IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
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N05/5/MATME/SP2/ENG/TZ0/XX/M+

MARKSCHEME

November 2005

MATHEMATICAL METHODS

Standard Level

Paper 2

22 pages

Instructions to Examiners

Note: **Where there are two marks (*e.g. M2, A2*) for an answer do not split the marks unless otherwise instructed.**

1 Method of marking

(a) All marking must be done using a **red** pen.

(b) Marks should be noted on candidates' scripts as in the markscheme:

- show the breakdown of individual marks using the abbreviations ***(M1)***, ***(A2)*** *etc*., unless a part is completely correct;
- write down each part mark total, indicated on the markscheme (for example, ***[3 marks]***) – it is suggested that this be written at the end of each part, and underlined;
- write down and circle the total for each question at the end of the question.

2 Abbreviations

The markscheme may make use of the following abbreviations:

(M) Marks awarded for **Method**

(A) Marks awarded for an **Answer** or for **Accuracy**

(N) Marks awarded for correct answers, if **no** working (or no relevant working) shown: they may not necessarily be all the marks for the question. Examiners should only award these marks for correct answers where there is no working.

(R) Marks awarded for clear **Reasoning**

(AG) **Answer Given** in the question and consequently marks are **not** awarded

Note: Unless otherwise stated, it is not possible to award ***(M0)(A1).***

Follow through (ft) marks should be awarded where a correct method has been attempted but error(s) are made in subsequent working which is essentially correct.

- Penalize the error when it first occurs
- Accept the incorrect result as the appropriate quantity in all subsequent working
- If the question becomes much simpler then use discretion to award fewer marks

Examiners should use **(d)** to indicate where discretion has been used. It should only be used for decisions on follow through and alternative methods. It must be accompanied by a brief note to explain the decision made

3 Using the Markscheme

(a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme. Indicate the awarding of these marks by (**d**).

Where alternative methods for complete questions or parts of questions are included, they are indicated by **METHOD 1**, **METHOD 2**, *etc*. Other alternative part solutions are indicated by **EITHER**….**OR.** It should be noted that ***G*** marks have been removed, and GDC solutions will not be indicated using the **OR** notation as on previous markschemes.

Candidates are expected to show working on this paper, and examiners should not award full marks for just the correct answer. Where it is appropriate to award marks for correct answers with no working (or no relevant working), it will be shown on the markscheme using the ***N*** notation. All examiners will be expected to award marks accordingly in these situations.

(b) Unless the question specifies otherwise, accept **equivalent** forms. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.
On the markscheme, these equivalent numerical or algebraic forms will generally be written in brackets after the required answer. Paper setters will indicate the required answer, by allocating full marks at that point. Further working should be ignored, even if it is incorrect. For example: if candidates are asked to factorize a quadratic expression, and they do so correctly, they are awarded full marks. If they then continue and find the roots of the corresponding equation, do not penalize, even if those roots are incorrect, *i.e.* once the correct answer is seen, ignore further working.

(c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7 , 1·7, 1,7; different forms of vector notation such as $\vec{u}$, $\bar{u}$, $\underline{u}$; $\tan^{-1} x$ for arctan x.

4 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

There are two types of accuracy error. Candidates should be penalized **once only IN THE PAPER** for an accuracy error **(AP)**.

Award the marks as usual then write –1(**AP**) against the answer and also on the **front** cover

Rounding errors: only applies to final answers not to intermediate steps.

Level of accuracy: when this is not specified in the question the general rule *unless otherwise stated in the question all numerical answers must be given exactly or to three significant figures* applies.

- If a final correct answer is incorrectly rounded, apply the **AP**
  OR
- If the level of accuracy is not specified in the question, apply the **AP** for answers not given to 3 significant figures. (Please note that this has changed from 2003).

Note: **If there is no working shown**, and answers are given to the correct two significant figures, apply the **AP**. However, do **not** accept answers to one significant figure without working.

5 Graphic Display Calculators

Many candidates will be obtaining solutions directly from their calculators, often without showing any working. They have been advised that they must use mathematical notation, not calculator commands when explaining what they are doing. Incorrect answers without working will receive no marks. However, if there is written evidence of using a graphic display calculator correctly, method marks may be awarded. Where possible, examples will be provided to guide examiners in awarding these method marks.

Examples

1. Accuracy

A question leads to the answer 4.6789….

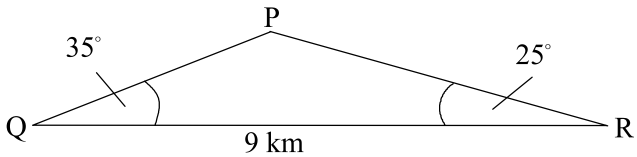
- 4.68 is the correct 3 s.f. answer.
- 4.7, 4.679 are to the wrong level of accuracy : both should be penalised the first time this type of error occurs.
- 4.67 is incorrectly rounded – penalise on the first occurrence.

Note: All these "incorrect" answers may be assumed to come from 4.6789..., even if that value is not seen, but previous correct working is shown. However, 4.60 is wrong, as is 4.5, 4.8, and these should be penalized as being incorrect answers, not as examples of accuracy errors.

2. Alternative solutions

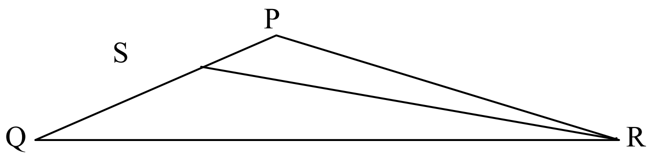
The points P, Q, R are three markers on level ground, joined by straight paths PQ, QR, PR as shown in the diagram. $\text{QR} = 9\text{ km}$, $\text{P}\hat{\text{Q}}\text{R} = 35^\circ$, $\text{P}\hat{\text{R}}\text{Q} = 25^\circ$.
(Note: in the original question, the first part was to find PR = 5.96)

diagram not to scale



(a) Tom sets out to walk from Q to P at a steady speed of 8 km h^{-1}. At the same time, Alan sets out to jog from R to P at a steady speed of $a\text{ km h}^{-1}$. They reach P at the same time. Calculate the value of a. ***[7 marks]***

(b) The point S is on [PQ], such that $\text{RS} = 2\text{QS}$, as shown in the diagram.



Find the length QS. ***[6 marks]***

MARKSCHEME

(a) **EITHER**

Sine rule to find PQ

$PQ = \frac{9\sin 25}{\sin 120}$ *(M1)(A1)*

$PQ = 4.39$ km *(A1)*

OR

Cosine rule: $PQ^2 = 5.96^2 + 9^2 - (2)(5.96)(9)\cos 25$ *(M1)(A1)*

$= 19.29$

$PQ = 4.39$ km *(A1)*

THEN

Time for Tom $= \frac{4.39}{8}$ *(A1)*

Time for Alan $= \frac{5.96}{a}$ *(A1)*

Then $\frac{4.39}{8} = \frac{5.96}{a}$ *(M1)*

$a = 10.9$ *(A1)* *(N5)*

[7 marks]

Note that the **THEN** part follows both **EITHER** and **OR** solutions, and this is shown by the alignment.

(b) **METHOD 1**

$RS^2 = 4QS^2$ *(A1)*

$4QS^2 = QS^2 + 81 - 18 \times QS \times \cos 35$ *(M1)(A1)*

$3QS^2 + 14.74QS - 81 = 0$ (or $3x^2 + 14.74x - 81 = 0$) *(A1)*

$\Rightarrow QS = -8.20$ or $QS = 3.29$ *(A1)*

therefore $QS = 3.29$ *(A1)*

METHOD 2

$\frac{QS}{\sin S\hat{R}Q} = \frac{2QS}{\sin 35}$ *(M1)*

$\Rightarrow \sin S\hat{R}Q = \frac{1}{2}\sin 35$ *(A1)*

$S\hat{R}Q = 16.7^\circ$ *(A1)*

Therefore, $Q\hat{S}R = 180 - (35 + 16.7) = 128.3^\circ$ *(A1)*

$\frac{9}{\sin 128.3} = \frac{QS}{\sin 16.7} \left(= \frac{SR}{\sin 35}\right)$ *(M1)*

$QS = \frac{9\sin 16.7}{\sin 128.3} \left(= \frac{9\sin 35}{2\sin 128.3}\right) = 3.29$ *(A1)* *(N2)*

If candidates have shown no working, award ***(N5)*** for the correct answer 10.9 in part (a), and ***(N2)*** for the correct answer 3.29 in part (b). ***[6 marks]***

3. Follow through

Question

Calculate the acute angle between the lines with equations

$$\boldsymbol{r} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + s\begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad \text{and} \quad \boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Markscheme

Angle between lines = angle between direction vectors (may be implied)	***(A1)***	
Direction vectors are $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (may be implied)	***(A1)***	
$\begin{pmatrix} 4 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \left\| \begin{pmatrix} 4 \\ 3 \end{pmatrix} \right\| \left\| \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\| \cos\theta$	***(M1)***	
$4 \times 1 + 3 \times (-1) = \sqrt{(4^2 + 3^2)}\sqrt{(1^2 + (-1)^2)} \cos\theta$	***(A1)***	
$\cos\theta = \frac{1}{5\sqrt{2}}$ $(= 0.1414\ldots)$	***(A1)***	
$\theta = 81.9^\circ$ (1.43 radians)	***(A1)***	***(N3)***

Examples of solutions and marking

	Solutions	**Marks allocated**	
1.	$\begin{pmatrix} 4 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \left\| \begin{pmatrix} 4 \\ 3 \end{pmatrix} \right\| \left\| \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\| \cos\theta$	***(A1)(A1)*** implied ***(M1)***	
	$\cos\theta = \frac{7}{5\sqrt{2}}$	***(A0)(A1)***	
	$\theta = 8.13^\circ$	***(A1)*ft**	Total 5 marks
2.	$\cos\theta = \frac{\begin{pmatrix} 4 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 4 \end{pmatrix}}{\sqrt{17}\sqrt{20}}$	***(A0)(A0)*** wrong vectors implied ***(M1)*** for correct method, ***(A1)*ft**	
	$= 0.2169$	***(A1)*ft**	
	$\theta = 77.5^\circ$	***(A1)*ft**	Total 4 marks
3.	$\theta = 81.9^\circ$	***(N3)***	Total 3 marks

Note that this candidate has obtained the correct answer, but not shown any working. The way the markscheme is written means that the first 2 marks may be implied by subsequent correct working, but the other marks are only awarded if the relevant working is seen. Thus award the first 2 implied marks, plus the final mark for the correct answer.

END OF EXAMPLES

QUESTION 1

(i) (a) **METHOD 1**

Finding gradient $m = \frac{53-13}{10-2}$ $(=5)$ *(A1)*

$y - 13 = 5(x-2)$ *(M1)*

$y = 5x + 3$ *(AG)* *(N0)*

METHOD 2

$u_3 = 13$ and $u_{11} = 53$ *(M1)*

$u_1 = 3$ and $d = 5$ *(A1)*

$y = 5x + 3$ *(AG)* *(N0)*

Note: Award no marks for showing that $(2, 13)$ and $(10, 53)$ satisfy $y = 5x + 3$.

[2 marks]

(b) 3 kg *(A1)* *(N1)*

[1 mark]

(c) Increase is 5 kg (per week) *(A1)* *(N1)*

[1 mark]

(d) $98 = 5x + 3$ *(M1)*

$5x = 95$

$x = 19$ *(A1)* *(N2)*

[2 marks]

(ii) (a) 76 (mice) *(A1)* *(N1)*

[1 mark]

(b) 11.2 (seconds) *(A1)* *(N1)*

[1 mark]

(c) (i) $p = 76 - (16 + 22) = 38$ (allow **ft** from (ii) (a)) *(A1)* *(N1)*

$q = 132 - 76 = 56$ *(A1)* *(N1)*

(ii) $x = \frac{7.5 \times 16 + ...14.5 \times 23}{16 + ...23}$ $\left(= \frac{3363}{300}\right)$ *(M1)*

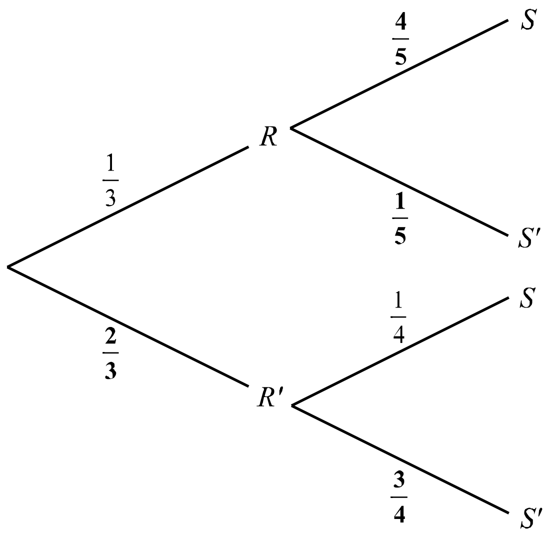
$= 11.2$ (accept 11.21) *(A1)* *(N2)*

[4 marks]

Total [12 marks]

QUESTION 2

(a)



(A1)(A1)(A1)

[3 marks]

(b) (i) $\mathrm{P}(R \cap S) = \frac{1}{3} \times \frac{4}{5} \quad \left(= \frac{4}{15} = 0.267 \right)$ ***(A1)*** ***(N1)***

(ii) $\mathrm{P}(S) = \frac{1}{3} \times \frac{4}{5} + \frac{2}{3} \times \frac{1}{4}$ ***(A1)(A1)***

$= \frac{13}{30} \quad (= 0.433)$ ***(A1)*** ***(N3)***

(iii) $\mathrm{P}(R \mid S) = \frac{\frac{4}{15}}{\frac{13}{30}}$ ***(A1)(A1)***

$= \frac{8}{13} \quad (= 0.615)$ ***(A1)*** ***(N3)***

[7 marks]

Total [10 marks]

QUESTION 3

(a) (i) $x = 10$ *(A1)* *(N1)*

(ii) $y = 8$ *(A1)* *(N1)*

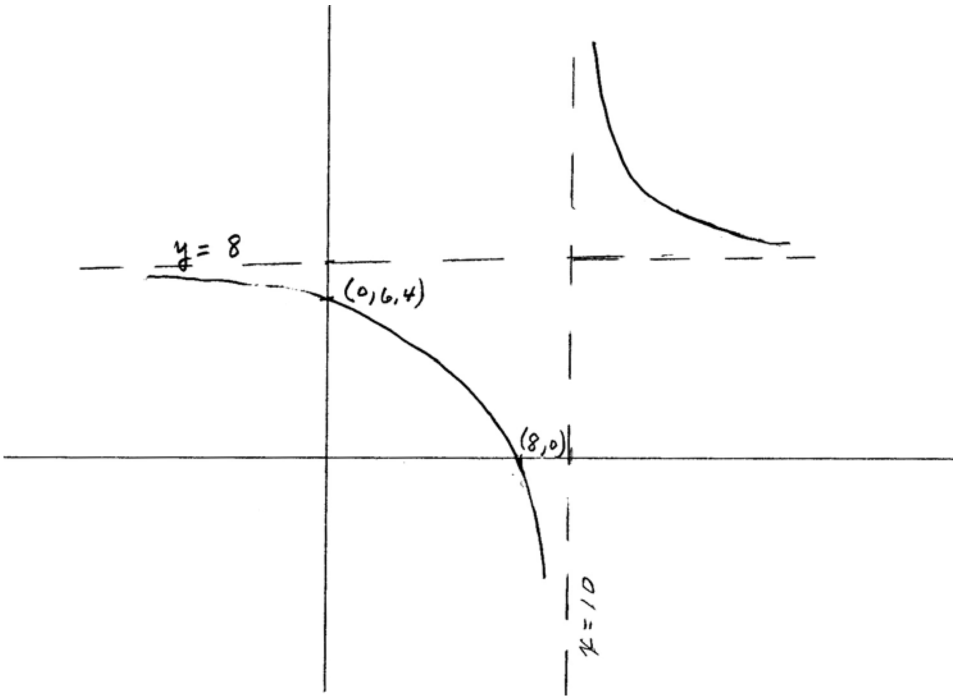
[2 marks]

(b) (i) 6.4 (or $(0, 6.4)$) *(A1)* *(N1)*

(ii) 8 (or $(8, 0)$) *(A1)* *(N1)*

[2 marks]

(c)



(A1)(A1)(A1)(A1) ***(N4)***

Note:	Award ***(A1)*** for both asymptotes correctly drawn, ***(A1)*** for both intercepts correctly marked, ***(A1)(A1)*** for each branch drawn in approximately correct positions. Asymptotes and intercepts need not be labelled.

[4 marks]

(d) There is a vertical translation of 8 units. (accept translation of $\begin{pmatrix} 0 \\ 8 \end{pmatrix}$) *(A2)* *(N2)*

[2 marks]

Total [10 marks]

QUESTION 4

(i) (a) $\vec{DE} = \begin{pmatrix} 12-4 \\ 11-5 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$ ***(M1)(A1)*** ***(N2)***

[2 marks]

(b) $\left|\vec{DE}\right| = \sqrt{8^2+6^2} \quad \left(= \sqrt{64+36}\right)$ ***(M1)***

$= 10$ ***(A1)*** ***(N2)***

[2 marks]

(c) **Vector geometry approach**

Using $DG = 10$ ***(M1)***

$(x-4)^2 + (y-5)^2 = 100$ ***(A1)***

Using (DG) perpendicular to (DE) ***(M1)***

Leading to $\vec{DG} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$, $\vec{DG} = \begin{pmatrix} 6 \\ -8 \end{pmatrix}$ ***(A1)(A1)***

Using $\vec{DG} = \vec{DO} + \vec{OG}$ (O is the origin) ***(M1)***

$G(-2, 13)$, $G(10, -3)$ (accept position vectors) ***(A1)(A1)***

Algebraic approach

gradient of DE $= \frac{6}{8}$ ***(A1)***

gradient of DG $= -\frac{8}{6}$ ***(A1)***

equation of line DG is $y - 5 = -\frac{4}{3}(x-4)$ ***(A1)***

Using $DG = 10$ ***(M1)***

$(x-4)^2 + (y-5)^2 = 100$ ***(A1)***

Solving simultaneous equation ***(M1)***

$G(-2, 13)$, $G(10, -3)$ (accept position vectors) ***(A1)(A1)***

Note:	Award full marks for an appropriately labelled diagram (e.g. showing that $DG = 10$, displacements of 6 and 8), or an **accurate** diagram leading to the correct answers.

[8 marks]

continued …

Question 4 continued

(ii) (a) $p = 2 \Rightarrow \begin{pmatrix} 0 \\ 12 \end{pmatrix} + 2\begin{pmatrix} 5 \\ -3 \end{pmatrix}$ ***(A1)***

$= \begin{pmatrix} 10 \\ 6 \end{pmatrix}$ (accept any other vector notation, including $(10, 6)$) ***(A1)*** ***(N2)***

[2 marks]

(b) **METHOD 1**

(i) equating components ***(M1)***

$0 + 5p = 14 + q$, $12 - 3p = 0 + 3q$ ***(A1)***

$p = 3$, $q = 1$ ***(A1)(A1)*** ***(N1)(N1)***

(ii) The coordinates of P are $(15, 3)$ (accept $x = 15$, $y = 3$) ***(A1)(A1)*** ***(N1)(N1)***

METHOD 2

(i) Setting up Cartesian equations ***(M1)***

$x = 5p$ $x = 14 + q$

$y = 12 - 3p$ $y = 3q$

giving $3x + 5y = 60$ $3x - y = 42$ ***(A1)***

Solving simultaneously gives $x = 15$, $y = 3$

Substituting to find p and q

$\begin{pmatrix} 15 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 12 \end{pmatrix} + p\begin{pmatrix} 5 \\ -3 \end{pmatrix}, \begin{pmatrix} 15 \\ 3 \end{pmatrix} = \begin{pmatrix} 14 \\ 0 \end{pmatrix} + q\begin{pmatrix} 1 \\ 3 \end{pmatrix},$

$p = 3$ $q = 1$ ***(A1)(A1)*** ***(N1)(N1)***

(ii) From above, P is $(15, 3)$ (accept $x = 15$, $y = 3$ seen above) ***(A1)(A1)*** ***(N1)(N1)***

[6 marks]

Total [20 marks]

QUESTION 5

(i) (a) $s = 25t - \frac{4}{3}t^3 + c$ *(M1)(A1)(A1)*

Note: Award no further marks if "c" is missing.

Substituting $s = 10$ and $t = 3$ *(M1)*

$10 = 25 \times 3 - \frac{4}{3}(3)^3 + c$

$10 = 75 - 36 + c$

$c = -29$ *(A1)*

$s = 25t - \frac{4}{3}t^3 - 29$ *(A1)* *(N3)*

[6 marks]

(b) **METHOD 1**

s is a maximum when $v = \frac{ds}{dt} = 0$ (may be implied) *(M1)*

$25 - 4t^2 = 0$ *(A1)*

$t^2 = \frac{25}{4}$

$t = \frac{5}{2}$ *(A1)* *(N2)*

[3 marks]

METHOD 2

Using maximum of s ($12\frac{2}{3}$, may be implied) *(M1)*

$25t - \frac{4}{3}t^3 - 29 = 12\frac{2}{3}$ *(A1)*

$t = 2.5$ *(A1)* *(N2)*

(c) $25t - \frac{4}{3}t^3 - 29 > 0$ (accept equation) *(M1)*

$m = 1.27$, $n = 3.55$ *(A1)(A1)* *(N3)*

[3 marks]

continued …

Question 5 continued

(ii)

Note:	There are many approaches possible. However, there must be some evidence of their method.

Area $= \int_0^k \sin 2x \, \mathrm{d}x$ (must be seen somewhere) ***(A1)***

Using area $= 0.85$ (must be seen somewhere) ***(M1)***

EITHER

Integrating $\left[\frac{-1}{2}\cos 2x\right]_0^k$ $\left(=\frac{-1}{2}\cos 2k + \frac{1}{2}\cos 0\right)$ ***(A1)***

Simplifying $\frac{-1}{2}\cos 2k + 0.5$ ***(A1)***

Equation $\frac{-1}{2}\cos 2k + 0.5 = 0.85$ $\left(\cos 2k = -0.7\right)$

OR

Evidence of using trial and error on a GDC ***(M1)(A1)***

E.g. $\int_0^{\frac{\pi}{2}} \sin 2x \, \mathrm{d}x = 0.5$, $\frac{\pi}{2}$ too small etc

OR

Using GDC and solver, starting with $\int_0^k \sin 2x \, \mathrm{d}x - 0.85 = 0$ ***(M1)(A1)***

THEN

$k = 1.17$ ***(A2)*** ***(N3)***

[6 marks]

Total [18 marks]

QUESTION 6

(i) (a) E *(A1)* *(N1)*

(b) C *(A1)* *(N1)*

(c) F *(A1)* *(N1)*

(d) A *(A1)* *(N1)*

(e) D *(A1)* *(N1)*

[5 marks]

(ii) (a) $z = \frac{185-170}{20} = 0.75$ *(M1)(A1)*

$\text{P}(Z < 0.75) = 0.773$ *(A1)* *(N3)*

[3 marks]

(b) $z = -0.47$ (may be implied) *(A1)*

$-0.47 = \frac{d-170}{20}$ *(M1)*

$d = 161$ *(A1)* *(N3)*

[3 marks]

continued …

Question 6 continued

(iii) (a) Together, home goals $= \dfrac{95}{160}$ of total goals ***(A1)***

EITHER

Expected *Atletico* home goals $= \dfrac{95}{160} \times 65 \quad \left(\text{or } \dfrac{95}{160} \times \dfrac{65}{160} \times 160\right)$ ***(A1)***

$= 38.6$ ***(AG)*** ***(N0)***

OR

Expected *Atletico* home goals $= \dfrac{65}{160}$ of total goals

$= \dfrac{65}{160} \times 95 \quad \left(\text{or } \dfrac{65}{160} \times \dfrac{95}{160} \times 160\right)$ ***(A1)***

$= 38.6$ ***(AG)*** ***(N0)***

[2 marks]

(b) **EITHER**

$\chi^2 = \dfrac{(38.6 - 45)^2}{38.6} + \cdots + \dfrac{(16.2 - 20)^2}{16.2}$ ***(M1)***

$= 4.66$ ***(A1)*** ***(N2)***

OR

From GDC $\chi^2 = 4.607 = 4.61$ (3 s.f.) ***(A2)*** ***(N2)***

[2 marks]

(c) (i) Yes (The hypothesis is accepted) ***(A1)*** ***(N1)***

(ii) **EITHER** – from tables

At 5 % significance level with 2 degrees of freedom the value of chi-squared must be ***(A1)***
greater than 5.991 to reject hypothesis. ***(A1)*** ***(N0)***

OR – from GDC

The probability of obtaining this value of chi-squared or greater is 0.0999 (9.99 %) ***(A1)***
This value is greater than 0.0500 (5 %). ***(A1)*** ***(N0)***

[3 marks]

continued ...

Question 6 continued

(iv) (a) **METHOD 1**

For normal populations approximately 95 % of population is within 2 standard deviations of mean. ***(R1)***

$260-200=2\sigma \Rightarrow \sigma=30$ ***(M1)(AG)*** ***(N0)***

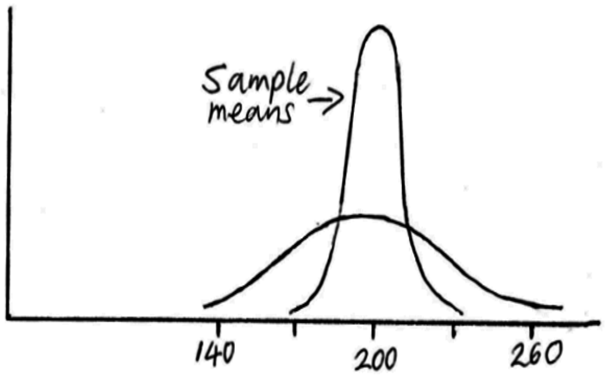
METHOD 2

$$\frac{260-200}{\sigma}=1.96$$ ***(M1)***

$\sigma=30.6$ (which is approximately 30) ***(R1)(AG)*** ***(N0)***

[2 marks]

(b) (i)



(A1)(A1)(A1) ***(N3)***

Note: Award ***(A1)*** for normal curve centred at 200, ***(A1)*** for showing curve narrower than original, ***(A1)*** for showing curve higher than original.

(ii) $z=1.96$ (seen anywhere) ***(A1)***

$$1.96=\frac{X-\mu}{\frac{\sigma}{\sqrt{n}}}$$

$$1.96=\frac{10}{\frac{30}{\sqrt{n}}}$$ ***(M1)***

$\sqrt{n}=3\times 1.96$

$n=35$ (accept 34.6) ***(A1)*** ***(N2)***

[6 marks]

(c) Assuming the mean is 200 ***(M1)***

$$\text{P}(M>212)=\text{P}\left(Z>\frac{212-200}{\frac{30}{\sqrt{25}}}\right)=\text{P}(Z>2.00)$$ (may be implied) ***(M1)***

$=1-0.9772=0.0228$ (accept 0.0227) ***(A1)***

As $\text{P}(M>212)<0.05$ we conclude that mean has increased. ***(R1)*** ***(N0)***

[4 marks]

Total [30 marks]

QUESTION 7

(i) (a) $x = 1.43$ *(A2)* *(N2)*

[2 marks]

(b) $f'(x) = 0$

$f'(x) = 12x^3 - 12x^2 - 60x - 36$ (may be implied) *(A1)*

Setting first derivative equal to zero *(M1)*

$f'(x) = 12x^3 - 12x^2 - 60x - 36 = 0$

$x = -1$ (is other solution) *(A1)* *(N2)*

[3 marks]

(c) $f''(x) = 0$

$f''(x) = 36x^2 - 24x - 60$ (may be implied) *(A1)*

Setting second derivative equal to zero *(M1)*

$f''(x) = 36x^2 - 24x - 60 = 0$

$x = \frac{5}{3}, -1$ *(A1)(A1)* *(N3)*

[4 marks]

(d) $(-1, 125)$ (or $x = -1$, $y = 125$) *(A1)(A1)* *(N2)*

Note: Award no marks if this answer is seen together with extra answers.

[2 marks]

(e) $x = 4$, $x = 1.43$ (allow **ft** from part (a)) *(A1)(A1)* *(N2)*

[2 marks]

(f) tangent to graph of $\frac{1}{f}$ horizontal $\Rightarrow$ tangent to graph of f is horizontal *(M1)*

$\Rightarrow x = 3$ *(A1)* *(N2)*

[2 marks]

(ii) (a) $g'(x) = \frac{x^3(2^x \ln 2) - 3x^2(2^x)}{(x^3)^2}$ *(M1)(A1)(A1)*

$= \frac{x^2 2^x (x \ln 2 - 3)}{x^6}$ *(A1)*

$= \frac{2^x (x \ln 2 - 3)}{x^4}$ *(AG)* *(N0)*

[4 marks]

(b) (i) $p = 9.9404$ (9.94 to 3 s.f.) *(A2)* *(N2)*

(ii) $p - \frac{g(p)}{g'(p)}$ *(A1)* *(N1)*

[3 marks]

continued …

Question 7 continued

(iii) (a) $h = \frac{4-0}{2} = 2$ ***(A1)***

Finding y values ***(M1)***

$y_0 = 6$; $y_1 = \sqrt{44}$; $y_2 = 10$ ***(A1)***

$I \approx \frac{2}{2}\left[(6+10)+2\left(2\sqrt{11}\right)\right]$ ***(A1)***

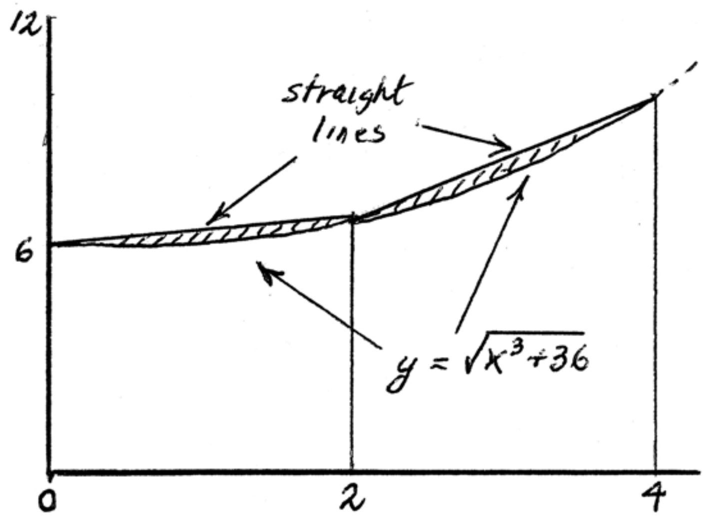
$= 16 + 4\sqrt{11}$ ***(AG)*** ***(N0)***

Note:	Award no marks for using a trapezium rule program on GDC leading to $I = 29.2(665)$.

[4 marks]

(b) (i) Trapezium rule overestimates value of I. ***(A1)*** ***(N1)***

(ii)



(A1)(A1)(A1) ***(N3)***

[4 marks]

Total [30 marks]

QUESTION 8

(i) (a) $A^{-1} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{7}{3} & \frac{5}{3} \end{pmatrix}$ or $\frac{1}{3}\begin{pmatrix} 2 & -1 \\ -7 & 5 \end{pmatrix}$ or $\begin{pmatrix} 0.667 & -0.333 \\ -2.33 & -1.67 \end{pmatrix}$ ***(A1)(A1)*** ***(N2)***

[2 marks]

(b) $AX = C - B$ (may be implied) ***(A1)***

$X = A^{-1}(C - B)$ ***(A1)***

$D = C - B$

$= \begin{pmatrix} 7 & -11 \\ 11 & -13 \end{pmatrix}$ ***(A1)*** ***(N3)***

[3 marks]

(c) $X = \begin{pmatrix} 1 & -3 \\ 2 & 4 \end{pmatrix}$ ***(A2)*** ***(N2)***

[2 marks]

(ii)

Matrix	M_1	M_2	M_3	M_4	M_5
Transformation	C	E	B	H	J

(A1)(A1)(A1)(A1)(A1) ***(N5)***

[5 marks]

(iii) For recognizing $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ is invariant (may be implied) ***(M1)***

$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ ***(A1)***

$\Rightarrow c = 4$ and $d = -2$ ***(A1)(A1)*** ***(N2)***

[4 marks]

continued …

Question 8 continued

(iv) **METHOD 1**

(a) $M'N' = \sqrt{[10-(-2)]^2 + [5-(-11)]^2} = 20$ *(M1)(A1)*

$M'L' = \sqrt{(10-2)^2 + (5-11)^2} = 10$ *(A1)*

Area of $K'M'L'N' = 20 \times 10 = 200$ *(A1)* *(N1)*

[4 marks]

(b) The sequence KLMN goes round the square in a counter-clockwise direction whereas the sequence K′L′M′N′ goes round the rectangle in a clockwise direction. (Any statement that the rotational order of the vertices has been reversed). ***(R2)*** *(N2)*

[2 marks]

(c) (i) Area scale factor $= \frac{200}{4} = 50$. *(A1)* *(N1)*

(ii) Reflection involved means determinant of transformation matrix $= -50$. *(A1)*

$\det \boldsymbol{T} = 6q - 32 = -50$ *(A1)*

$\Rightarrow q = -3$ *(AG)* *(N0)*

[3 marks]

METHOD 2

(a) Let matrix for $\boldsymbol{T}$ be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

leading to $\left.\begin{matrix} a+b=10 \\ c+d=5 \end{matrix}\right\}$ and $\left.\begin{matrix} a+b=10 \\ c+d=5 \end{matrix}\right\}$ *(A1)*

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 6 & 4 \\ 8 & -3 \end{pmatrix}$ *(A1)*

$\det = -50$ *(A1)*

Area $= 50 \times 4 = 200$ *(A1)* *(N1)*

[4 marks]

(b) Determinant negative $\Rightarrow$ reflection ***(R2)*** *(N2)*

[2 marks]

(c) (i) Area scale factor $= 50$ *(A1)* *(N1)*

(ii) From above $q = -3$ *(A2)(AG)*

[3 marks]

continued ...

Question 8 (iv) continued

(d) (i) $\boldsymbol{T} = \boldsymbol{QRP}$ ***(A1) (N1)***

(ii) **EITHER**

$\boldsymbol{R} = \boldsymbol{Q}^{-1}\boldsymbol{T}\boldsymbol{P}^{-1}$ ***(A1)***

$= \begin{pmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{pmatrix} \begin{pmatrix} 6 & 4 \\ 8 & -3 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$ ***(A1)***

$= \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{pmatrix}$ ***(A2) (N2)***

OR

$\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ ***(A1)***

$= \begin{pmatrix} 10a & 5b \\ 10c & 5d \end{pmatrix} = \begin{pmatrix} 6 & 4 \\ 8 & -3 \end{pmatrix}$ ***(A1)***

$\Rightarrow \boldsymbol{R} = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{pmatrix}$ ***(A2) (N2)***

[5 marks]

Total [30 marks]