N15/5/MATME/SP2/ENG/TZ0/XX

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Thursday 12 November 2015 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

12 pages

8815–7302

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

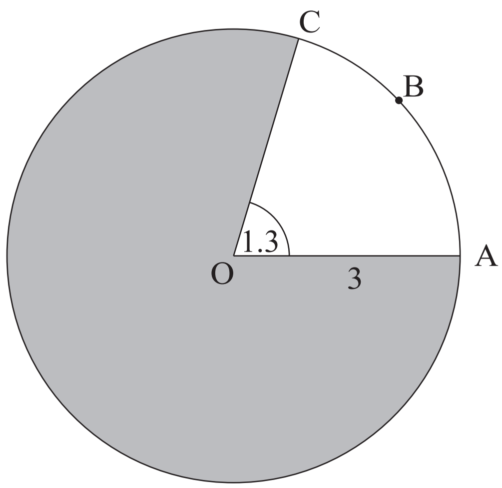
Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius 3 cm.

diagram not to scale



Points A, B, and C lie on the circle, and $\hat{AOC} = 1.3$ radians.

(a) Find the length of arc ABC. [2]

(b) Find the area of the shaded region. [4]

2. [Maximum mark: 5]

The following table shows the probability distribution of a discrete random variable X.

x	0	1	2	3
$\mathrm{P}(X=x)$	0.15	k	0.1	$2k$

(a) Find the value of k. [3]

(b) Find $\mathrm{E}(X)$. [2]

....

....

....

....

....

....

....

....

....

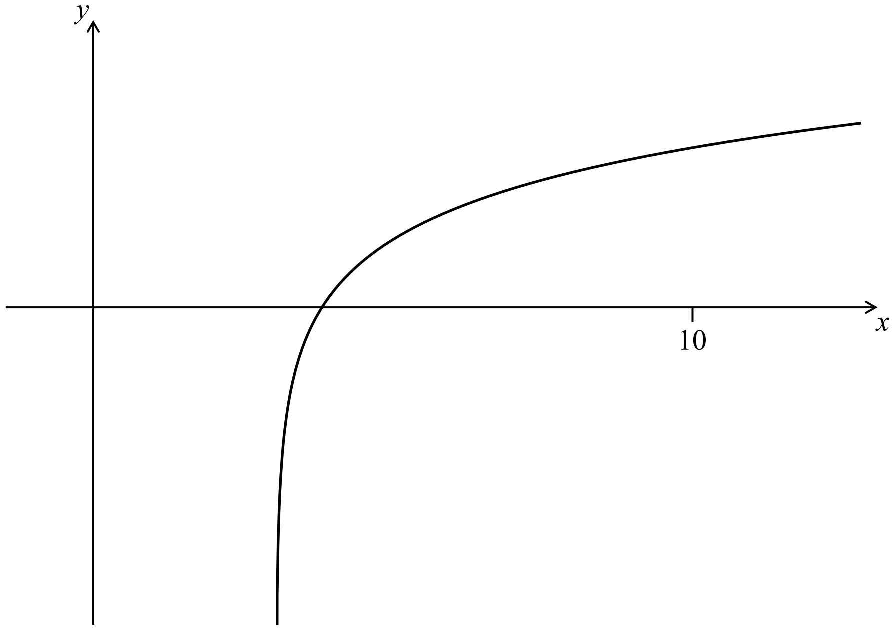
....

....

....

3. [Maximum mark: 7]

Let $f(x) = 2\ln(x - 3)$, for $x > 3$. The following diagram shows part of the graph of f.



(a) Find the equation of the vertical asymptote to the graph of f. [2]

(b) Find the x-intercept of the graph of f. [2]

(c) The region enclosed by the graph of f, the x-axis and the line $x = 10$ is rotated 360° about the x-axis. Find the volume of the solid formed. [3]

(This question continues on the following page)

(Question 3 continued)

4. [Maximum mark: 7]

The first three terms of a geometric sequence are $u_1 = 0.64$, $u_2 = 1.6$, and $u_3 = 4$.

(a) Find the value of r . [2]

(b) Find the value of S_6 . [2]

(c) Find the least value of n such that $S_n > 75\,000$. [3]

5. [Maximum mark: 7]

Let C and D be independent events, with $\mathrm{P}(C) = 2k$ and $\mathrm{P}(D) = 3k^2$, where $0 < k < 0.5$.

(a) Write down an expression for $\mathrm{P}(C \cap D)$ in terms of k. [2]

(b) Given that $\mathrm{P}(C \cap D) = 0.162$, find k. [2]

(c) Find $\mathrm{P}\left(C' \mid D\right)$. [3]

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

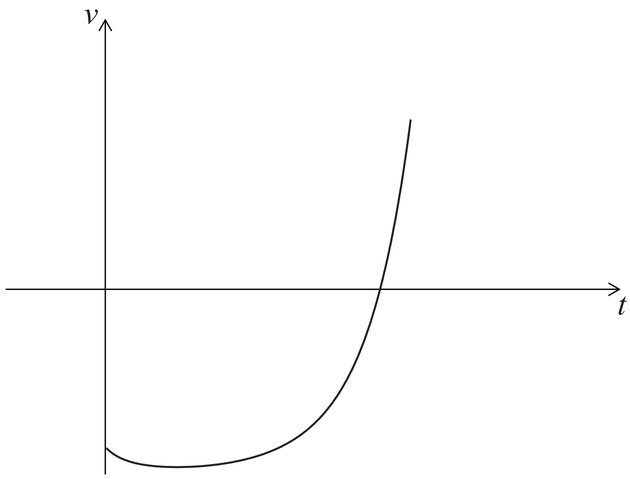
. . . .

6. [Maximum mark: 6]

The velocity $v\,\text{m}\,\text{s}^{-1}$ of a particle after t seconds is given by

$$v(t) = (0.3t + 0.1)^t - 4\text{, for } 0 \le t \le 5.$$

The following diagram shows the graph of v.



(a) Find the value of t when the particle is at rest. [3]

(b) Find the value of t when the acceleration of the particle is 0. [3]

7. [Maximum mark: 8]

Let $f(x) = \ln(x^2)$, for $x \neq 0$.

(a) Show that $f'(x) = \dfrac{2}{x}$. [2]

(b) The tangent to the graph of f at a point $\mathrm{P}\left(d, f(d)\right)$ passes through another point $\mathrm{Q}(1, -3)$. Find the value of d. [6]

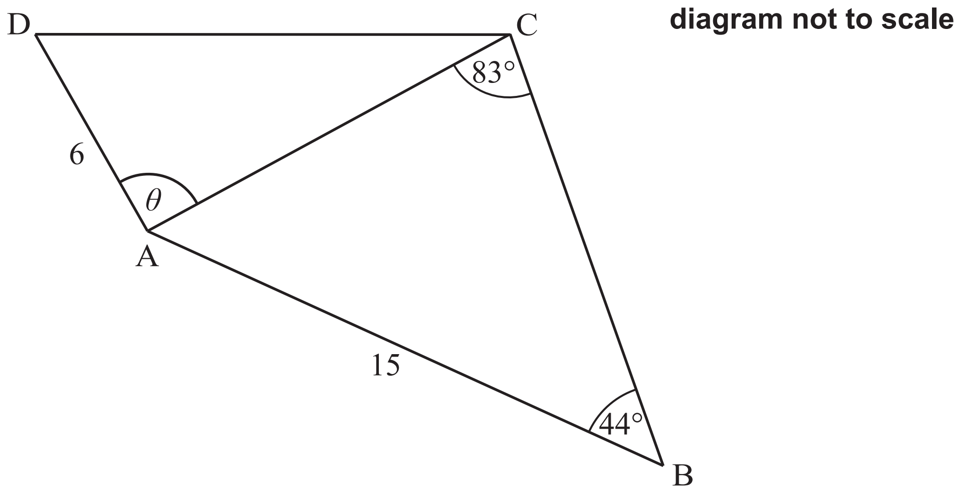
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

The following diagram shows the quadrilateral ABCD.



$AD = 6\,cm$, $AB = 15\,cm$, $A\hat{B}C = 44^\circ$, $A\hat{C}B = 83^\circ$ and $D\hat{A}C = \theta$

(a) Find AC. [3]

(b) Find the area of triangle ABC. [3]

The area of triangle ACD is half the area of triangle ABC.

(c) Find the possible values of θ. [5]

(d) Given that θ is obtuse, find CD. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

An environmental group records the numbers of coyotes and foxes in a wildlife reserve after t years, starting on 1 January 1995.

Let c be the number of coyotes in the reserve after t years. The following table shows the number of coyotes after t years.

number of years (t)	0	2	10	15	19
number of coyotes (c)	115	197	265	320	406

The relationship between the variables can be modelled by the regression equation $c = at + b$.

(a) Find the value of a and of b. [3]

(b) Use the regression equation to estimate the number of coyotes in the reserve when $t = 7$. [3]

Let f be the number of foxes in the reserve after t years. The number of foxes can be modelled by the equation $f = \dfrac{2000}{1+99e^{-kt}}$, where k is a constant.

(c) Find the number of foxes in the reserve on 1 January 1995. [3]

(d) After five years, there were 64 foxes in the reserve. Find k. [3]

(e) During which year were the number of coyotes the same as the number of foxes? [4]

Do **not** write solutions on this page.

10. [Maximum mark: 14]

The masses of watermelons grown on a farm are normally distributed with a mean of $10\,\text{kg}$. The watermelons are classified as small, medium or large.

A watermelon is small if its mass is less than $4\,\text{kg}$. Five percent of the watermelons are classified as small.

(a) Find the standard deviation of the masses of the watermelons. [4]

The following table shows the percentages of small, medium and large watermelons grown on the farm.

small	medium	large
$5\,\%$	$57\,\%$	$38\,\%$

A watermelon is large if its mass is greater than w kg.

(b) Find the value of w. [2]

All the medium and large watermelons are delivered to a grocer.

(c) The grocer selects a watermelon at random from **this** delivery. Find the probability that it is medium. [3]

(d) The grocer sells all the medium watermelons for $\$1.75$ each, and all the large watermelons for $\$3.00$ each. His costs on this delivery are $\$300$, and his total profit is $\$150$. Find the number of watermelons in the delivery. [5]