Diploma Programme
Programme du diplôme
Programa del Diploma

International Baccalaureate®
Baccalauréat International
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Mathematical studies
Standard level
Paper 2

Tuesday 14 May 2019 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

7 pages

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 15]

Sila High School has 110 students. They each take exactly one language class from a choice of English, Spanish or Chinese. The following table shows the number of female and male students in the three different language classes.

		Language Choice			
		English	**Spanish**	**Chinese**	**Total**
Student Gender	**Female**	25	8	10	43
	Male	21	14	32	67
	Total	46	22	42	110

A χ^2 test was carried out at the 5% significance level to analyse the relationship between gender and student choice of language class.

(a) Write down the null hypothesis, H_0, for this test. [1]

(b) State the number of degrees of freedom. [1]

(c) Use your graphic display calculator to write down

   (i) the expected frequency of female students who chose to take the Chinese class;

   (ii) the χ^2 statistic. [3]

The critical value at the 5% significance level for this test is 5.99.

(d) State whether or not H_0 should be rejected. Justify your statement. [2]

(e) One student is chosen at random from this school.

   (i) Find the probability that the student does not take the Spanish class.

   Another student is chosen at random from this school.

   (ii) Find the probability that neither of the two students take the Spanish class.

   (iii) Find the probability that at least one of the two students is female. [8]

2. [Maximum mark: 13]

An archaeological site is to be made accessible for viewing by the public. To do this, archaeologists built two straight paths from point A to point B and from point B to point C as shown in the following diagram. The length of path AB is 185 m, the length of path BC is 250 m, and angle AB̂C is 125°.

diagram not to scale
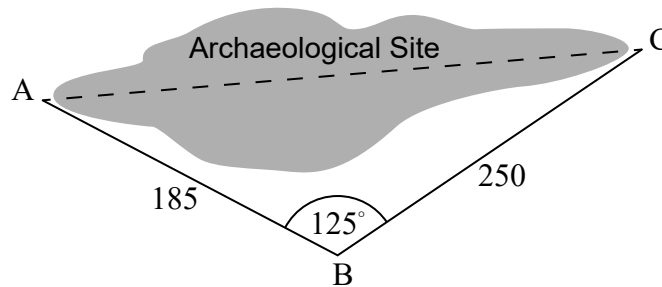
(a) Find the distance from A to C. [3]

The archaeologists plan to build two more straight paths, AD and DC. For the paths to go around the site, angle BÂD is to be made equal to 85° and angle BĈD is to be made equal to 70° as shown in the following diagram.

diagram not to scale
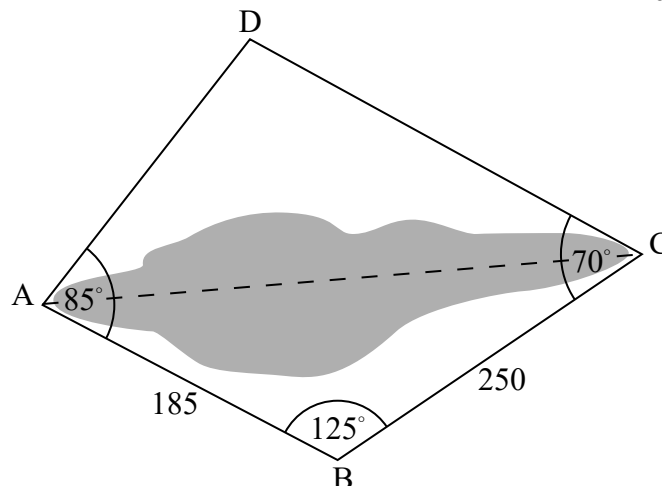
(b) Find the size of

    (i) angle BÂC;

    (ii) angle CÂD. [4]

(c) Find the size of angle AĈD. [2]

The length of path AD is 287 m.

(d) Find the area of the region ABCD. [4]

3.   [Maximum mark: 15]

A factory packages coconut water in cone-shaped containers with a base radius of $5.2\,\text{cm}$ and a height of $13\,\text{cm}$.

(a)   Find the volume of one cone-shaped container.   [2]

(b)   Find the slant height of the cone-shaped container.   [2]

(c)   Show that the total surface area of the cone-shaped container is $314\,\text{cm}^2$, correct to three significant figures.   [3]

The factory designers are currently investigating whether a cone-shaped container can be replaced with a cylinder-shaped container with the same radius and the same total surface area.

(d)   Find the height, h, of this cylinder-shaped container.   [4]

The factory director wants to increase the volume of coconut water sold per container.

(e)   State whether or not they should replace the cone-shaped containers with cylinder-shaped containers. Justify your conclusion.   [4]

4.   [Maximum mark: 16]

On her first day in a hospital, Kiri receives u_1 milligrams (mg) of a therapeutic drug. The amount of the drug Kiri receives increases by the same amount, d, each day. On the seventh day, she receives $21\,\text{mg}$ of the drug, and on the eleventh day she receives $29\,\text{mg}$.

(a)   Write down an equation, in terms of u_1 and d, for the amount of the drug that she receives

   (i)   on the seventh day;

   (ii)   on the eleventh day.   [2]

(b)   Write down the value of d and the value of u_1.   [2]

Kiri receives the drug for 30 days.

(c)   Calculate the total amount of the drug, in mg, that she receives.   [3]

Ted is also in a hospital and on his first day he receives a $20\,\text{mg}$ antibiotic injection. The amount of the antibiotic Ted receives decreases by 50% each day. On the second day, Ted receives a $10\,\text{mg}$ antibiotic injection, on the third day he receives $5\,\text{mg}$, and so on.

(d)   (i)   Find the amount of antibiotic, in mg, that Ted receives on the fifth day.

   (ii)   The daily amount of antibiotic Ted receives will first be less than $0.06\,\text{mg}$ on the kth day. Find the value of k.

   (iii) Hence find the total amount of antibiotic, in mg, that Ted receives during the first k days.   [9]

Turn over

5.   [Maximum mark: 20]

Consider the function $f(x) = \dfrac{1}{3}x^3 + \dfrac{3}{4}x^2 - x - 1$.

(a)   Find $f(2)$. [2]

(b)   Write down the y-intercept of the graph of $y = f(x)$. [1]

(c)   Sketch the graph of $y = f(x)$ for $-3 \le x \le 3$ and $-4 \le y \le 12$. [4]

(d)   Find $f'(x)$. [3]

(e)   Find the gradient of the graph of $y = f(x)$ at $x = 2$. [2]

(f)   Find the equation of the tangent line to the graph of $y = f(x)$ at $x = 2$. Give the
equation in the form $ax + by + d = 0$ where a , b and $d \in \mathbb{Z}$. [2]

The function has one local maximum at $x = p$ and one local minimum at $x = q$.

(g)   Use the derivative of $f(x)$ to find the value of p and the value of q . [3]

(h)   Determine the range of $f(x)$ for $p \le x \le q$. [3]

6.   [Maximum mark: 11]

Tommaso plans to compete in a regional bicycle race after he graduates, however he needs to buy a racing bicycle. He finds a bicycle that costs 1100 euro (EUR). Tommaso has 950 EUR and invests this money in an account that pays 5% interest per year, **compounded monthly**.

(a)   Determine the amount that he will have in his account after 3 years. Give your answer correct to two decimal places.   [3]

The cost of the bicycle, C, can be modelled by $C = 20x + 1100$, where x is the number of years since Tommaso invested his money.

(b)   Find the difference between the cost of the bicycle and the amount of money in Tommaso's account after 3 years. Give your answer correct to two decimal places.   [3]

After m complete **months** Tommaso will, for the first time, have enough money in his account to buy the bicycle.

(c)   Find the value of m.   [5]