IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

M06/5/MATME/SP1/ENG/TZ2/XX

22067303

MATHEMATICS
STANDARD LEVEL
PAPER 1

Wednesday 3 May 2006 (afternoon)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working. Working may be continued below the lines, if necessary.

1. Consider the infinite geometric series $405 + 270 + 180 + \ldots$.

(a) For this series, find the common ratio, giving your answer as a fraction in its simplest form.

(b) Find the fifteenth term of this series.

(c) Find the **exact** value of the sum of the infinite series.

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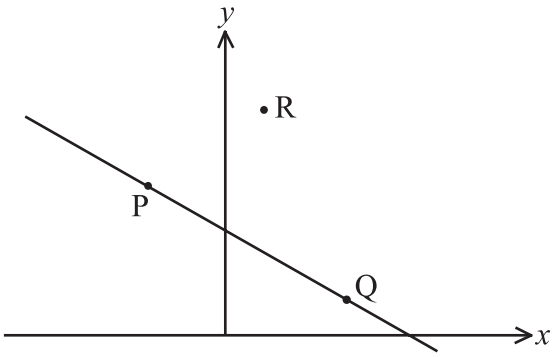
2. The points P(–2, 4), Q(3, 1) and R(1, 6) are shown in the diagram below.



(a) Find the vector $\vec{PQ}$.

(b) Find a vector equation for the line through R parallel to the line (PQ).

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3. The population below is listed in ascending order.

$$5,\ 6,\ 7,\ 7,\ 9,\ 9,\ r,\ 10,\ s,\ 13,\ 13,\ t$$

The median of the population is 9.5. The upper quartile Q_3 is 13.

(a) Write down the value of

(i) r;

(ii) s.

(b) The mean of the population is 10. Find the value of t.

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4. Solve the following equations.

(a) $\ln(x+2)=3$.

(b) $10^{2x}=500$.

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5. The probability distribution of the discrete random variable X is given by the following table.

x	1	2	3	4	5
$\mathrm{P}(X = x)$	0.4	p	0.2	0.07	0.02

(a) Find the value of p.

(b) Calculate the expected value of X.

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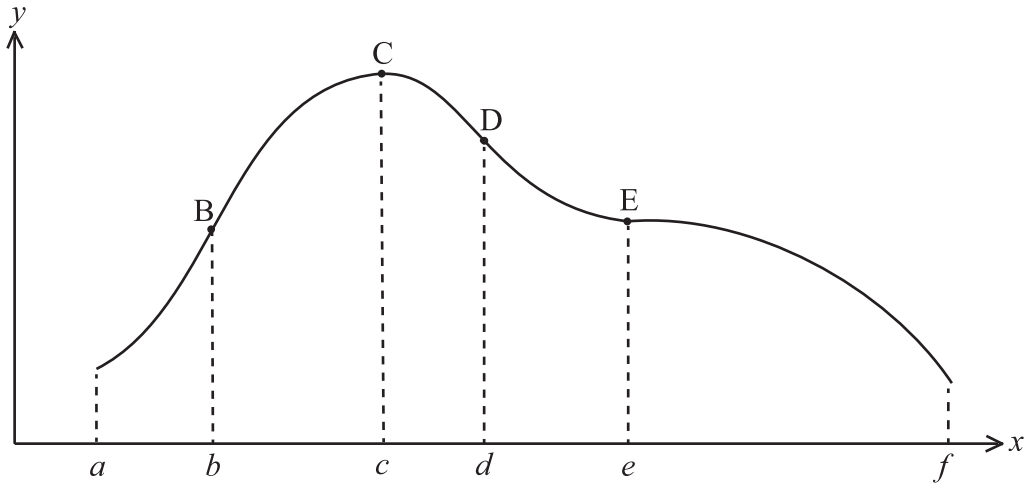
6. The graph of a function g is given in the diagram below.



The gradient of the curve has its maximum value at point B and its minimum value at point D. The tangent is horizontal at points C and E.

(a) Complete the table below, by stating whether the first derivative g' is positive or negative, and whether the second derivative g'' is positive or negative.

Interval	g'	g''
$a < x < b$		
$e < x < f$		

(b) Complete the table below by noting the points on the graph described by the following conditions.

Conditions	Point
$g'(x) = 0,\ g''(x) < 0$	
$g'(x) < 0,\ g''(x) = 0$	

7. (a) Express $y = 2x^2 - 12x + 23$ in the form $y = 2(x - c)^2 + d$.

The graph of $y = x^2$ is transformed into the graph of $y = 2x^2 - 12x + 23$ by the transformations

a vertical stretch with scale factor k **followed by**
a horizontal translation of p units **followed by**
a vertical translation of q units.

(b) Write down the value of

(i) k;

(ii) p;

(iii) q.

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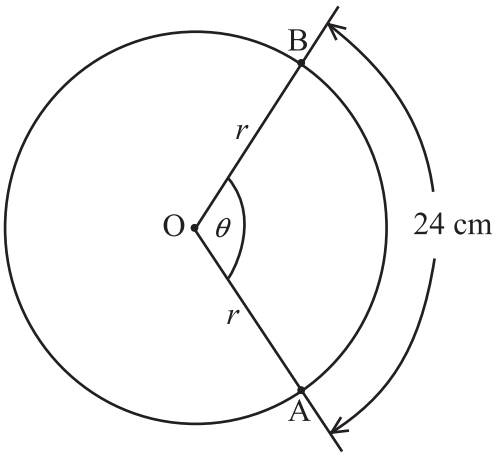
8. The diagram below shows a circle of radius r and centre O. The angle $\text{AOB} = \theta$.



The length of the arc AB is 24 cm. The area of the sector OAB is 180 cm^2.

Find the value of r and of θ.

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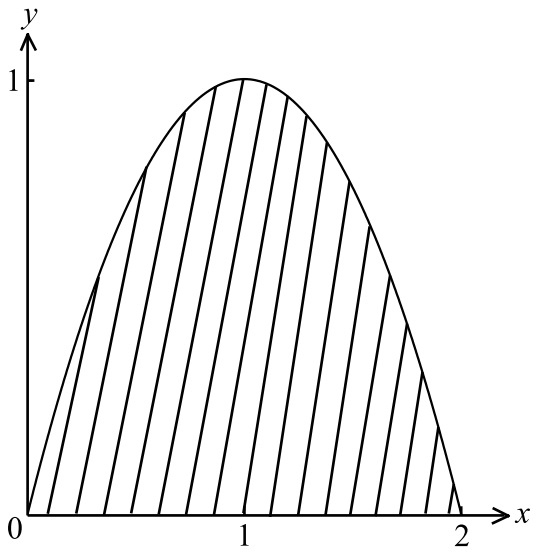
9. A part of the graph of $y = 2x - x^2$ is given in the diagram below.



The shaded region is revolved through 360° about the x-axis.

(a) Write down an expression for this volume of revolution.

(b) Calculate this volume.

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10. The matrix $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 0 \\ -3 & 1 & -1 \\ 2 & -2 & 1 \end{pmatrix}$ has inverse $\boldsymbol{A}^{-1} = \begin{pmatrix} -1 & -2 & -2 \\ 1 & 1 & 1 \\ a & 6 & b \end{pmatrix}$.

(a) Write down the value of

(i) a;

(ii) b.

Consider the simultaneous equations

$$\begin{aligned} x + 2y &= 7 \\ -3x + y - z &= 10 \\ 2x - 2y + z &= -12 \end{aligned}$$

(b) Write these equations as a matrix equation.

(c) Solve the matrix equation.

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11. Consider the function $f : x \mapsto 3x^2 - 5x + k$.

(a) Write down $f'(x)$.

The equation of the tangent to the graph of f at $x = p$ is $y = 7x - 9$. Find the value of

(b) p;

(c) k.

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12. In a class, 40 students take chemistry only, 30 take physics only, 20 take both chemistry and physics, and 60 take neither.

(a) Find the probability that a student takes physics given that the student takes chemistry.

(b) Find the probability that a student takes physics given that the student does **not** take chemistry.

(c) State whether the events "taking chemistry" and "taking physics" are mutually exclusive, independent, or neither. Justify your answer.

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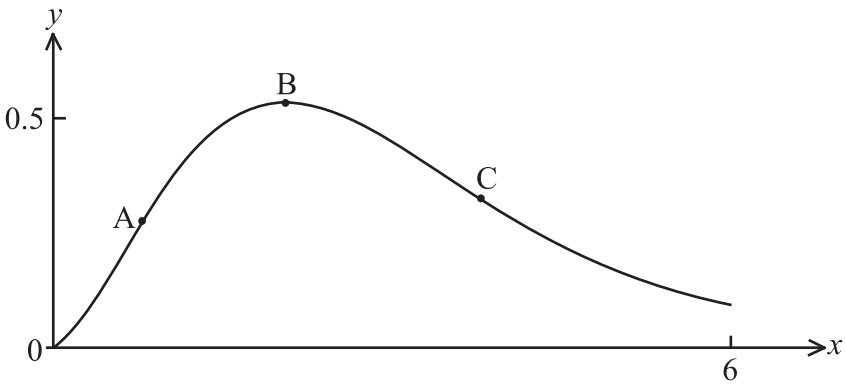
13. The diagram below shows the graph of $f(x) = x^2 e^{-x}$ for $0 \leq x \leq 6$. There are points of inflexion at A and C and there is a maximum at B.



(a) Using the product rule for differentiation, find $f'(x)$.

(b) Find the **exact** value of the ***y*-coordinate** of B.

(c) The second derivative of f is $f''(x) = (x^2 - 4x + 2)e^{-x}$. Use this result to find the **exact** value of the x-coordinate of C.

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14. The displacement s metres at time t seconds is given by

$$s = 5\cos 3t + t^2 + 10, \text{ for } t \geq 0.$$

(a) Write down the minimum value of s.

(b) Find the acceleration, a, at time t.

(c) Find the value of t when the **maximum** value of a first occurs.

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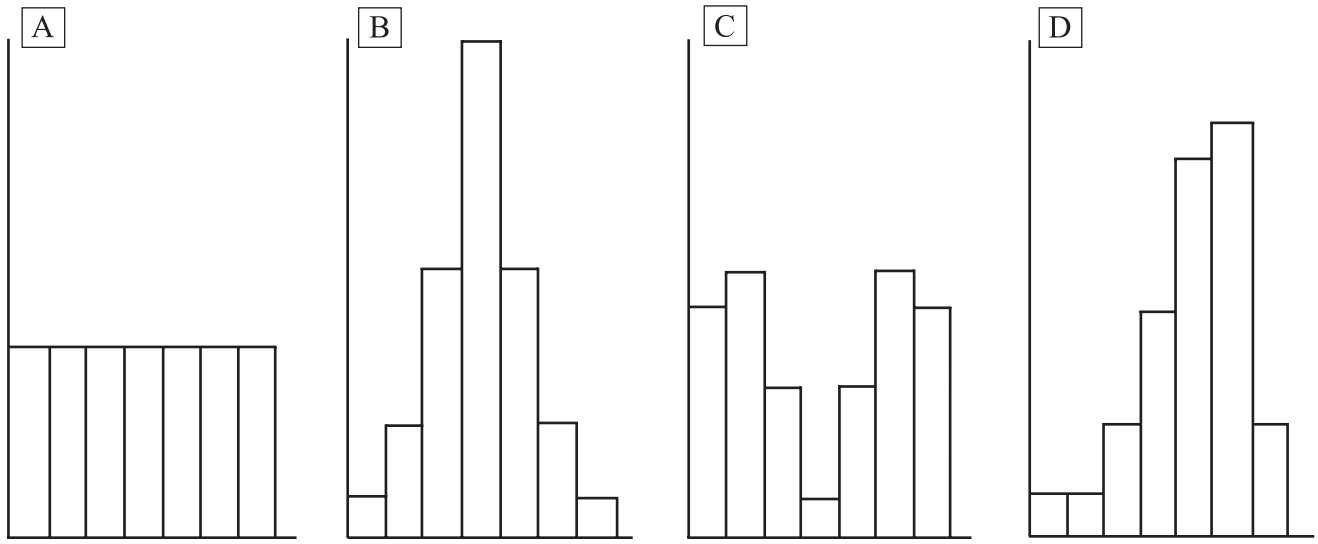
15. The four populations A, B, C and D are the same size and have the same range. Frequency histograms for the four populations are given below.



(a) Each of the three box and whisker plots below corresponds to one of the four populations. Write the letter of the correct population under each plot.

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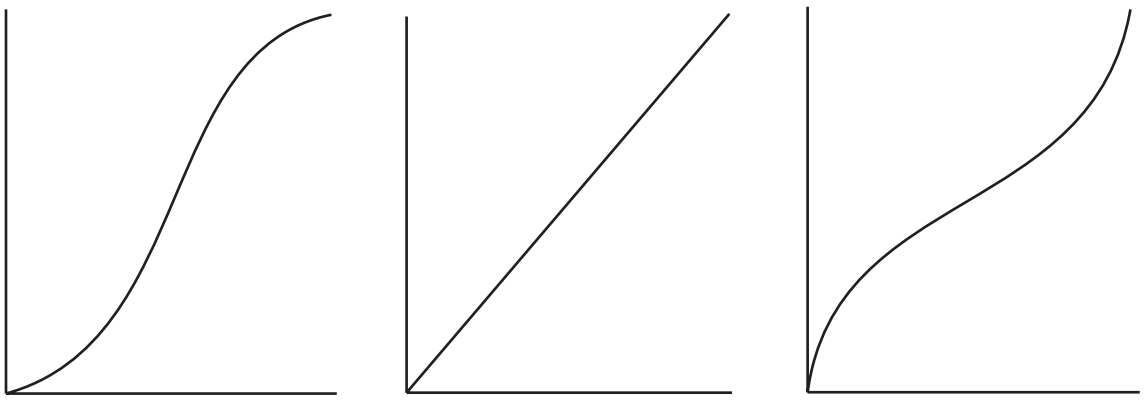
(b) Each of the three cumulative frequency diagrams below corresponds to one of the four populations. Write the letter of the correct population under each diagram.



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