c) Calculate how long it will take for the number of bacteria to exceed five million. Give your answer to the nearest hour. [4]

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

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