Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Wednesday 13 May 2015 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2215–7304

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 7]

The following table shows the average number of hours per day spent watching television by seven mothers and each mother's youngest child.

Hours per day that a mother watches television (x)	2.5	3.0	3.2	3.3	4.0	4.5	5.8
Hours per day that her child watches television (y)	1.8	2.2	2.6	2.5	3.0	3.2	3.5

The relationship can be modelled by the regression line with equation $y = ax + b$.

(a) (i) Find the correlation coefficient.

(ii) Write down the value of a and of b. [4]

Elizabeth watches television for an average of 3.7 hours per day.

(b) Use your regression line to predict the average number of hours of television watched per day by Elizabeth's youngest child. Give your answer correct to one decimal place. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. [Maximum mark: 5]

Consider the expansion of $(2x + 3)^8$.

(a) Write down the number of terms in this expansion. [1]

(b) Find the term in x^3. [4]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. [Maximum mark: 6]

In an arithmetic sequence $u_{10} = 8$, $u_{11} = 6.5$.

(a) Write down the value of the common difference. [1]

(b) Find the first term. [3]

(c) Find the sum of the first 50 terms of the sequence. [2]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

4. [Maximum mark: 7]

Let $f(x) = \frac{2x-6}{1-x}$, for $x \neq 1$.

(a) For the graph of f

(i) find the x-intercept;

(ii) write down the equation of the vertical asymptote;

(iii) find the equation of the horizontal asymptote. [5]

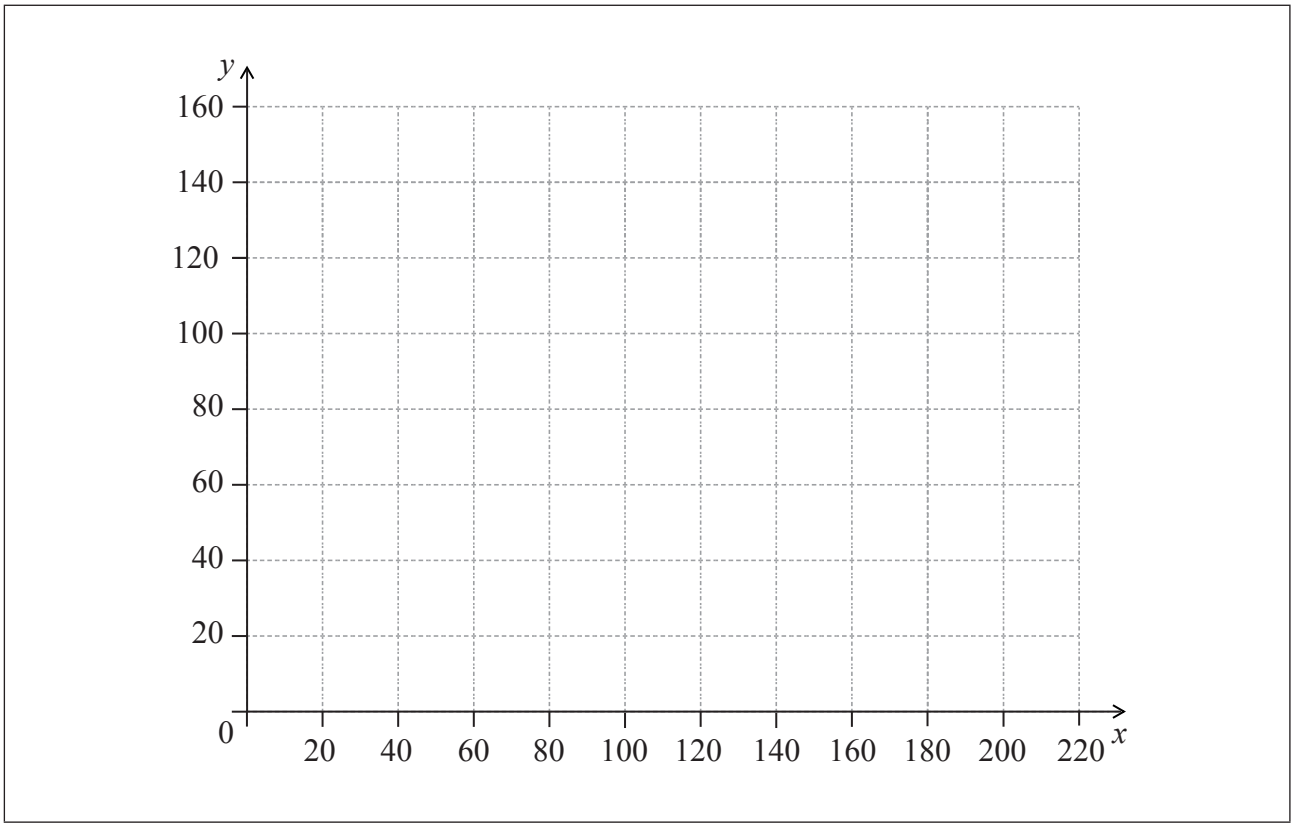
(b) Find $\lim_{x \to \infty} f(x)$. [2]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

5. [Maximum mark: 6]

Let $G(x) = 95e^{(-0.02x)} + 40$, for $20 \leq x \leq 200$.

(a) On the following grid, sketch the graph of G. [3]



(b) Robin and Pat are planning a wedding banquet. The cost per guest, G dollars, is modelled by the function $G(n) = 95e^{(-0.02n)} + 40$, for $20 \leq n \leq 200$, where n is the number of guests.

Calculate the **total** cost for 45 guests. [3]

(This question continues on the following page)

(Question 5 continued)

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

6. [Maximum mark: 7]

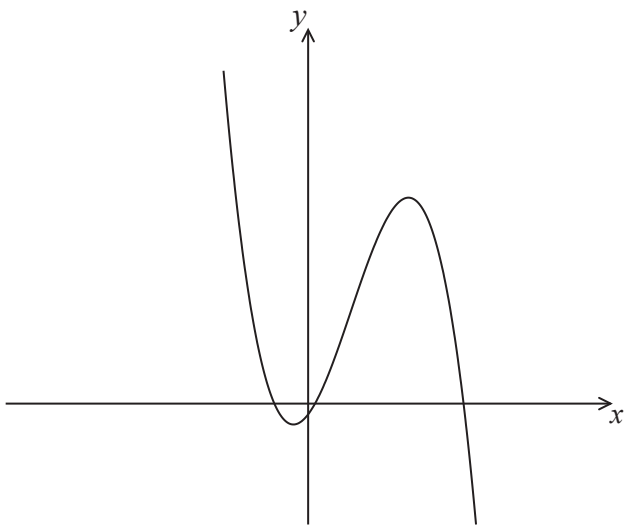
Let $f(x) = \frac{\ln(4x)}{x}$, for $0 < x \leq 5$.

Points $\mathrm{P}(0.25, 0)$ and Q are on the curve of f. The tangent to the curve of f at P is perpendicular to the tangent at Q. Find the coordinates of Q.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. [Maximum mark: 7]

The following diagram shows part of the graph of $f(x) = -2x^3 + 5.1x^2 + 3.6x - 0.4$.



(a) Find the coordinates of the local minimum point. [2]

(b) The graph of f is translated to the graph of g by the vector $\begin{pmatrix} 0 \\ k \end{pmatrix}$. Find all values of k so that $g(x) = 0$ has exactly one solution. [5]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

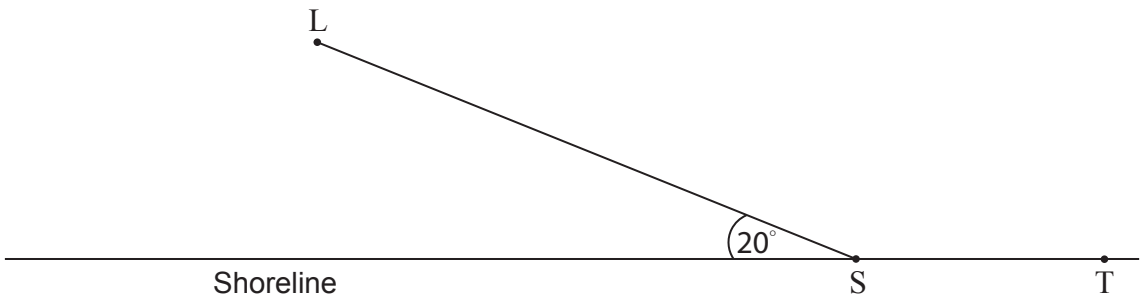
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

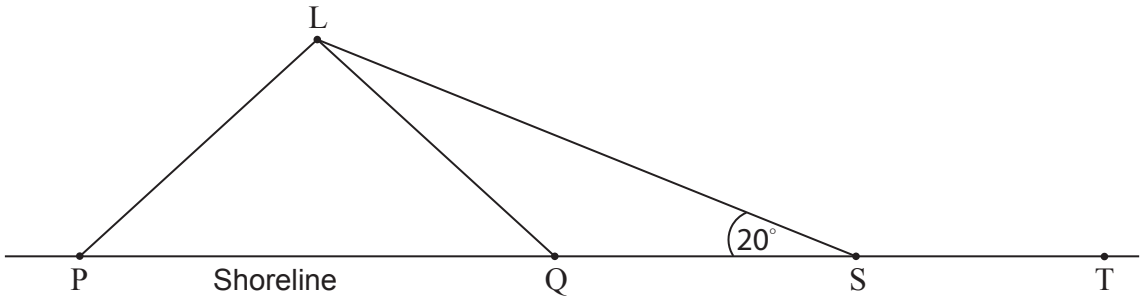
The following diagram shows a straight shoreline, with a supply store at S, a town at T, and an island L.



A boat delivers supplies to the island. The boat leaves S, and sails to the island. Its path makes an angle of 20° with the shoreline.

(a) The boat sails at $6\,\text{km}$ per hour, and arrives at L after 1.5 hours. Find the distance from S to L. [2]

It is decided to change the position of the supply store, so that its distance from L is $5\,\text{km}$. The following diagram shows the two possible locations P and Q for the supply store.



(b) Find the size of $\text{S}\hat{\text{P}}\text{L}$ and of $\text{S}\hat{\text{Q}}\text{L}$. [5]

(c) The town wants the new supply store to be as near as possible to the town.

(i) State which of the points P or Q is chosen for the new supply store.

(ii) Hence find the distance between the old supply store and the new one. [6]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

A company makes containers of yogurt. The volume of yogurt in the containers is normally distributed with a mean of $260\,\text{ml}$ and standard deviation of $6\,\text{ml}$.

A container which contains less than $250\,\text{ml}$ of yogurt is **underfilled**.

(a) A container is chosen at random. Find the probability that it is underfilled. [2]

The company decides that the probability of a container being underfilled should be reduced to 0.02. It decreases the standard deviation to σ and leaves the mean unchanged.

(b) Find σ. [4]

The company changes to the new standard deviation, σ, and leaves the mean unchanged. A container is chosen at random for inspection. It passes inspection if its volume of yogurt is between 250 and $271\,\text{ml}$.

(c) (i) Find the probability that it passes inspection.

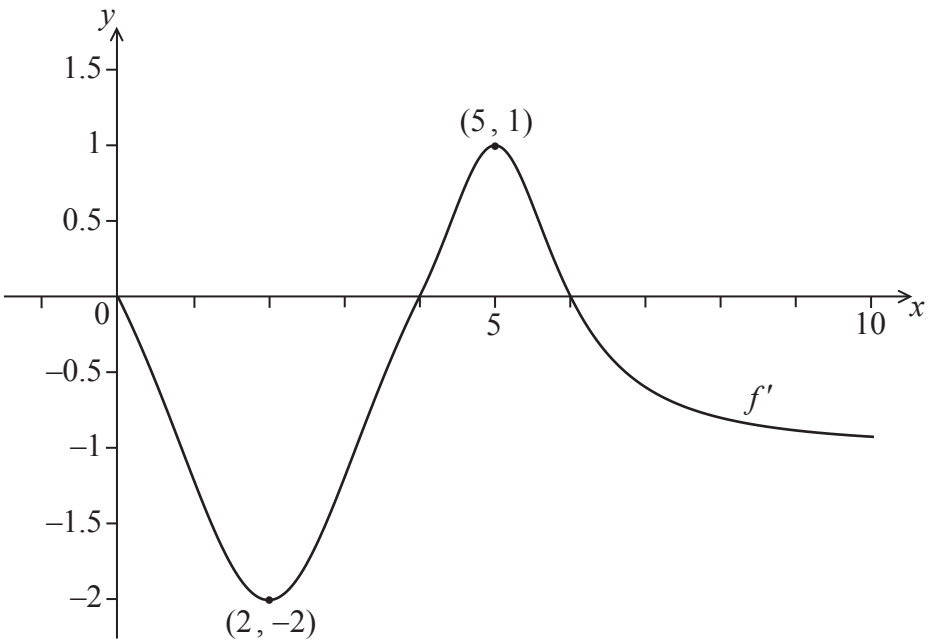
(ii) Given that the container is **not** underfilled, find the probability that it passes inspection. [6]

(d) A sample of 50 containers is chosen at random. Find the probability that 48 or more of the containers pass inspection. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 16]

Consider a function f, for $0 \le x \le 10$. The following diagram shows the graph of f', the derivative of f.



The graph of f' passes through $(2, -2)$ and $(5, 1)$, and has x-intercepts at $0, 4$ and 6.

(a) The graph of f has a local maximum point when $x = p$. State the value of p, and justify your answer . [3]

(b) Write down $f'(2)$. [1]

Let $g(x) = \ln\big(f(x)\big)$ and $f(2) = 3$.

(c) Find $g'(2)$. [4]

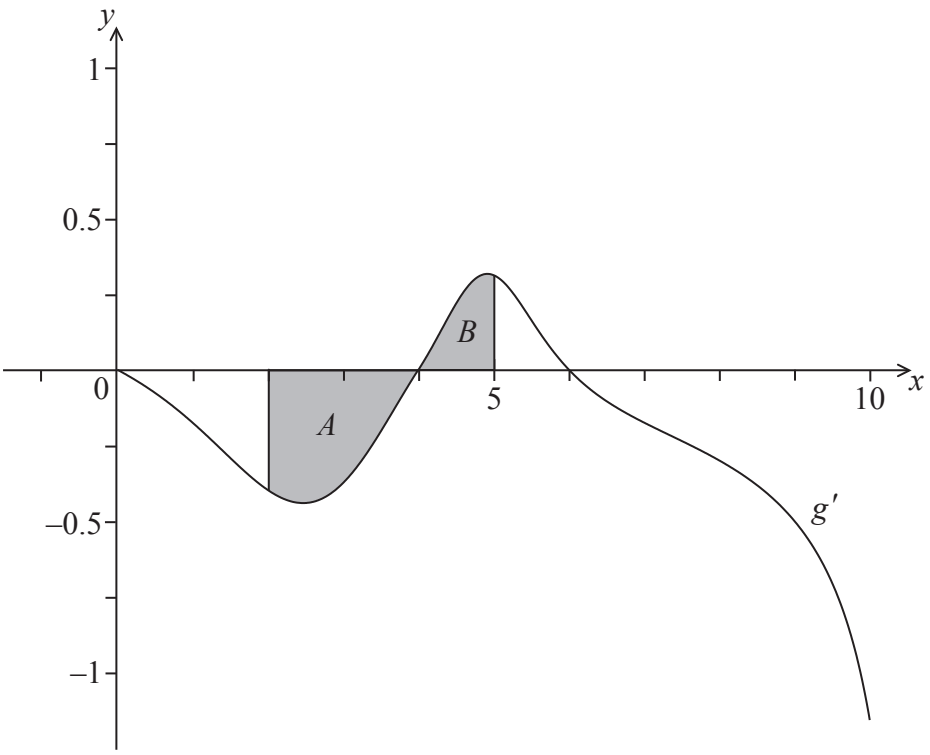
(d) Verify that $\ln 3 + \int_2^a g'(x)\,\mathrm{d}x = g(a)$, where $0 \le a \le 10$. [4]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 10 continued)

(e) The following diagram shows the graph of g', the derivative of g.



The shaded region A is enclosed by the curve, the x-axis and the line $x = 2$, and has area 0.66 units^2.

The shaded region B is enclosed by the curve, the x-axis and the line $x = 5$, and has area 0.21 units^2.

Find $g(5)$. [4]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.