Diploma Programme
Programme du diplôme
Programa del Diploma

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Mathematics
Standard level
Paper 2

Tuesday 14 May 2019 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

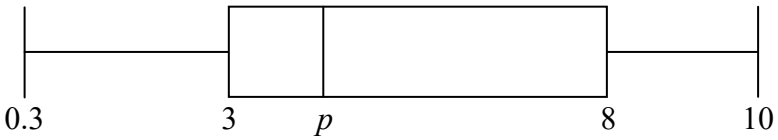
1. [Maximum mark: 5]

Ten students were asked for the distance, in km, from their home to school. Their responses are recorded below.

0.3 0.4 3 3 3.5 5 7 8 8 10

(a) For these data, find the mean distance from a student's home to school. [2]

The following box-and-whisker plot represents this data.



(b) Find the value of p. [1]

(c) Find the interquartile range. [2]

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2. [Maximum mark: 6]

Consider the graph of the function $f(x) = a(x + 10)^2 + 15$, $x \in \mathbb{R}$.

(a) Write down the coordinates of the vertex. [2]

(b) The graph of f has a y-intercept at -20. Find a. [2]

(c) Point $\mathrm{P}(8, b)$ lies on the graph of f. Find b. [2]

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3. [Maximum mark: 6]

Consider the function $f(x) = x^2 e^{3x}$, $x \in \mathbb{R}$.

(a) Find $f'(x)$. [4]

(b) The graph of f has a horizontal tangent line at $x = 0$ and at $x = a$. Find a. [2]

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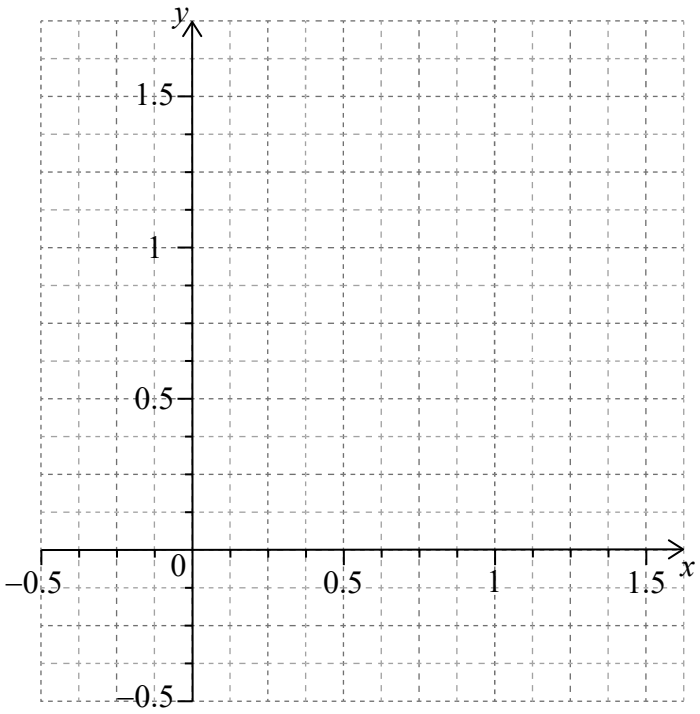
4. [Maximum mark: 8]

Let $f''(x) = (\cos 2x)(\sin 6x)$, for $0 \le x \le 1$.

(a) Sketch the graph of f'' on the grid below: [3]



(b) Find the x-coordinates of the points of inflexion of the graph of f. [3]

(c) Hence find the values of x for which the graph of f is concave-down. [2]

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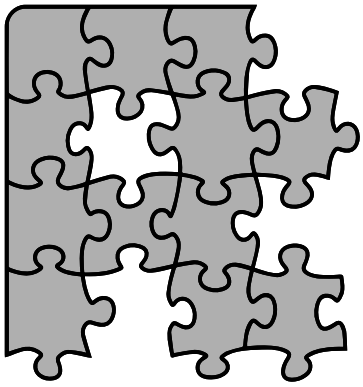
5. [Maximum mark: 6]

A jigsaw puzzle consists of many differently shaped pieces that fit together to form a picture.



Jill is doing a 1000-piece jigsaw puzzle. She started by sorting the edge pieces from the interior pieces. Six times she stopped and counted how many of each type she had found. The following table indicates this information.

Edge pieces (x)	16	31	39	55	84	115
Interior pieces (y)	89	239	297	402	580	802

Jill models the relationship between these variables using the regression equation $y = ax + b$.

(a) Write down the value of a and of b. [3]

(b) Use the model to predict how many edge pieces she had found when she had sorted a **total** of 750 pieces. [3]

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6. [Maximum mark: 7]

Consider the expansion of $\left(x^2+1.2\right)^n$ where $n\in\mathbb{Z}$, $n\geq 3$. Given that the coefficient of the term containing x^6 is greater than $200\,000$, find the smallest possible value of n.

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7. [Maximum mark: 7]

The first terms of an infinite geometric sequence, u_n, are $2, 6, 18, 54, \ldots$.
The first terms of a second infinite geometric sequence, v_n, are $2, -6, 18, -54, \ldots$.

The terms of a third sequence, w_n, are defined as $w_n = u_n + v_n$.

(a) Write down the first three **non-zero** terms of w_n. [3]

The finite series, $\sum_{k=1}^{225} w_k$, can also be written in the form $\sum_{k=0}^{m} 4r^k$.

(b) Find the value of

(i) r;

(ii) m. [4]

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

Let $f(x) = 2\sin(3x) + 4$ for $x \in \mathbb{R}$.

(a) The range of f is $k \leq f(x) \leq m$. Find k and m. [3]

Let $g(x) = 5f(2x)$.

(b) Find the range of g. [2]

The function g can be written in the form $g(x) = 10\sin(bx) + c$.

(c) (i) Find the value of b and of c.

(ii) Find the period of g. [5]

(d) The equation $g(x) = 12$ has two solutions where $\pi \leq x \leq \frac{4\pi}{3}$. Find both solutions. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

Let $f(x) = \frac{16}{x}$. The line L is tangent to the graph of f at $x = 8$.

(a) Find the gradient of L. [2]

L can be expressed in the form $\boldsymbol{r} = \begin{pmatrix} 8 \\ 2 \end{pmatrix} + t\boldsymbol{u}$.

(b) Find $\boldsymbol{u}$. [2]

The direction vector of $y = x$ is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

(c) Find the acute angle between $y = x$ and L. [5]

(d) (i) Find $(f \circ f)(x)$.

(ii) Hence, write down $f^{-1}(x)$.

(iii) Hence or otherwise, find the obtuse angle formed by the tangent line to f at $x = 8$ and the tangent line to f at $x = 2$. [7]

Do **not** write solutions on this page.

10. [Maximum mark: 16]

There are three fair six-sided dice. Each die has two green faces, two yellow faces and two red faces.

All three dice are rolled.

(a) (i) Find the probability of rolling exactly one red face.

(ii) Find the probability of rolling two or more red faces. [5]

Ted plays a game using these dice. The rules are:

- Having a turn means to roll all three dice.
- He wins $\$10$ for each green face rolled and adds this to his winnings.
- After a turn Ted can either:
  - end the game (and keep his winnings), or
  - have another turn (and try to increase his winnings).
- If two or more red faces are rolled in a turn, all winnings are lost and the game ends.

(b) Show that, after a turn, the probability that Ted adds exactly $\$10$ to his winnings is $\frac{1}{3}$. [5]

The random variable D $(\$)$ represents how much is added to his winnings after a turn.

The following table shows the distribution for D, where $\$w$ represents his winnings in the game so far.

D ($)	$-w$	0	10	20	30
$\mathrm{P}(D=d)$	x	y	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{27}$

(c) (i) Write down the value of x.

(ii) Hence, find the value of y. [3]

Ted will always have another turn if he expects an increase to his winnings.

(d) Find the least value of w for which Ted should end the game instead of having another turn. [3]

Please **do not** write on this page.

Answers written on this page
will not be marked.