M15/5/MATME/SP2/ENG/TZ2/XX

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Wednesday 13 May 2015 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2215–7306

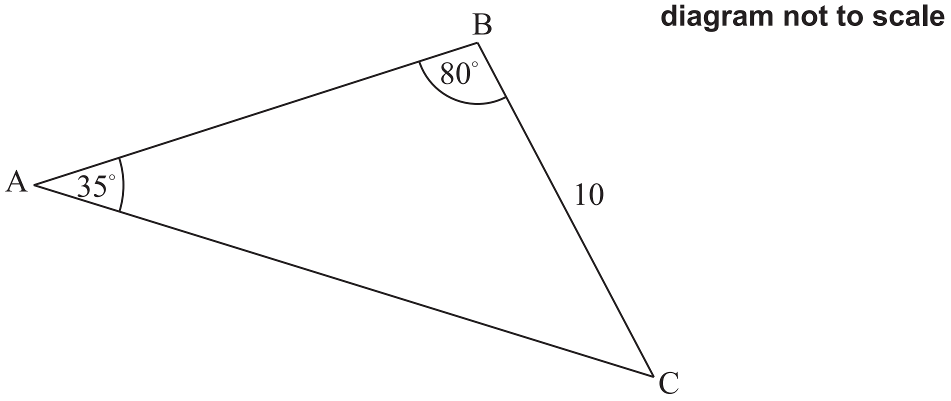
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following diagram shows triangle ABC.



$\text{BC} = 10\,\text{cm}$, $\text{A}\hat{\text{B}}\text{C} = 80^\circ$ and $\text{B}\hat{\text{A}}\text{C} = 35^\circ$.

(a) Find AC. [3]

(b) Find the area of triangle ABC. [3]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 7]

Let $\boldsymbol{u} = 6\boldsymbol{i} + 3\boldsymbol{j} + 6\boldsymbol{k}$ and $\boldsymbol{v} = 2\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$.

(a) Find

(i) $\boldsymbol{u} \bullet \boldsymbol{v}$;

(ii) $|\boldsymbol{u}|$;

(iii) $|\boldsymbol{v}|$. [5]

(b) Find the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$. [2]

3. [Maximum mark: 6]

The following table shows the sales, y millions of dollars, of a company, x years after it opened.

Time after opening (x years)	2	4	6	8	10
Sales (y millions of dollars)	12	20	30	36	52

The relationship between the variables is modelled by the regression line with equation $y = ax + b$.

(a) (i) Find the value of a and of b.

(ii) Write down the value of r. [4]

(b) Hence estimate the sales in millions of dollars after seven years. [2]

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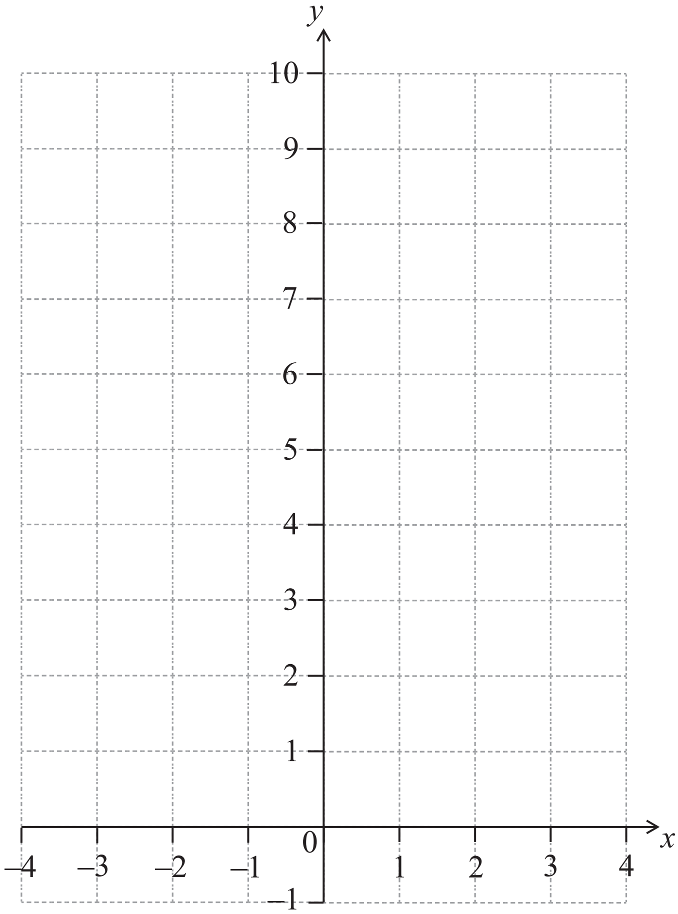
4. [Maximum mark: 5]

The third term in the expansion of $(x+k)^8$ is $63x^6$. Find the possible values of k.

5. [Maximum mark: 6]

Let $f(x) = e^{x+1} + 2$, for $-4 \le x \le 1$.

(a) On the following grid, sketch the graph of f. [3]



(b) The graph of f is translated by the vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ to obtain the graph of a function g.

Find an expression for $g(x)$. [3]

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6. [Maximum mark: 7]

Ramiro walks to work each morning. During the first minute he walks 80 metres. In each subsequent minute he walks $90\,\%$ of the distance walked during the previous minute. The distance between his house and work is 660 metres. Ramiro leaves his house at $08{:}00$ and has to be at work by $08{:}15$.

Explain why he will not be at work on time.

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7. [Maximum mark: 8]

Let $f(x) = kx^2 + kx$ and $g(x) = x - 0.8$. The graphs of f and g intersect at two distinct points. Find the possible values of k.

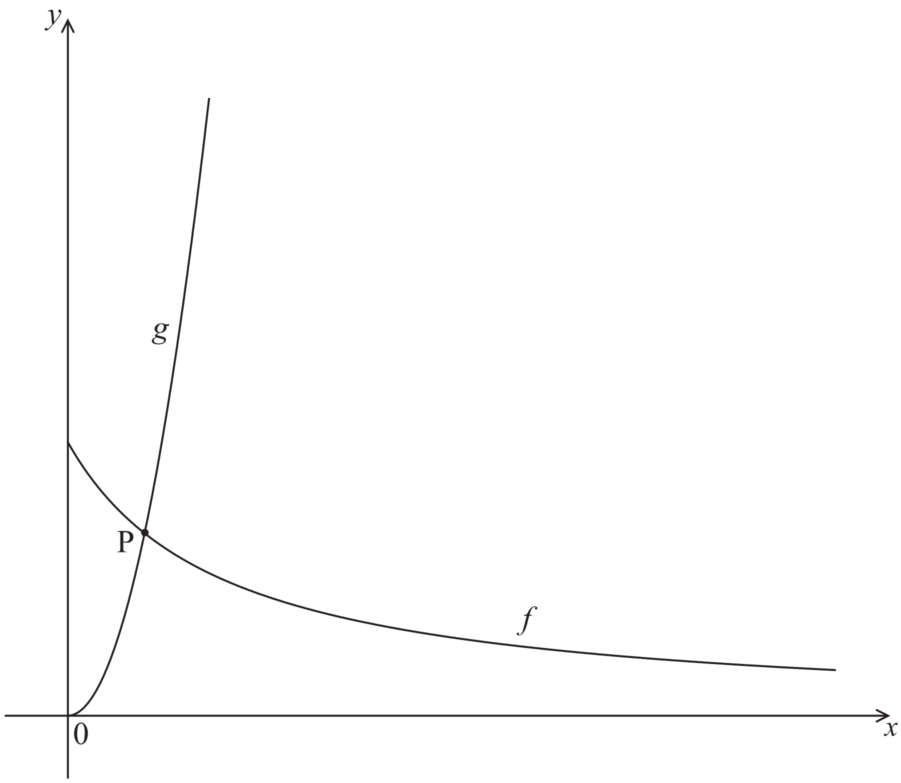
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

Let $f(x) = \dfrac{9}{x+2}$ and $g(x) = 3x^2$, for $x \geq 0$. Parts of the graphs of f and g are shown in the following diagram.



The graphs of f and g intersect at the point $\mathrm{P}(p, q)$.

(a) Find the value of p and of q. [3]

(b) Write down $f'(p)$. [2]

Let L be the normal to the graph of f at P.

(c) (i) Find the equation of L, giving your answer in the form $y = ax + b$.

(ii) Write down the y-intercept of L. [5]

(d) Let R be the region enclosed by the y-axis, the graph of g and the line L. Find the area of R. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

A machine manufactures a large number of nails. The length, L mm, of a nail is normally distributed, where $L \sim \mathrm{N}\left(50, \sigma^2\right)$.

(a) Find $\mathrm{P}(50 - \sigma < L < 50 + 2\sigma)$. [3]

(b) The probability that the length of a nail is less than 53.92 mm is 0.975.
Show that $\sigma = 2.00$ (correct to three significant figures). [2]

All nails with length at least t mm are classified as large nails.

(c) A nail is chosen at random. The probability that it is a large nail is 0.75.
Find the value of t. [3]

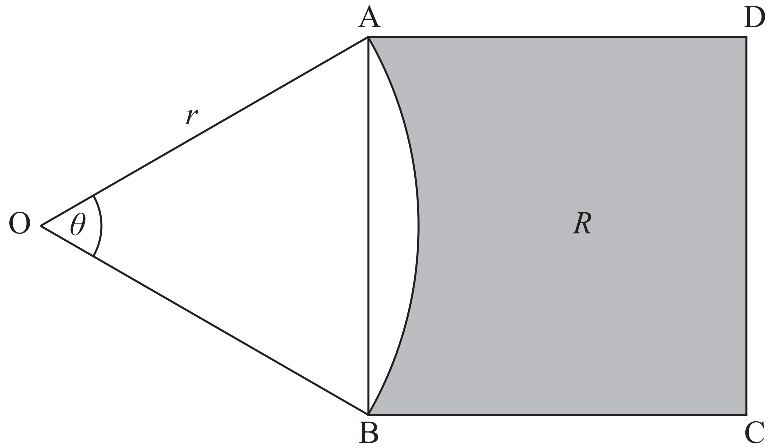
(d) (i) A nail is chosen at random from the large nails. Find the probability that the length of this nail is less than 50.1 mm.

(ii) Ten nails are chosen at random from the large nails. Find the probability that at least two nails have a length that is less than 50.1 mm. [8]

Do **not** write solutions on this page.

10. [Maximum mark: 16]

The following diagram shows a square ABCD, and a sector OAB of a circle centre O, radius r. Part of the square is shaded and labelled R.



$A\hat{O}B = \theta$, where $0.5 \le \theta < \pi$.

(a) Show that the area of the square ABCD is $2r^2(1-\cos\theta)$. [4]

(b) When $\theta = \alpha$, the area of the square ABCD is equal to the area of the sector OAB.

(i) Write down the area of the sector when $\theta = \alpha$.

(ii) Hence find α. [4]

(c) When $\theta = \beta$, the area of R is more than twice the area of the sector.
Find all possible values of β. [8]