N15/5/MATHL/HP2/ENG/TZ0/XX

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Thursday 12 November 2015 (afternoon)

2 hours

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks]**.

16 pages

8815 – 7202

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

The events A and B are such that $\mathrm{P}(A) = 0.65$, $\mathrm{P}(B) = 0.48$ and $\mathrm{P}(A \cup B) = 0.818$.

(a) Find $\mathrm{P}(A \cap B)$. [2]

(b) Hence show that the events A and B are independent. [2]

2. [Maximum mark: 4]

The three planes having Cartesian equations $2x + 3y - z = 11$, $x + 2y + z = 3$ and $5x - y - z = 10$ meet at a point P. Find the coordinates of P.

3. [Maximum mark: 6]

The data of the goals scored by players in a football club during a season are given in the following table.

Goals	Frequency
0	4
1	k
2	3
3	2
4	3
8	1

(a) Given that the mean number of goals scored per player is 1.95, find the value of k. [3]

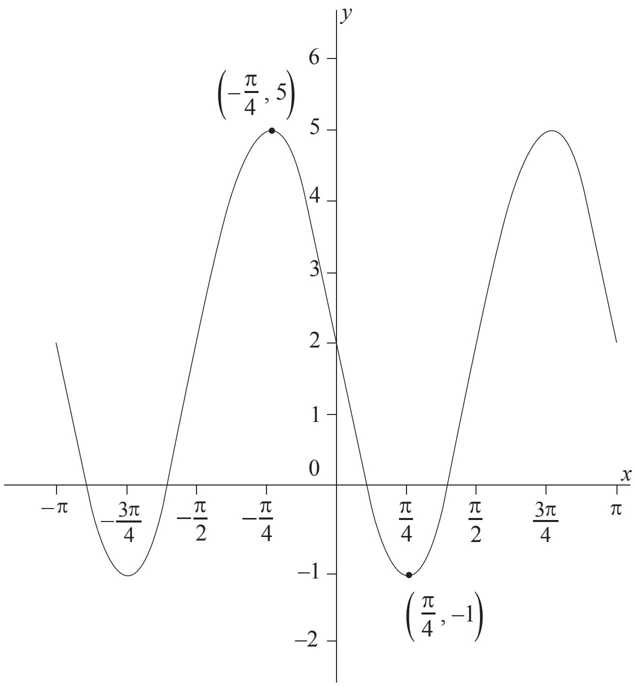
It is discovered that there is a mistake in the data and that the top scorer, who scored 22 goals, has not been included in the table.

(b) (i) Find the correct mean number of goals scored per player.

(ii) Find the correct standard deviation of the number of goals scored per player. [3]

4. [Maximum mark: 6]

A function is defined by $f(x) = A\sin(Bx) + C$, $-\pi \le x \le \pi$, where $A, B, C \in \mathbb{Z}$. The following diagram represents the graph of $y = f(x)$.



(a) Find the value of

(i) A;

(ii) B;

(iii) C. [4]

(b) Solve $f(x) = 3$ for $0 \le x \le \pi$. [2]

5. [Maximum mark: 6]

A function is defined by $f(x) = x^2 + 2$, $x \geq 0$. A region R is enclosed by $y = f(x)$, the y-axis and the line $y = 4$.

(a) (i) Express the area of the region R as an integral with respect to y.

(ii) Determine the area of R, giving your answer correct to four significant figures. [3]

(b) Find the exact volume generated when the region R is rotated through 2π radians about the y-axis. [3]

6. [Maximum mark: 6]

Josie has three ways of getting to school. 30% of the time she travels by car, 20% of the time she rides her bicycle and 50% of the time she walks.
When travelling by car, Josie is late 5% of the time. When riding her bicycle she is late 10% of the time. When walking she is late 25% of the time. Given that she was on time, find the probability that she rides her bicycle.

7. [Maximum mark: 6]

Triangle ABC has area $21\,\mathrm{cm}^2$. The sides AB and AC have lengths $6\,\mathrm{cm}$ and $11\,\mathrm{cm}$ respectively. Find the two possible lengths of the side BC.

8. [Maximum mark: 6]

The continuous random variable X has the probability distribution function $f(x) = A\sin(\ln(x))$, $1 \le x \le 5$.

(a) Find the value of A to three decimal places. [2]

(b) Find the mode of X. [2]

(c) Find the value $\mathrm{P}(X \le 3 \mid X \ge 2)$. [2]

9. [Maximum mark: 8]

A particle can move along a straight line from a point O. The velocity v, in m s^{-1}, is given by the function $v(t) = 1 - e^{-\sin t^2}$ where time $t \ge 0$ is measured in seconds.

(a) Write down the first two times $t_1, t_2 > 0$, when the particle changes direction. [2]

(b) (i) Find the time $t < t_2$ when the particle has a maximum velocity.

(ii) Find the time $t < t_2$ when the particle has a minimum velocity. [4]

(c) Find the distance travelled by the particle between times $t = t_1$ and $t = t_2$. [2]

Please **do not** write on this page.

Answers written on this page will not be marked.

10. [Maximum mark: 8]

Ed walks in a straight line from point $\mathrm{P}(-1, 4)$ to point $\mathrm{Q}(4, 16)$ with constant speed. Ed starts from point P at time $t = 0$ and arrives at point Q at time $t = 3$, where t is measured in hours.
Given that, at time t, Ed's position vector, relative to the origin, can be given in the form, $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$,

(a) find the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. [3]

Roderick is at a point $\mathrm{C}(11, 9)$. During Ed's walk from P to Q Roderick wishes to signal to Ed. He decides to signal when Ed is at the closest point to C.

(b) Find the time when Roderick signals to Ed. [5]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 18]

A survey is conducted in a large office building. It is found that 30% of the office workers weigh less than 62 kg and that 25% of the office workers weigh more than 98 kg.
The weights of the office workers may be modelled by a normal distribution with mean μ and standard deviation σ.

(a) (i) Determine two simultaneous linear equations satisfied by μ and σ.

(ii) Find the values of μ and σ. [6]

(b) Find the probability that an office worker weighs more than 100 kg. [1]

There are elevators in the office building that take the office workers to their offices.
Given that there are 10 workers in a particular elevator,

(c) find the probability that at least four of the workers weigh more than 100 kg. [2]

Given that there are 10 workers in an elevator and at least one weighs more than 100 kg,

(d) find the probability that there are fewer than four workers exceeding 100 kg. [3]

The arrival of the elevators at the ground floor between $08{:}00$ and $09{:}00$ can be modelled by a Poisson distribution. Elevators arrive on average every 36 seconds.

(e) Find the probability that in any half hour period between $08{:}00$ and $09{:}00$ more than 60 elevators arrive at the ground floor. [3]

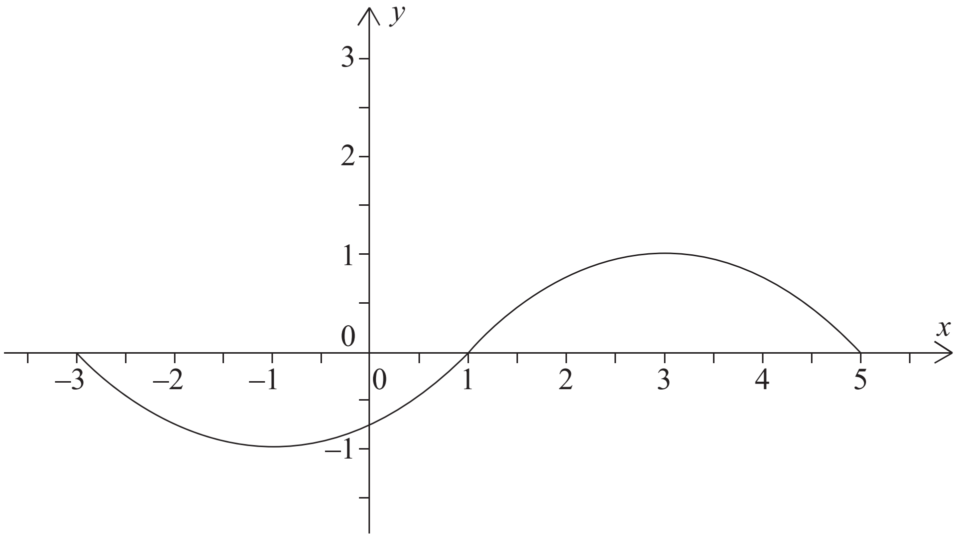
An elevator can take a maximum of 10 workers. Given that 400 workers arrive in a half hour period independently of each other,

(f) find the probability that there are sufficient elevators to take them to their offices. [3]

Do **not** write solutions on this page.

12. [Maximum mark: 21]

The following graph represents a function $y = f(x)$, where $-3 \le x \le 5$.
The function has a maximum at $(3, 1)$ and a minimum at $(-1, -1)$.



(a) The functions u and v are defined as $u(x) = x - 3$, $v(x) = 2x$ where $x \in \mathbb{R}$.

(i) State the range of the function $u \circ f$.

(ii) State the range of the function $u \circ v \circ f$.

(iii) Find the largest possible domain of the function $f \circ v \circ u$. [7]

(b) (i) Explain why f does not have an inverse.

(ii) The domain of f is restricted to define a function g so that it has an inverse g^{-1}. State the largest possible domain of g.

(iii) Sketch a graph of $y = g^{-1}(x)$, showing clearly the y-intercept and stating the coordinates of the endpoints. [6]

Consider the function defined by $h(x) = \dfrac{2x-5}{x+d}$, $x \ne -d$ and $d \in \mathbb{R}$.

(c) (i) Find an expression for the inverse function $h^{-1}(x)$.

(ii) Find the value of d such that h is a self-inverse function.

For this value of d, there is a function k such that $h \circ k(x) = \dfrac{2x}{x+1}$, $x \ne -1$.

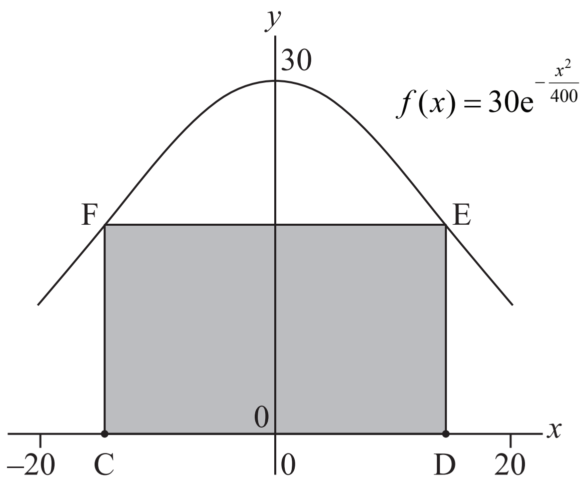
(iii) Find $k(x)$. [8]

Do **not** write solutions on this page.

13. [Maximum mark: 21]

The following diagram shows a vertical cross section of a building. The cross section of the roof of the building can be modelled by the curve $f(x) = 30\mathrm{e}^{-\frac{x^2}{400}}$, where $-20 \le x \le 20$.

Ground level is represented by the x-axis.



(a) Find $f''(x)$. [4]

(b) Show that the gradient of the roof function is greatest when $x = -\sqrt{200}$. [3]

The cross section of the living space under the roof can be modelled by a rectangle CDEF with points $\mathrm{C}(-a, 0)$ and $\mathrm{D}(a, 0)$, where $0 < a \le 20$.

(c) Show that the maximum area A of the rectangle CDEF is $600\sqrt{2}\mathrm{e}^{-\frac{1}{2}}$. [5]

(d) A function I is known as the Insulation Factor of CDEF. The function is defined as $I(a) = \frac{P(a)}{A(a)}$ where P = Perimeter and A = Area of the rectangle.

(i) Find an expression for P in terms of a.

(ii) Find the value of a which minimizes I.

(iii) Using the value of a found in part (ii) calculate the percentage of the cross sectional area under the whole roof that is not included in the cross section of the living space. [9]