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International Baccalaureate®  
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**MATHEMATICS  
HIGHER LEVEL  
PAPER 3 – SERIES AND DIFFERENTIAL EQUATIONS**

Tuesday 19 November 2013 (afternoon)

1 hour

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**INSTRUCTIONS TO CANDIDATES**

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **Mathematics HL and Further Mathematics SL information booklet** is required for this paper.
- The maximum mark for this examination paper is [60 marks].

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 10]

Consider the infinite series  $\sum_{n=1}^{\infty} \frac{2}{n^2 + 3n}$ .

(a) Use a comparison test to show that the series converges. [2]

(b) (i) Express  $\frac{2}{n^2 + 3n}$  in partial fractions.

(ii) Hence find the value of  $\sum_{n=1}^{\infty} \frac{2}{n^2 + 3n}$ . [8]

2. [Maximum mark: 9]

The general term of a sequence  $\{a_n\}$  is given by the formula  $a_n = \frac{e^n + 2^n}{2e^n}$ ,  $n \in \mathbb{Z}^+$ .

(a) Determine whether the sequence  $\{a_n\}$  is decreasing or increasing. [3]

(b) Show that the sequence  $\{a_n\}$  is convergent and find the limit  $L$ . [2]

(c) Find the smallest value of  $N \in \mathbb{Z}^+$  such that  $|a_n - L| < 0.001$ , for all  $n \geq N$ . [4]

3. [Maximum mark: 19]

Consider the differential equation  $\frac{dy}{dx} = \frac{y}{x + \sqrt{xy}}$ , for  $x, y > 0$ .

(a) Use Euler's method starting at the point  $(x, y) = (1, 2)$ , with interval  $h = 0.2$ , to find an approximate value of  $y$  when  $x = 1.6$ . [7]

(b) Use the substitution  $y = vx$  to show that  $x \frac{dv}{dx} = \frac{v}{1 + \sqrt{v}} - v$ . [3]

(c) (i) Hence find the solution of the differential equation in the form  $f(x, y) = 0$ , given that  $y = 2$  when  $x = 1$ .

(ii) Find the value of  $y$  when  $x = 1.6$ . [9]

## 4. [Maximum mark: 13]

Let  $g(x) = \sin x^2$ , where  $x \in \mathbb{R}$ .

(a) Using the result  $\lim_{t \rightarrow 0} \frac{\sin t}{t} = 1$ , or otherwise, calculate  $\lim_{x \rightarrow 0} \frac{g(2x) - g(3x)}{4x^2}$ . [4]

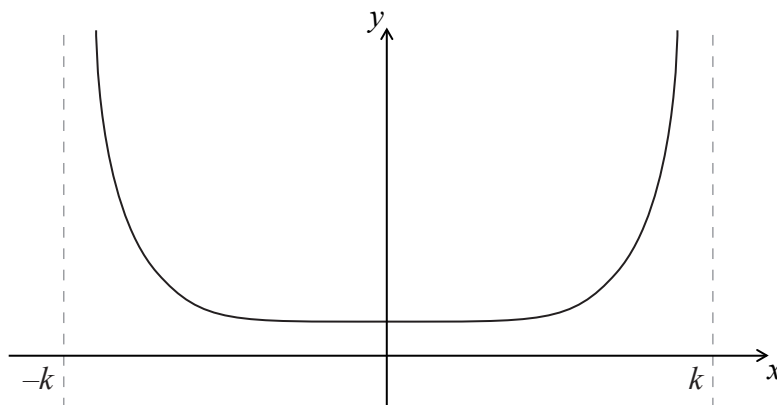
(b) Use the Maclaurin series of  $\sin x$  to show that  $g(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{(2n+1)!}$ . [2]

(c) Hence determine the minimum number of terms of the expansion of  $g(x)$  required to approximate the value of  $\int_0^1 g(x) dx$  to four decimal places. [7]

## 5. [Maximum mark: 9]

A function  $f$  is defined in the interval  $] -k, k[$ , where  $k > 0$ . The gradient function  $f'$  exists at each point of the domain of  $f$ .

The following diagram shows the graph of  $y = f(x)$ , its asymptotes and its vertical symmetry axis.



(a) Sketch the graph of  $y = f'(x)$ . [2]

Let  $p(x) = a + bx + cx^2 + dx^3 + \dots$  be the Maclaurin expansion of  $f(x)$ .

(b) (i) Justify that  $a > 0$ .  
 (ii) Write down a condition for the largest set of possible values for each of the parameters  $b$ ,  $c$  and  $d$ . [5]

(c) State, with a reason, an upper bound for the radius of convergence. [2]