N18/5/MATHL/HP2/ENG/TZ0/XX

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Tuesday 13 November 2018 (morning)

2 hours

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

14 pages

8818 – 7202

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Consider a geometric sequence with a first term of 4 and a fourth term of -2.916.

(a) Find the common ratio of this sequence. [3]

(b) Find the sum to infinity of this sequence. [2]

2. [Maximum mark: 7]

A function f satisfies the conditions $f(0) = -4$, $f(1) = 0$ and its second derivative is $f''(x) = 15\sqrt{x} + \frac{1}{(x+1)^2}$, $x \geq 0$.

Find $f(x)$.

3. [Maximum mark: 8]

It is known that 56 % of Infiglow batteries have a life of less than 16 hours, and 94 % have a life less than 17 hours. It can be assumed that battery life is modelled by the normal distribution $N(\mu, \sigma^2)$.

(a) Find the value of μ and the value of σ. [6]

(b) Find the probability that a randomly selected Infiglow battery will have a life of at least 15 hours. [2]

4. [Maximum mark: 5]

Find the value of the constant term in the expansion of $x^4\left(x+\dfrac{3}{x^2}\right)^5$.

5. [Maximum mark: 5]

Differentiate from first principles the function $f(x) = 3x^3 - x$.

6. [Maximum mark: 6]

Let $P(x) = 2x^4 - 15x^3 + ax^2 + bx + c$, where $a, b, c \in \mathbb{R}$.

(a) Given that $(x - 5)$ is a factor of $P(x)$, find a relationship between a, b and c. [2]

(b) Given that $(x - 5)^2$ is a factor of $P(x)$, write down the value of $P'(5)$. [1]

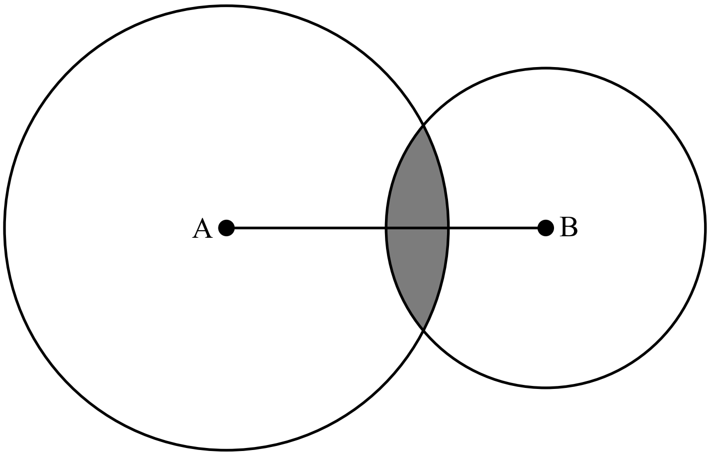
(c) Given that $(x - 5)^2$ is a factor of $P(x)$, and that $a = 2$, find the values of b and c. [3]

Please **do not** write on this page.

Answers written on this page
will not be marked.

7. [Maximum mark: 6]

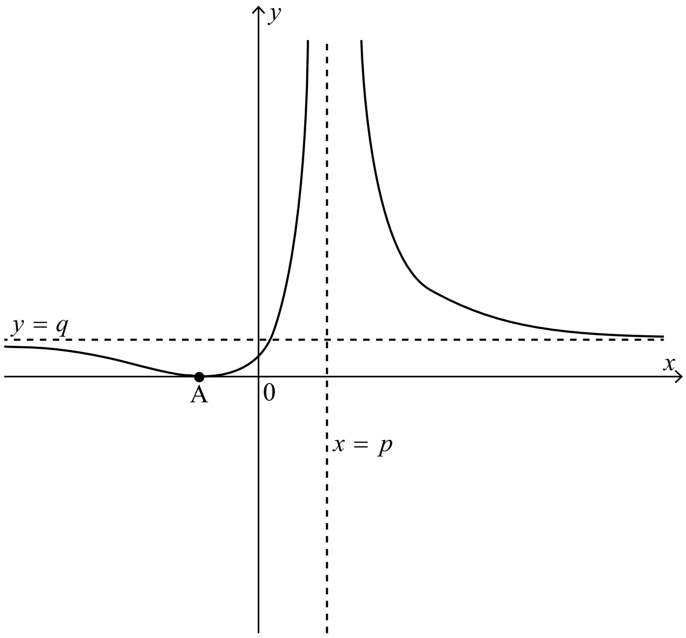
Boat A is situated 10km away from boat B, and each boat has a marine radio transmitter on board. The range of the transmitter on boat A is 7km, and the range of the transmitter on boat B is 5km. The region in which both transmitters can be detected is represented by the shaded region in the following diagram. Find the area of this region.



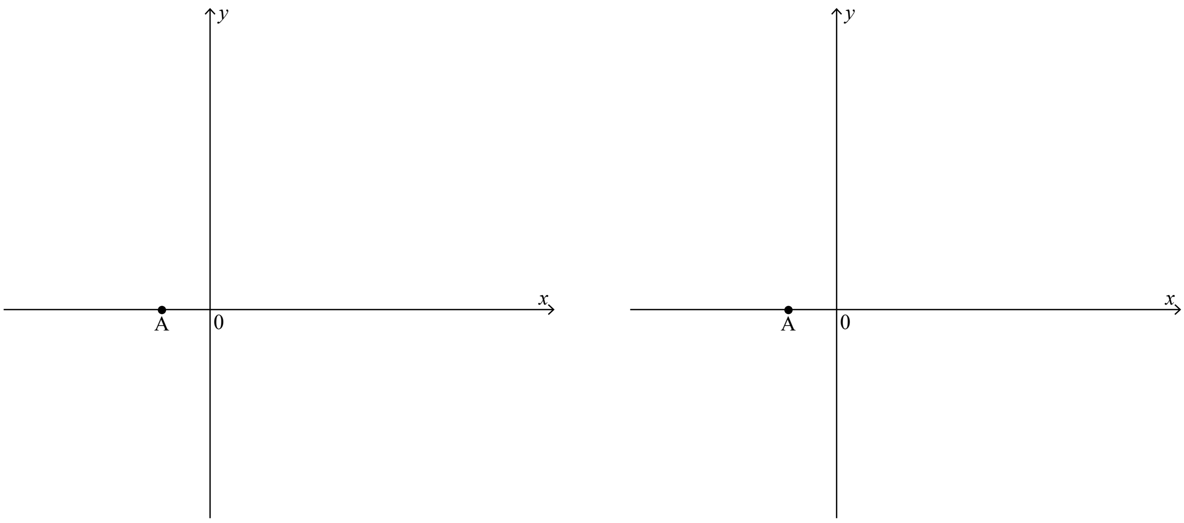
8. [Maximum mark: 8]

Consider the function $f(x) = \dfrac{ax + 1}{bx + c}$, $x \neq -\dfrac{c}{b}$, where $a, b, c \in \mathbb{Z}$.

The following graph shows the curve $y = (f(x))^2$. It has asymptotes at $x = p$ and $y = q$ and meets the x-axis at A.



(a) On the following axes, sketch the two possible graphs of $y = f(x)$ giving the equations of any asymptotes in terms of p and q. [4]



(This question continues on the following page)

(Question 8 continued)

(b) Given that $p = \frac{4}{3}$, $q = \frac{4}{9}$ and A has coordinates $\left(-\frac{1}{2}, 0\right)$, determine the possible sets of values for a, b and c. [4]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 19]

The function f is defined by $f(x) = \dfrac{2\ln x + 1}{x - 3}$, $0 < x < 3$.

(a) Find $f'(x)$. [4]

(b) Hence, or otherwise, find the coordinates of the point of inflexion on the graph of $y = f(x)$. [4]

(c) Draw a set of axes showing x and y values between -3 and 3. On these axes

(i) sketch the graph of $y = f(x)$, showing clearly any axis intercepts and giving the equations of any asymptotes.

(ii) sketch the graph of $y = f^{-1}(x)$, showing clearly any axis intercepts and giving the equations of any asymptotes. [8]

(d) Hence, or otherwise, solve the inequality $f(x) > f^{-1}(x)$. [3]

Do **not** write solutions on this page.

10. [Maximum mark: 18]

Willow finds that she receives approximately 70 emails per working day.
She decides to model the number of emails received per working day using the random variable X, where X follows a Poisson distribution with mean 70.

(a) Using this distribution model, find

(i) $\mathrm{P}(X < 60)$

(ii) the standard deviation of X. [4]

(b) In order to test her model, Willow records the number of emails she receives per working day over a period of 6 months. The results are shown in the following table.

Number of emails received (x)	**Number of days**
$40 \le x \le 49$	2
$50 \le x \le 59$	15
$60 \le x \le 69$	40
$70 \le x \le 79$	53
$80 \le x \le 89$	0
$90 \le x \le 99$	1
$100 \le x \le 109$	3
$110 \le x \le 119$	6

From the table, calculate

(i) an estimate for the mean number of emails received per working day;

(ii) an estimate for the standard deviation of the number of emails received per working day. [5]

(c) Give one piece of evidence that suggests Willow's Poisson distribution model is not a good fit. [1]

Archie works for a different company and knows that he receives emails according to a Poisson distribution, with a mean of λ emails per day.

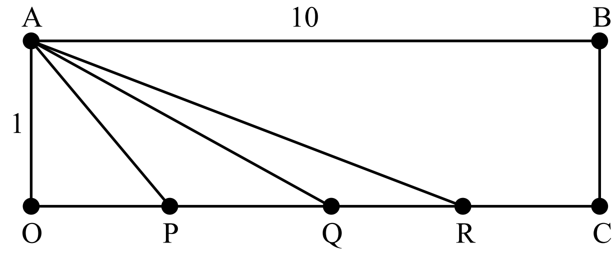
(d) Suppose that the probability of Archie receiving more than 10 emails in total on any one day is 0.99. Find the value of λ. [3]

(e) Now suppose that Archie received exactly 20 emails in total in a consecutive two day period. Show that the probability that he received exactly 10 of them on the first day is independent of λ . [5]

Do **not** write solutions on this page.

11. [Maximum mark: 13]

Consider the rectangle OABC such that $AB = OC = 10$ and $BC = OA = 1$, with the points P, Q and R placed on the line OC such that $OP = p$, $OQ = q$ and $OR = r$, such that $0 < p < q < r < 10$.



Let θ_p be the angle APO, θ_q be the angle AQO and θ_r be the angle ARO.

(a) Find an expression for θ_p in terms of p. [3]

Consider the case when $\theta_p = \theta_q + \theta_r$ and $QR = 1$.

(b) Show that $p = \dfrac{q^2 + q - 1}{2q + 1}$. [6]

(c) By sketching the graph of p as a function of q, determine the range of values of p for which there are possible values of q. [4]

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