M15/5/MATSD/SP2/ENG/TZ1/XX/M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2015

Mathematical studies

Standard level

Paper 2

22 pages

Paper 2 Markscheme
Instructions to Examiners

Notes: **If in doubt about these instructions or any other marking issues, contact your team leader for clarification.**

1 Abbreviations

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

R Marks awarded for clear **Reasoning**

G Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, when no working shown.

AG **Answer Given** in the question and consequently, marks not awarded.

ft Marks that can be awarded as **follow through** from previous results in the question.

2 Method of Marking

(a) All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.

(b) If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the ***A0*** annotation, otherwise full annotations must be shown.

(c) Working crossed out by the candidate should not be awarded any marks.

(d) Where candidates have written two solutions to a question, only the first solution should be marked.

(e) If correct working results in a correct answer but then further working is developed, indicating a lack of mathematical understanding full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal.

Example:

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	Award the final ***(A1)*** *(ignore the further working)*
2.	$(x-6)(x+1)$	$x=6$ and -1	Do **not** award the final ***(A1)***

Example: Calculate the gradient of the line passing through the points $(5, 3)$ and $(0, 9)$.

Markscheme	Candidates' Scripts	Marking
$\frac{9-3}{0-5}$ ***(M1)*** *Award **(M1)** for correct substitution in gradient formula* $=-\frac{6}{5}$ ***(A1)***	(i) $\frac{9-3}{0-5}=-\frac{6}{5}$ Gradient is $=-\frac{6}{5}$ *(There is clear understanding of the gradient.)* $y=-\frac{6}{5}x+9$ (ii) $\frac{9-3}{0-5}=-\frac{6}{5}$ $y=-\frac{6}{5}x+9$ *(There is confusion about what is required.)*	***(M1)*** ***(A1)*** ***(M1)*** ***(A0)***

3 Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with **'(ft)'**.

(a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.

(b) If an answer resulting from follow through is extremely unrealistic (*eg*, negative distances or incorrect by large order of magnitude) then the final ***A*** mark should not be awarded.

(c) If a question is transformed by an error into a **different**, **much simpler question** then follow through may not apply.

(d) To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.

(e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**

(f) Inadvertent use of radians will be penalized the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

Markscheme	Candidates' Scripts	Marking
(a) $\frac{\sin A}{3}=\frac{\sin 30}{4}$ ***(M1)(A1)***	(a) $\frac{\sin A}{4}=\frac{\sin 30}{3}$	***(M1)(A0)***
*Award **(M1)** for substitution in sine rule formula, **(A1)** for correct substitutions.*		*(use of sine rule but with wrong values)*
$A=22.0^\circ$ (22.0243…) ***(A1)(G2)***	$A=41.8^\circ$	***(A0)***
	*(**Note:** the 2nd **(A1)** here was not marked **(ft)** and cannot be awarded because there was an earlier error in the **same** question part.)*	
(b) $x=7\tan(22.0243\ldots^\circ)$ ***(M1)***	(b) case (i) $x=7\tan 41.8^\circ$	***(M1)***
$=2.83$ (2.83163…) ***(A1)*(ft)**	$=6.26$	***(A1)*(ft)**
	but case (ii) 6.26	***(G0)***
		since no working shown

4 Using the Markscheme

(a) ***A*** marks are **dependent** on the preceding ***M*** mark being awarded, it is **not** possible to award ***(M0)(A1)***. Once an ***(M0)*** has been awarded, all subsequent ***A*** marks are lost in that part of the question, even if calculations are performed correctly, until the next ***M*** mark.
The only exception to this will be for an answer where the accuracy is specified in the question – see section 5.

(b) ***A*** marks are **dependent** on the ***R*** mark being awarded, it is **not** possible to award ***(A1)(R0)***. Hence the ***(A1)*** cannot be awarded for an answer which is correct when no reason or the wrong reason is given.

(c) In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated ***G*** in the mark scheme as an alternative to the full marks. Example ***(M1)(A1)(A1)(G2)***.

Example: Using trigonometry to calculate an angle in a triangle.

Markscheme	Candidates' Scripts	Marking
(a) $\frac{\sin A}{3}=\frac{\sin 30}{4}$ ***(M1)(A1)*** *Award **(M1)** for substitution in sine rule formula, **(A1)** for correct substitutions.* $A=22.0^\circ$ $(22.0243\ldots)$ ***(A1)(G2)***	(i) $\frac{\sin A}{3}=\frac{\sin 30}{4}$ $A=22.0^\circ$ (ii) $A=22.0^\circ$ ***Note:** **G** marks are used only if no working has been shown and the answer is correct.*	***(M1)(A1)*** ***(A1)*** ***(G2)***

(d) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.
Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' *etc*.

(e) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.
On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.
Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:
the 3 significant figure answer worked through from full calculator display;
the exact value $\left(\text{for example } \frac{2}{3} \text{ if applicable}\right)$;
the full calculator display in the form $2.83163\ldots$ as in the example above.
Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

(f) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; $1'7$; $1 \cdot 7$; $1{,}7$.

Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or $.49$.

Different descriptions of an interval: $3 < x < 5$; $(3, 5)$; $]\,3, 5\,[$.

Different forms of notation for set properties (*eg,* complement): A'; $\overline{A}$; A^c; $U - A$; $(A$; $U \setminus A$.

Different forms of logic notation: $\neg p$; p' ; $\tilde{p}$; $\overline{p}$; $\sim p$.

$p \Rightarrow q$; $p \to q$; $q \Leftarrow p$.

Significance level may be written as α .

(g) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt and exception should be raised through RM Assessor to the team leader.

As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

5 Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award ***(A1)*** and ignore subsequent rounding.

2. If the candidate's unrounded answer is **not** seen then award ***(A1)*** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise ***(A0)***.

Note: If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples f

	If candidates final answer is given …					
	Exact or to 4 or more sf (and would **round to the correct 3 sf**)	**Correct to 3 sf**	**Incorrect to 3 sf**	Correct to 2 sf [3]	Incorrect to 2 sf	Correct or incorrect to 1 sf
Unrounded answer seen[1]	Award the final ***(A1)*** irrespective of correct or incorrect rounding					
Unrounded answer not seen[2]	***(A1)***	***(A1)***	***(A0)***	***(A1)***	***(A0)***	***(A0)***
Treatment of subsequent parts	As per MS		Treat as follow through, only if working is seen.[3]			

Examples:

Markscheme	Candidates' Scripts	Marking
9.43 (9.43398…) ***(A1)***	(i) 9.43398… is seen followed by 9; 9.4; 9.43; 9.434 *etc*. (correctly rounded)	***(A1)***
	(ii) 9.43398… is seen followed by 9.433; 9.44 *etc*. (incorrectly rounded)	***(A1)***
	(iii) 9.4	***(A1)***
	(iv) 9	***(A0)*** *(correct to 1 sf)*
	(v) 9.3	***(A0)*** *(incorrectly rounded to 2 sf)*
	(vi) 9.44	***(A0)*** *(incorrectly rounded to 3 sf)*

Markscheme	Candidates' Scripts	Marking
7.44 (7.43798…) ***(A1)***	(i) 7.43798… is seen followed by 7; 7.4; 7.44; 7.438 *etc.* (correctly rounded)	***(A1)***
	(ii) 7.43798… is seen followed by 7.437; 7.43 *etc*. (incorrectly rounded)	***(A1)***
	(iii) 7.4	***(A1)***
	(iv) 7	***(A0)*** *(correct to 1 sf)*
	(v) 7.5	***(A0)*** *(incorrectly rounded to 2 sf)*
	(vi) 7.43	***(A0)*** *(incorrectly rounded to 3 sf)*

Example: ABC is a right angled triangle with angle $ABC = 90^\circ$, $AC = 32\,\text{cm}$ and $AB = 30\,\text{cm}$. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
(a) $BC = \sqrt{32^2 - 30^2}$ ***(M1)***	(a) $BC = \sqrt{32^2 - 30^2}$	***(M1)***
*Award **(M1)** for correct substitution in Pythagoras' formula*	$11\,(\text{cm})$	***(A1)***
$= 11.1\ \left(\sqrt{124}, 11.1355...\right)(\text{cm})$ ***(A1)***	*(2 sf answer only seen, but correct)*	
(b) $\text{Area} = \frac{1}{2} \times 30 \times 11.1355...$ ***(M1)***	(b) case (i) $\text{Area} = \frac{1}{2} \times 30 \times 11$	***(M1)***
*Award **(M1)** for correct substitution in area of triangle formula*	*(working shown)*	
$= 167(167.032...)(\text{cm}^2)$ ***(A1)*(ft)***	$= 165\,(\text{cm}^2)$	***(A1)*(ft)***
	case (ii) $= 165\,(\text{cm}^2)$	***(M0)(A0)*(ft)***
	(No working shown, the answer 11 is treated as a ft, so no marks awarded here)	

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

eg, Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared	(a) 7.7	***(G2)***
7.68 (7.67543…) ***(A2)***	(b) 7.67	***(G1)***
	(c) 7.6	***(G1)***
	(d) 8	***(G0)***
	(e) 7	***(G0)***
	(e) 7.66	***(G0)***

Regression line

Markscheme	Candidates' Scripts	Marking
$y = 0.888x + 13.5$ ***(A2)*** $(y = 0.887686\ldots x + 13.4895\ldots)$ If an answer is not in the form of an equation award at most ***(A1)(A0)***.	(a) $y = 0.89x + 13$	***(G2)*** *(both accepted)*
	(b) $y = 0.88x + 13$	***(G1)*** *(one rounding error)*
	(c) $y = 0.88x + 14$	***(G1)*** *(rounding error repeated)*
	(d) (i) $y = 0.9x + 13$	
	(ii) $y = 0.8x + 13$	***(G1)*** *(1 sf not accepted)*
	(e) $0.88x + 13$	***(G0)*** *(one rounding error and not an equation)*

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts	Marking
$(2.06, 4.49)$ ***(A1)(A1)*** $(2.06020\ldots, 4.49253\ldots)$	(a) $(2.1, 4.5)$	***(A1)(A1)*** *(both accepted)*
	(b) $(2.0, 4.4)$	***(A1)*** *(same rounding error twice)*
	(c) $(2.06, 4.4)$	***(A1)*** *(one rounding error)*
	(d) $(2, 4.4)$	***(A0)*** *(1sf not accepted, one rounding error)*

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. Exact answers such as $\frac{1}{4}$ can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected.

Ratios of π and answers taking the form of square roots of integers or any rational power of an integer (*eg,* $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$,) may be accepted as exact answers. All other powers (*eg,* of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding ***(M0)***.

6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final ***A*** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts	Marking
\$231.62 (231.6189) ***(A1)***	(i) 231.6	***(A0)***
	(ii) 232	***(A0)*** *(Correct rounding to incorrect level)*
	(iii) 231.61	***(A0)***
	(iv) 232.00	***(A0)*** *(Parts (iii) and (iv) are both incorrect rounding to correct level)*

7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final ***A*** mark. The markscheme will give clear instructions to ensure that only one or two mark per paper can be lost for lack of units or incorrect units.
The units are considered only when the numerical answer is awarded ***(A1)*** under the accuracy rules given in Section 5.

Example:

Markscheme	Candidates' Scripts	Marking
(a) $37000\ m^2$ ***(A1)***	(a) $36000\ m^2$	***(A0)*** *(Incorrect answer so units not considered)*
(b) $3200\ m^3$ ***(A1)***	(b) $3200\ m^2$	***(A0)*** *(Incorrect units)*

If no method is shown and the answer is correct but with incorrect or missing units award G marks with a one mark penalty.

8 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

1. (a) (i) H_0: age and opinion (about the reduction) are independent. ***(A1)***

Notes: Accept "not associated" instead of independent.

(ii) H_1: age and opinion are not independent. ***(A1)*****(ft)** ***[2 marks]***

Notes: Follow through from part (a)(i). Accept "associated" or "dependent". Award ***(A1)*****(ft)** for their correct H_1 worded consistently with their part (a)(i).

(b) 2 ***(A1)*** ***[1 mark]***

(c) $\frac{80}{130}\times\frac{35}{130}\times 130$ **OR** $\frac{80\times 35}{130}$ ***(M1)***

Note: Award ***(M1)*** for $\frac{80}{130}\times\frac{35}{130}\times 130$ **OR** $\frac{80\times 35}{130}$ seen. The following ***(A1)*** cannot be awarded without this statement.

$= 21.5384\ldots$ ***(A1)***
$= 21.5$ ***(AG)*** ***[2 marks]***

Note: Both an unrounded answer that rounds to the given answer and rounded must be seen for the ***(A1)*** to be awarded. Accept 21.54 or 21.53 as an unrounded answer.

(d) (i) χ^2 statistic $= 10.3$ $(10.3257\ldots)$ ***(G2)***

Note: Accept 10 as a correct 2 significant figure answer.

(ii) p-value $= 0.00573$ $(0.00572531\ldots)$ ***(G1)*** ***[3 marks]***

(e) since p-value < 0.01, H_0 should not be accepted ***(R1)(A1)*****(ft)**

OR

since χ^2 statistic $> \chi^2$ critical value, H_0 should not be accepted ***(R1)(A1)*****(ft)** ***[2 marks]***

Note: Do not award ***(R0)(A1)***. Follow through from their answer to part (d). Award ***(R0)(A0)*** if part (d) is unanswered. Award ***(R1)*** for a correct comparison of either their p-value to the test level or their χ^2 statistic to the χ^2 critical value, award ***(A1)*** for the correct result from that comparison.

Total [10 marks]

2. (a) $(p \wedge q) \Rightarrow r$ ***(A1)(A1)(A1)*** ***[3 marks]***

Notes: Award ***(A1)*** for conjunction seen, award ***(A1)*** for implication seen, award ***(A1)*** for correct simple propositions in correct order (the parentheses **are** required). Accept $r \Leftarrow (p \wedge q)$.

(b)

p	q	r	$(p \wedge q)$	$(p \wedge q) \Rightarrow r$
T	T	T	**T**	**T**
T	T	F	**T**	**F**
T	F	T	**F**	**T**
T	F	F	**F**	**T**
F	T	T	**F**	**T**
F	T	F	**F**	**T**
F	F	T	**F**	**T**
F	F	F	**F**	**T**

(A1)**(ft)*****(A1)*****(ft)*** ***[2 marks]***

Notes: Award ***(A1)*****(ft)** for each correct column, follow through to the final column from **their** $(p \wedge q)$ column. For the second ***(A1)*****(ft)** to be awarded there must be an implication in part (a).
Follow through from part (a).

(c) The argument is not valid since not all entries in the final column are T. ***(A1)*****(ft)*****(R1)*** ***[2 marks]***

Notes: Do not award ***(A1)*****(ft)*****(R0)***. Follow through from part (b).
Accept "The argument is not valid since $(p \wedge q) \Rightarrow r$ is not a tautology".

(d) (i) $\neg(p \wedge q) \Rightarrow \neg r$ ***(A1)*****(ft)*****(A1)*****(ft)**

OR

$(\neg p \vee \neg q) \Rightarrow \neg r$ ***(A1)*****(ft)*****(A1)*****(ft)**

Notes: Award ***(A1)*****(ft)** for the negation of their antecedent and the negation of their consequent, ***(A1)*****(ft)** for their fully correct answer. Follow through from part (a). Accept $\neg r \Leftarrow \neg(p \wedge q)$ or $\neg r \Leftarrow (\neg p \vee \neg q)$. Follow through from part (a).

continued…

Question 2 continued

(ii) if it is **not the case** that the land has been purchased **and** the building permit has been obtained then the land can **not** be used for residential purposes. ***(A1)(A1)*****(ft)**

OR

if (either) the land has **not** been purchased **or** the building permit has **not** been obtained then the land can **not** be used for residential purposes. ***(A1)(A1)*****(ft)**

[4 marks]

Notes:	Award ***(A1)*** for "if… then…" seen, ***(A1)*****(ft)** for correct statements in correct order. Follow through from part (d)(i).

Total [11 marks]

3. (a) $10\ (\text{km h}^{-1})$ ***(A1)*** ***[1 mark]***

(b) 36 ***(G2)*** ***[2 marks]***

(c) 41.5 ***(G1)*** ***[1 mark]***

(d) $41.5 - 32.5$ ***(M1)***
$= 9\ (\pm 1)$ ***(A1)*****(ft)*****(G2)*** ***[2 marks]***

Notes: Award ***(M1)*** for quartiles seen. Follow through from part (c).

(e) $120 - 110$ ***(M1)***
$= 10$ ***(A1)(G2)*** ***[2 marks]***

Note: Award ***(M1)*** for 110 seen.

(f) $p = 4 \quad q = 10$ ***(A1)*****(ft)*****(A1)*****(ft)*** ***[2 marks]***

Note: Follow through from part (e).

(g) (i) $30 < s \leq 40$ ***(A1)***

(ii) 35 ***(A1)*****(ft)*** ***[2 marks]***

Note: Follow through from part (g)(i).

(h) (i) $36.8\ (\text{km h}^{-1})\ (36.8333\ldots)$ ***(G2)*****(ft)***

Notes: Follow through from part (f).

(ii) $8.85\ (8.84904...)$ ***(G1)*****(ft)*** ***[3 marks]***

Note: Follow through from part (f), irrespective of working seen.

(i) $\frac{26}{120} \times 100$ ***(M1)***

Note: Award ***(M1)*** for $\frac{26}{120} \times 100$ seen.

$= 21.7\ (\%)\ \left(21.6666..., 21\frac{2}{3}, \frac{65}{3}\right)$ ***(A1)(G2)*** ***[2 marks]***

Total [17 marks]

4. (a) $AC^2 = 700^2 + 900^2 - 2 \times 700 \times 900 \times \cos 110^\circ$ ***(M1)(A1)***

$AC = 1315.65...$ ***(A1)(G2)***

length of course $= 2920$ (m) $(2915.65\ldots \text{m})$ ***(A1)*** ***[4 marks]***

Notes: Award ***(M1)*** for substitution into cosine rule formula, ***(A1)*** for correct substitution, ***(A1)*** for correct answer.
Award ***(G3)*** for 2920 $(2915.65\ldots)$ seen without working.
The final ***(A1)*** is awarded for adding 900 and 700 to their AC irrespective of working seen.

(b) $\dfrac{2915.65}{1.5}$ ***(M1)***

Note: Award ***(M1)*** for their length of course divided by 1.5.
Follow through from part (a).

$= 1943.76\ldots$ (seconds) ***(A1)*(ft)**

$= 32$ (minutes) ***(A1)*(ft)*(G2)*** ***[3 marks]***

Notes: Award the final ***(A1)*** for correct conversion of **their** answer in seconds to minutes, correct to the nearest minute.
Follow through from part (a).

(c) $\dfrac{700}{\sin ACB} = \dfrac{1315.65...}{\sin 110^\circ}$ ***(M1)(A1)*(ft)**

OR

$\cos ACB = \dfrac{900^2 + 1315.65...^2 - 700^2}{2 \times 900 \times 1315.65...}$ ***(M1)(A1)*(ft)**

$ACB = 30.0^\circ$ $(29.9979...^\circ)$ ***(A1)*(ft)*(G2)*** ***[3 marks]***

Notes: Award ***(M1)*** for substitution into sine rule or cosine rule formula, ***(A1)*** for their correct substitution, ***(A1)*** for correct answer.
Accept 29.9° for sine rule and 29.8° for cosine rule from use of correct three significant figure values. Follow through from their answer to (a).

continued…

Question 4 continued

(d) $\frac{1}{2}\times 700\times 900\times \sin 110^\circ$ ***(M1)(A1)***

Note: Accept $\frac{1}{2}\times$ their $\text{AC}\times 900\times \sin(\text{their ACB})$. Follow through from parts (a) and (c).

$= 296000\,\text{m}^2$ $(296003\,\text{m}^2)$ ***(A1)(G2)*** ***[3 marks]***

Notes: Award ***(M1)*** for substitution into area of triangle formula, ***(A1)*** for correct substitution, ***(A1)*** for correct answer.
Award ***(G1)*** if 296000 is seen without units or working.

(e) $\sin 29.9979... = \frac{\text{distance}}{900}$ ***(M1)***

$(\text{distance} =)\ 450\ (\text{m})\ (449.971...)$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (c).

OR

$\frac{1}{2}\times \text{distance}\times 1315.65... = 296003$ ***(M1)***

$(\text{distance} =)\ 450\ (\text{m})\ (449.971...)$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (a) and part (d).

450 is greater than 375, thus the course complies with the safety regulations ***(R1)*** ***[3 marks]***

Notes: A comparison of their area from (d) and the area resulting from the use of 375 as the perpendicular distance is a valid approach and should be given full credit. Similarly a comparison of angle ACB and $\sin^{-1}\left(\frac{375}{900}\right)$ should be given full credit.
Award ***(R0)*** for correct answer without any working seen. Award ***(R1)*(ft)*** for a justified reason consistent with their working.
Do not award ***(M0)(A0)(R1)***.

continued…

Question 4 continued

(f) $\tan 15^\circ = \frac{AH}{700}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into trig formula.

$AH = 188 \text{ (m)} \ (187.564...)$ ***(A1)*****(ft)*****(G2)*** ***[2 marks]***

(g) $HC^2 = 187.564...^2 + 1315.65...^2$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into Pythagoras, ***(A1)*** for their $1315.65...$ and their $187.564\ldots$ correctly substituted in formula.

$HC = 1330\ldots \text{ (m)} (1328.95...)$ ***(A1)*****(ft)*****(G2)*** ***[3 marks]***

Note: Follow through from their answer to parts (a) and (f).

Total [21 marks]

5. (a) $\frac{-192}{x^3}+k$ ***(A1)(A1)(A1)*** ***[3 marks]***

Note: Award ***(A1)*** for -192, ***(A1)*** for x^{-3}, ***(A1)*** for k (only).

(b) at local minimum $f'(x)=0$ ***(M1)***

Note: Award ***(M1)*** for seeing $f'(x)=0$ (may be implicit in their working).

$\frac{-192}{4^3}+k=0$ ***(A1)***

$k=3$ ***(AG)*** ***[2 marks]***

Note: Award ***(A1)*** for substituting $x=4$ in their $f'(x)$, provided it leads to $k=3$. The conclusion $k=3$ must be seen for the ***(A1)*** to be awarded.

(c) $\frac{96}{2^2}+3(2)$ ***(M1)***

Note: Award ***(M1)*** for substituting $x=2$ and $k=3$ in $f(x)$.

$=30$ ***(A1)(G2)*** ***[2 marks]***

(d) $\frac{-192}{2^3}+3$ ***(M1)***

Note: Award ***(M1)*** for substituting $x=2$ and $k=3$ in their $f'(x)$.

$=-21$ ***(A1)*****(ft)*****(G2)*** ***[2 marks]***

Note: Follow through from part (a).

continued…

Question 5 continued

(e) $y-30=\frac{1}{21}(x-2)$ ***(A1)*****(ft)*****(M1)***

Notes: Award ***(A1)*****(ft)** for their $\frac{1}{21}$ seen, ***(M1)*** for the correct substitution of their point and their normal gradient in equation of a line.
Follow through from part (c) and part (d).

OR

gradient of normal $=\frac{1}{21}$ ***(A1)*****(ft)**

$30=\frac{1}{21}\times 2+c$ ***(M1)***

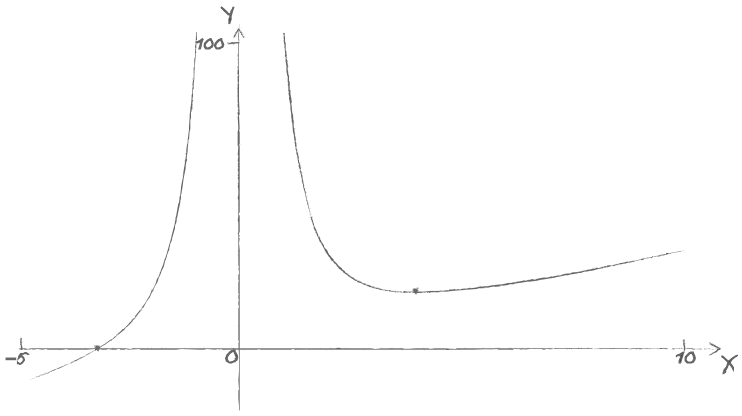
$c=29\frac{19}{21}$

$y=\frac{1}{21}x+29\frac{19}{21}\quad (y=0.0476x+29.904)$

$x-21y+628=0$ ***(A1)*****(ft)*****(G2)*** ***[3 marks]***

Notes: Accept equivalent answers.

(f)



(A1)(A1)(A1)(A1) ***[4 marks]***

Notes: Award ***(A1)*** for correct window (at least one value, other than zero, labelled on each axis), the axes must also be labelled; ***(A1)*** for a smooth curve with the correct shape (graph should not touch y-axis and should not curve away from the y-axis), on the given domain; ***(A1)*** for axis intercept in approximately the correct position (nearer -5 than zero); ***(A1)*** for local minimum in approximately the correct position (first quadrant, nearer the y-axis than $x=10$).
If there is no scale, award a maximum of ***(A0)(A1)(A0)(A1)*** – the final ***(A1)*** being awarded for the zero and local minimum in approximately correct positions relative to each other.

continued…

Question 5 continued

(g) $(-3.17, 0)$ $\left((-3.17480\ldots, 0)\right)$ ***(G1)(G1)*** ***[2 marks]***

Notes: If parentheses are omitted award ***(G0)(G1)*****(ft)**.
Accept $x = -3.17$, $y = 0$. Award ***(G1)*** for -3.17 seen.

(h) $0 < x \leq 4$ or $0 < x < 4$ ***(A1)(A1)*** ***[2 marks]***

Notes: Award ***(A1)*** for correct end points of interval, ***(A1)*** for correct notation (note: lower inequality must be strict).
Award a maximum of ***(A1)(A0)*** if y or $f(x)$ used in place of x.

Total [20 marks]

6. (a) the temperature of the water cannot fall below room temperature ***(R1)***

an (informal) explanation that as $m \to \infty$, $k^{-m} \to 0$ ***(R1)***

OR

recognition that there is a horizontal asymptote at $y = a$ ***(R1)*** ***[2 marks]***

Note: Award ***(R1)*** for a contextual reason involving room temperature. Award ***(R1)*** for a mathematical reason similar to one of the two alternatives.

(b) $100 = 20 + b(k^0)$ ***(M1)***

Note: Award ***(M1)*** for substituting 100, 20 and 0.

$b = 80$ ***(A1)(G2)*** ***[2 marks]***

Note: The ***(A1)*** is awarded only if all working seen is consistent with the final answer of 80.

(c) $84 = 20 + 80k^{-1}$ ***(M1)***

Note: Substituting $k = 1.25$ at any stage is an invalid method and is awarded ***(M0)(M0)***. Award ***(M1)*** for correctly substituting 84, 20 and their 80.

$\frac{64}{80} = k^{-1}$ ***(M1)***

$k = 1.25$ ***(AG)*** ***[2 marks]***

Note: Award ***(M1)*** for correct rearrangement that isolates k; $k = 1.25$ must be consistent with their working **and** the conclusion $k = 1.25$ must be seen.

(d) $T = 20 + 80(1.25^{-3})$ ***(M1)***

Note: Award ***(M1)*** for their correct substitutions into T. Follow through from part (b) and $k = 1.25$.

$T = 61.0$ (60.96) ***(A1)*****(ft)*****(G2)*** ***[2 marks]***

(e) $35 = 20 + 80(1.25^{-m})$ ***(M1)***

Note: Award ***(M1)*** for their correct substitutions into T. Follow through from part (b). Accept graphical solutions. Award ***(M1)*** for sketch of function.

$(m =)$ 7.50 (minutes) $(7.50179...)$ ***(A1)*****(ft)*****(G2)***

7 minutes and 30 seconds ***(A1)*** ***[3 marks]***

Note: Award the final ***(A1)*** for correct conversion of **their** m in minutes to minutes and seconds, but only if answer in minutes is explicitly shown.

Total [11 marks]