Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Tuesday 12 May 2015 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks]**.

12 pages

2215 – 7205

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

A and B are two events such that $\mathrm{P}(A) = 0.25$, $\mathrm{P}(B) = 0.6$ and $\mathrm{P}(A \cup B) = 0.7$.

(a) Find $\mathrm{P}(A \cap B)$. [2]

(b) Determine whether events A and B are independent. [2]

2. [Maximum mark: 4]

Expand $(3-x)^4$ in ascending powers of x and simplify your answer.

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3. [Maximum mark: 6]

Find all solutions to the equation $\tan x + \tan 2x = 0$ where $0^\circ \le x < 360^\circ$.

4. [Maximum mark: 7]

Consider the function defined by $f(x) = x^3 - 3x^2 + 4$.

(a) Determine the values of x for which $f(x)$ is a decreasing function. [4]

There is a point of inflexion, P, on the curve $y = f(x)$.

(b) Find the coordinates of P. [3]

5. [Maximum mark: 6]

Show that $\int_{1}^{2} x^3 \ln x \, dx = 4\ln 2 - \frac{15}{16}$.

6. [Maximum mark: 8]

In triangle ABC, $\text{BC} = \sqrt{3}\,\text{cm}$, $\text{A}\hat{\text{B}}\text{C} = \theta$ and $\text{B}\hat{\text{C}}\text{A} = \frac{\pi}{3}$.

(a) Show that length $\text{AB} = \frac{3}{\sqrt{3}\cos\theta + \sin\theta}$. [4]

(b) Given that AB has a minimum value, determine the value of θ for which this occurs. [4]

7. [Maximum mark: 9]

(a) Find three distinct roots of the equation $8z^3 + 27 = 0$, $z \in \mathbb{C}$ giving your answers in modulus-argument form. [6]

The roots are represented by the vertices of a triangle in an Argand diagram.

(b) Show that the area of the triangle is $\frac{27\sqrt{3}}{16}$. [3]

8. [Maximum mark: 8]

By using the substitution $t = \tan x$, find $\int \frac{\mathrm{d}x}{1+\sin^2 x}$.

Express your answer in the form $m \arctan(n \tan x) + c$, where m, n are constants to be determined.

9. [Maximum mark: 8]

(a) State the set of values of a for which the function $x \mapsto \log_a x$ exists, for all $x \in \mathbb{R}^+$. [2]

(b) Given that $\log_x y = 4\log_y x$, find all the possible expressions of y as a function of x. [6]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 17]

The function f is defined by $f(x)=\dfrac{3x}{x-2}$, $x \in \mathbb{R}$, $x \neq 2$.

(a) Sketch the graph of $y = f(x)$, indicating clearly any asymptotes and points of intersection with the x and y axes. [4]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Find all values of x for which $f(x) = f^{-1}(x)$. [3]

(d) Solve the inequality $|f(x)| < \dfrac{3}{2}$. [4]

(e) Solve the inequality $f(|x|) < \dfrac{3}{2}$. [2]

11. [Maximum mark: 16]

Consider the functions $f(x) = \tan x$, $0 \le x < \dfrac{\pi}{2}$ and $g(x) = \dfrac{x+1}{x-1}$, $x \in \mathbb{R}$, $x \neq 1$.

(a) Find an expression for $g \circ f(x)$, stating its domain. [2]

(b) Hence show that $g \circ f(x) = \dfrac{\sin x + \cos x}{\sin x - \cos x}$. [2]

(c) Let $y = g \circ f(x)$, find an exact value for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at the point on the graph of $y = g \circ f(x)$ where $x = \dfrac{\pi}{6}$, expressing your answer in the form $a + b\sqrt{3}$, $a, b \in \mathbb{Z}$. [6]

(d) Show that the area bounded by the graph of $y = g \circ f(x)$, the x-axis and the lines $x = 0$ and $x = \dfrac{\pi}{6}$ is $\ln\left(1+\sqrt{3}\right)$. [6]

Do **not** write solutions on this page.

12. [Maximum mark: 14]

The cubic equation $x^3 + px^2 + qx + c = 0$, has roots α, β, γ. By expanding $(x-\alpha)(x-\beta)(x-\gamma)$ show that

(a) (i) $p = -(\alpha + \beta + \gamma)$;

(ii) $q = \alpha\beta + \beta\gamma + \gamma\alpha$;

(iii) $c = -\alpha\beta\gamma$. [3]

It is now given that $p = -6$ and $q = 18$ for parts (b) and (c) below.

(b) (i) In the case that the three roots α, β, γ form an arithmetic sequence, show that one of the roots is 2.

(ii) Hence determine the value of c. [5]

(c) In another case the three roots α, β, γ form a geometric sequence. Determine the value of c. [6]

13. [Maximum mark: 13]

(a) Show that $\frac{1}{\sqrt{n} + \sqrt{n+1}} = \sqrt{n+1} - \sqrt{n}$ where $n \geq 0,\ n \in \mathbb{Z}$. [2]

(b) Hence show that $\sqrt{2} - 1 < \frac{1}{\sqrt{2}}$. [2]

(c) Prove, by mathematical induction, that $\sum_{r=1}^{r=n} \frac{1}{\sqrt{r}} > \sqrt{n}$ for $n \geq 2,\ n \in \mathbb{Z}$. [9]