

Mathematical studies
Standard level
Paper 2

Thursday 12 November 2015 (afternoon)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 14]

A shop sells strawberry, coffee and orange flavoured chocolates. 50 customers were asked which flavoured chocolates they had bought.

The results are as follows:

7 bought only strawberry chocolates
6 bought only coffee chocolates
10 bought only orange chocolates
3 bought coffee and strawberry, **but not** orange
5 bought strawberry and orange, **but not** coffee
4 bought orange and coffee, **but not** strawberry
 x bought strawberry, coffee and orange.

- (a) Represent this information on a Venn diagram. [4]
- (b) Find the value of x , given that 13 of these 50 customers did not buy any chocolates. [2]
- (c) (i) Find the probability that a customer, chosen at random from these 50 customers, bought strawberry flavoured chocolates.
- (ii) Find the probability that a customer, chosen at random from these 50 customers, bought orange flavoured chocolates.
- (iii) Determine whether the events in parts (c)(i) and (c)(ii) are independent. Give a reason for your answer. [6]
- (d) A customer is chosen at random from these 50 customers. Given that the customer bought coffee flavoured chocolates, find the probability that they also bought strawberry flavoured chocolates. [2]

2. [Maximum mark: 19]

The following table shows the internal assessment marks and examination marks for six students.

| | | | | | | |
|---|----|----|----|----|----|----|
| Internal assessment marks (x) | 4 | 10 | 12 | 16 | 18 | 20 |
| Examination marks (y) | 35 | 45 | 52 | 55 | 65 | 70 |

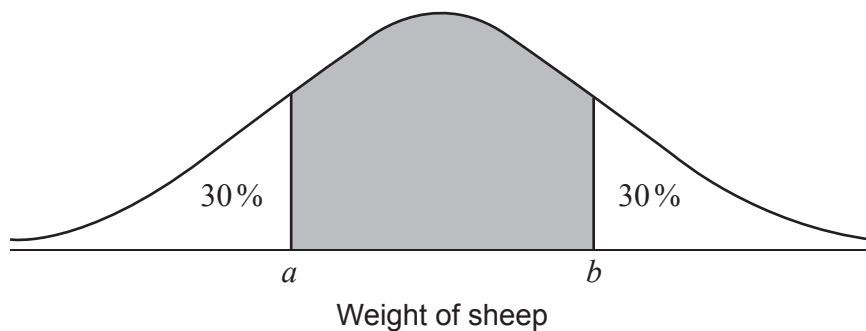
- (a) **On graph paper**, draw a scatter diagram for the above data. Use a scale of 1 cm to represent 2 marks on the x -axis and 1 cm to represent 10 marks on the y -axis. [3]
- (b) (i) Write down the Pearson's product-moment correlation coefficient, r , for the above data.
- (ii) Describe the correlation between internal assessment marks and examination marks. [4]
- (c) Find the
- (i) mean internal assessment mark, \bar{x} ;
- (ii) mean examination mark, \bar{y} . [2]
- (d) Plot and label the point $M(\bar{x}, \bar{y})$ on your scatter diagram from part (a). [2]
- (e) Write down the equation of the regression line for y on x . [2]
- (f) Use your **equation** from part (e) to estimate the examination mark of a student who scored 8 marks on their internal assessment. [2]
- (g) Draw the regression line y on x on your scatter diagram from part (a). [2]
- A **new student** scores 30 marks on her internal assessment and uses the above data to estimate that she will score 89 marks on the examination.
- (h) State whether this estimate is reliable and give a reason for your answer. [2]

3. [Maximum mark: 13]

The weights of sheep on a farm are normally distributed with a mean of 110 kg and a standard deviation of 8 kg.

- (a) Sketch a diagram of the distribution of the weights of these sheep. On your diagram, label the mean and label one standard deviation above and below the mean. [2]
- (b) (i) A sheep has a weight of 94 kg. Write down the number of standard deviations that this weight is below the mean. [2]
- (ii) Find the probability that a sheep, chosen at random, weighs more than 94 kg. [3]
- (c) (i) Find the probability that a sheep, chosen at random, weighs between 88 kg and 116 kg. [2]
- (ii) The farmer weighs 160 sheep. Find the number of sheep that he would expect to weigh between 88 kg and 116 kg. [4]
- (d) Given that 75% of the sheep weigh **less than** w kg, find the value of w . [2]

A sheep is chosen at random. Its weight is within the central shaded region of the following diagram.



- (e) Find the value of a and of b . [2]

4. [Maximum mark: 15]

A lake is in the shape of a triangle, ABC , where AB , BC and CA are paths around the lake. The paths have the following lengths: $AB = 140\text{ m}$, $BC = 110\text{ m}$ and $CA = 120\text{ m}$.

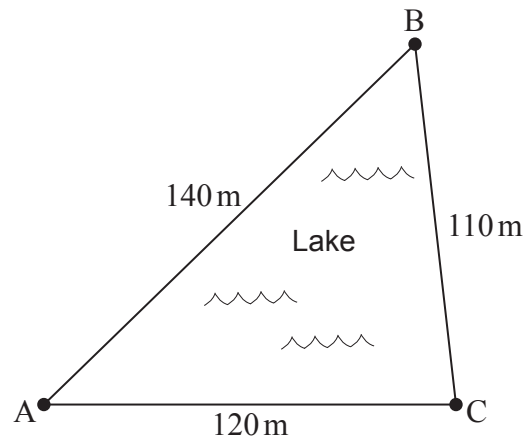


diagram not to scale

D
•

(a) Find the size of angle \hat{BAC} . [3]

(b) Find the surface area of the lake. [3]

A farmhouse is located at point D , away from the lake, such that angle \hat{DBC} is 80° and angle \hat{BCD} is 40° . The farmer has built straight paths from her farmhouse, D , to points B and C .

(c) Show that angle $\hat{BDC} = 60^\circ$. [1]

(d) Find the distance between C and D . [3]

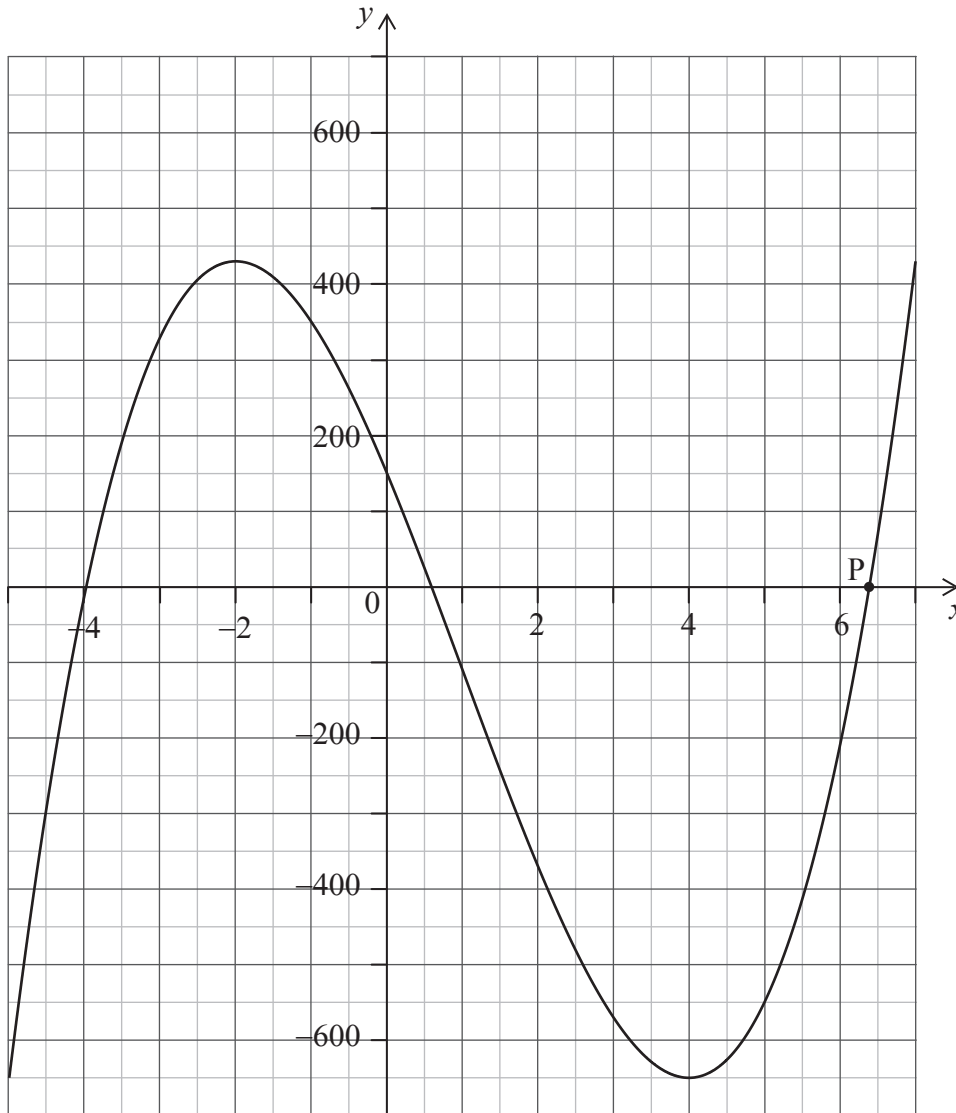
The farmer's tractor is located at point A , on the other side of the lake. The farmer walks along the straight paths from her farmhouse, D , and around the lake, to the tractor at A .

(e) Show that the **shortest** possible route that the farmer can walk passes through point B . [5]

5. [Maximum mark: 17]

The following diagram shows the graph of the function

$$f(x) = nx^3 + px^2 + qx + r, \quad n \neq 0, \quad \text{for } -5 \leq x \leq 7.$$



(a) State whether the function is increasing or decreasing at $x = -3$. Give a reason for your answer. [2]

(b) Write down the value of r . [1]

The values of p and q are such that $f(x) = nx^3 - 30x^2 - 240x + r$.

(c) Find $f'(x)$. [3]

(This question continues on the following page)

(Question 5 continued)

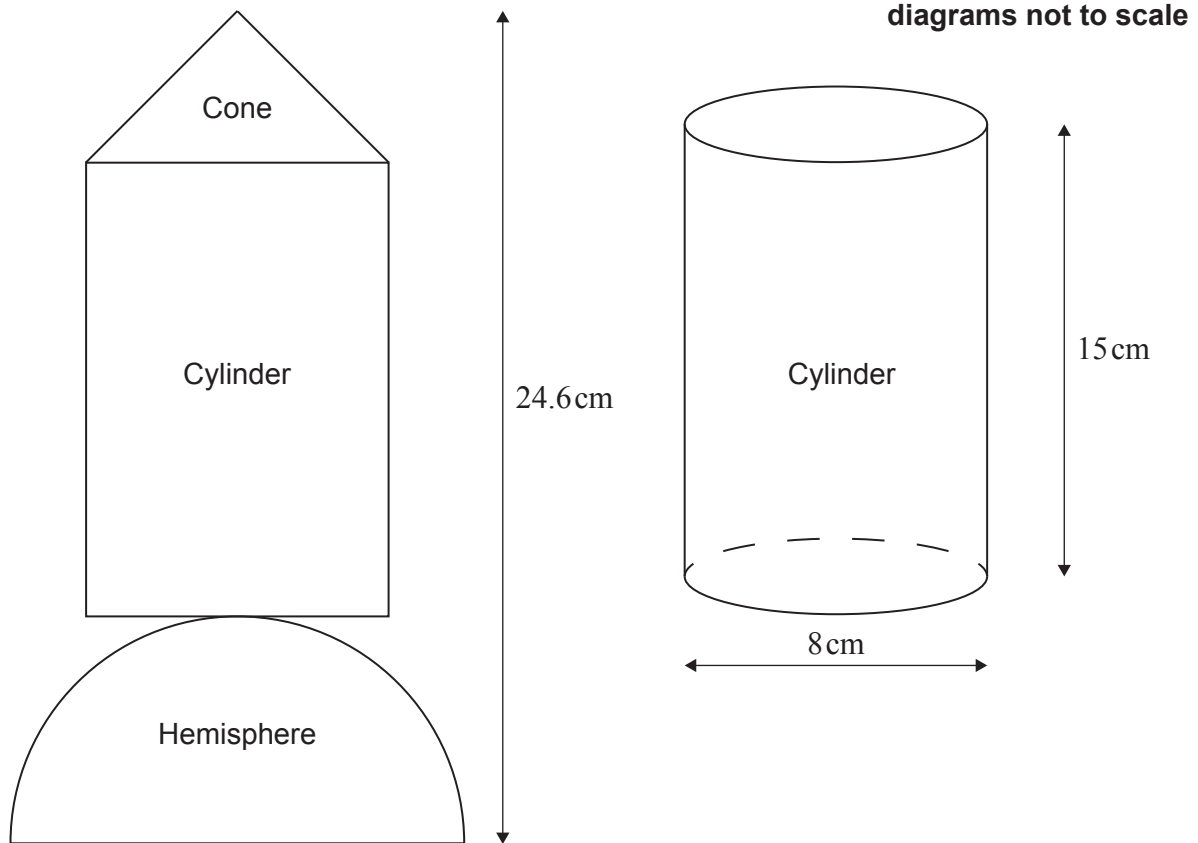
- (d) Write down the coordinates of the local minimum point. [1]
- (e) Show that the value of n is 10. [2]
- (f) (i) Calculate $f'(-1)$.
- (ii) Find the equation of the tangent to the graph at the point $(-1, 350)$. Give your answer in the form $ax + by + d = 0$.
- (iii) Write down the gradient of the normal to the graph at $x = -1$. [5]

The graph of the function intersects the x -axis at point P, as shown in the diagram.

- (g) Use your graphic display calculator to find the x -coordinate of P. [1]
- (h) Let $g(x) = 100x + 400$, for $-5 \leq x \leq 7$. Use your graphic display calculator to find the values of x where $f(x) = g(x)$, for the given domain. [2]

6. [Maximum mark: 12]

Yutaka makes a toy spaceship. It has three separate solid parts: a cone, a cylinder and a hemisphere. This toy is represented by the following two-dimensional diagram. The cylinder is also shown in a separate diagram.



The cylinder has height 15 cm and diameter 8 cm.

- (a) Find the volume of the cylinder. [2]

The cone has diameter 8 cm and volume 85 cm^3 .

- (b) Find the height of the cone. [2]

The toy spaceship's total height is 24.6 cm.

- (c) Find the volume of the hemisphere. [4]

Yutaka decides to paint the cone of the toy spaceship.

- (d) Calculate the curved surface area of the cone. Give your answer to the nearest cm^2 . [4]