M15/5/FURMA/HP1/ENG/TZ0/XX/M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2015

Further mathematics

Higher level

Paper 1

20 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions and the document "**Mathematics HL: Guidance for e-marking May 2015**". It is essential that you read this document before you start marking. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets eg (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**).*
*A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses one mark.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 3, but calculators with symbolic manipulation features (for example, TI-89) are not allowed.

Calculator notation

The Mathematics HL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

1. $\lim_{x\to 0}(\csc x - \cot x) = \lim_{x\to 0}\left(\frac{1-\cos x}{\sin x}\right)$ ***M1A1***

$= \lim_{x\to 0}\left(\frac{\sin x}{\cos x}\right)$ ***M1A1***

$= 0$ ***A1***

Total [5 marks]

2. (a) by any method including trial and error or the Euclidean algorithm,
a specific solution is, for example, $x = 4, y = -1$ ***(A1)(A1)***

$3(4) + 5(-1) = 7$ (equation i)

$3x + 5y = 7$ (equation ii)

equation ii – equation i: $3(x-4) + 5(y+1) = 0$

$\frac{4-x}{5} = \frac{y+1}{3} = N$ ***(M1)***

$x = 4 - 5N$ ***A1***

$y = 3N - 1$ ***A1***

[5 marks]

(b) smallest positive integer > 50 occurs when $N = -10$ ***(M1)***

$x = 54,\ y = -31$ ***A1***

[2 marks]

Total [7 marks]

3. (a)

	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

A2

Note: ***A1*** for one or two errors in the table, ***A0*** otherwise.

[2 marks]

(b) closed no new elements ***A1***

0 is identity (since $0 + a = a + 0 = a,\ a \in S$) ***A1***

0, 3 self inverse, $1 \Leftrightarrow 5$ inverse pair, $2 \Leftrightarrow 4$ inverse pair ***A1***

all elements have an inverse

associativity is assumed over addition ***A1***

since symmetry on leading diagonal in table or commutativity of addition ***A1***

$\Rightarrow \{S,\ +_6\}$ is an Abelian group ***AG***

[5 marks]

continued…

Question 3 continued

(c)

Element	Order
0	1
1	6
2	3
3	2
4	3
5	6

A2

Note: ***A1*** for one or two errors in the table, ***A0*** otherwise.

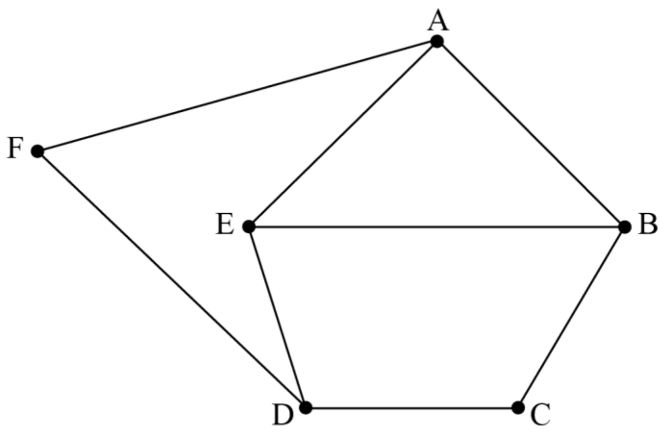
[2 marks]

(d) since there is an element with order 6 **OR** 1 or 5 are generators ***R1***
the group is cyclic ***A1***

[2 marks]

Total [11 marks]

4. (a)



A1

[1 mark]

(b) because it has vertices which are not of even degree. ***A1***

[1 mark]

(c) for example $D \to C \to B \to E \to A \to F \to D$ ***A2***

[2 marks]

(d) G is planar. ***A1***
either note from original graph in part (a) that there are no edges crossing
or a redrawing with statement that there are no edges crossing. ***R1***

Note: Do not accept an argument based on $e \le 3v - 6$

[2 marks]

continued…

Question 4 continued

(e) the graph is not bipartite. ***A1***

EITHER

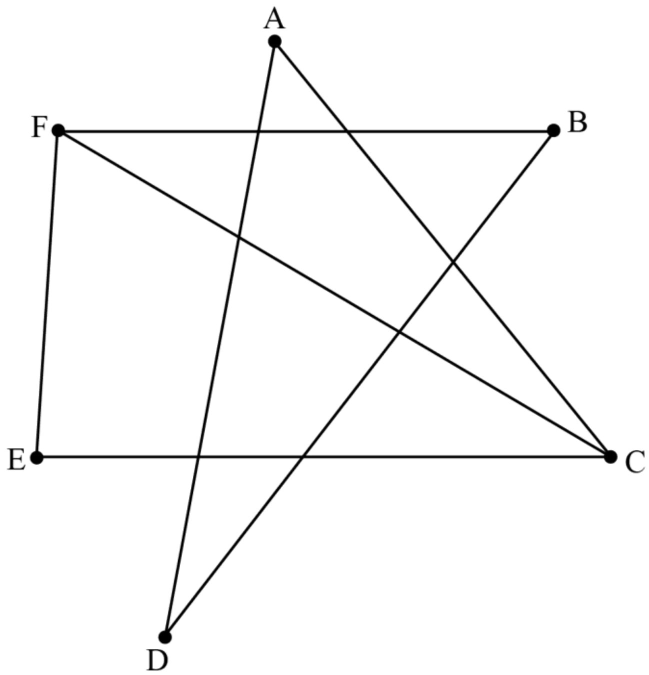
the graph contains a triangle ***R1***

OR

to be bipartite according to line 1 of the table A, C and D need to be one set as A connects to B, E and F. Since C and D are connected, this is contradicted. ***R1***

[2 marks]

(f)



A2

Note: Award ***A1*** if extra line seen or missed out.

[2 marks]

Total [10 marks]

5.

Note: In all parts accept answers which round to the correct 2sf answer.

(a) $r = 0.806$ ***A2***

[2 marks]

(b) 4.38×10^{-4} ***A2***

[2 marks]

continued…

Question 5 continued

(c) p-value represents strong evidence to indicate a (positive) association between height and foot length ***A1***

Note: FT the *p*-value

[1 mark]

(d) $y = 0.103x + 12.3$ ***A2***

[2 marks]

(e) attempted substitution of $x = 170$ ***(M1)***

$y = 29.7$ ***A1***

Note: Accept $y = 29.8$

[2 marks]

Total [9 marks]

6. ratio test $\lim_{k\to\infty}\left|\frac{u_{k+1}}{u_k}\right| = \lim_{k\to\infty}\left|\frac{(x-3)^{k+1}k^2}{(k+1)^2(x-3)^k}\right|$ ***M1A1***

$\lim_{k\to\infty}\left|(x-3)\frac{k^2}{(k+1)^2}\right|$

Note: Condone absence of limits and modulus signs in above.

$|x-3|\lim_{k\to\infty}\left|\left(\frac{1}{1+\frac{1}{k}}\right)^2\right| = |x-3|$ ***A1***

for convergence $|x-3| < 1$ ***(M1)***

$\Rightarrow -1 < x-3 < 1$

$\Rightarrow 2 < x < 4$ ***(A1)***

now we need to test end points. ***(M1)***

when $x = 4$ we have $\sum_{k=1}^{\infty}\frac{1}{k^2}$ which is a convergent series ***R1***

when $x = 2$ we have $\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2} = -1 + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} + \ldots$ which is convergent ***R1***

(alternating series/absolutely converging series)

hence the interval of convergence is $[2, 4]$ ***A1***

Total [9 marks]

7.

Note: In all parts accept answers which round to the correct 2sf answer.

(a) (i) contents: $X \sim N(750, 400)$

packaging: $Y \sim N(40, 25)$

consider $X+Y$ ***(M1)***

$\mathrm{E}(X+Y)=790$ ***A1***

$\mathrm{Var}(X+Y)=425$ ***A1***

$\mathrm{P}(X+Y>780)=0.686$ ***A1***

(ii) Let $X_1+X_2+X_3+X_4+X_5=A$ ***M1***

$\mathrm{E}(A)=5\mathrm{E}(X)=3750$ ***A1***

$\mathrm{Var}(A)=5\mathrm{Var}(X)=2000$ ***A1***

$\mathrm{P}(A>3800)=0.132$ ***A1***

Note: Condone the notation $A=5X$ if the variance is correct, ***M0*** if not

[8 marks]

(b) contents of Bright: $B \sim N(650, 256)$

let $G=B_1+B_2+B_3+B_4+B_5+B_6-(X_1+X_2+X_3+X_4+X_5)$ ***M1***

$\mathrm{E}(G)=6\times 650-5\times 750=150$ ***A1***

$\mathrm{Var}(G)=6\times 256+5\times 400=3536$ ***A1***

$\mathrm{P}(G>0)=0.994$ ***A1***

Note: Condone the notation $G=6B-5X$ if the variance is correct, ***M0*** if not

[4 marks]

Total [12 marks]

8. (a) $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2 \tan y\right)=2x\tan y+x^2\sec^2 y\frac{\mathrm{d}y}{\mathrm{d}x}$ ***M1A1A1***

[3 marks]

continued…

Question 8 continued

(b) $x^2\dfrac{\mathrm{d}y}{\mathrm{d}x}+2x\sin y\cos y=x^3\cos^2 y$ ***A1***

$\Rightarrow x^2\sec^2 y\dfrac{\mathrm{d}y}{\mathrm{d}x}+2x\tan y=x^3$ ***M1A1***

$\Rightarrow \dfrac{\mathrm{d}}{\mathrm{d}x}\left(x^2\tan y\right)=x^3$ ***A1***

$x^2\tan y=\dfrac{x^4}{4}+c$ ***A1***

Note: Condone the omission of *c* in the line above

when $x=1,\ y=0\Rightarrow c=-\dfrac{1}{4}$ ***M1***

$\tan y=\dfrac{x^2}{4}-\dfrac{1}{4x^2}$

$y=\arctan\left(\dfrac{x^2}{4}-\dfrac{1}{4x^2}\right)$ ***A1***

[7 marks]

Total [10 marks]

9. (a) $1031=1\times5^3+0\times5^2+3\times5+1$ ***M1***

$=125+0+15+1$

$=141$ ***A1***

[2 marks]

(b) $365=1\times7^3+0\times7^2+3\times7+1$ ***M1***

$\Rightarrow a_0=1,\ a_1=3,\ a_2=0,\ a_3=1$ ***A1***

[2 marks]

(c) (i) $899=6\times12^2+2\times12+11$ ***M1***

$\Rightarrow a_0=11,\ a_1=2,\ a_2=6$ ***A1***

(ii) $(899)_{10}=(62B)_{12}$

(where *B* represents the digit in base 12 given by $a_0=11$) ***A1***

Note: Accept any letter in place of *B* provided it is defined

[3 marks]

(d) 121 in base *r* is $1+2r+r^2$ ***M1A1***

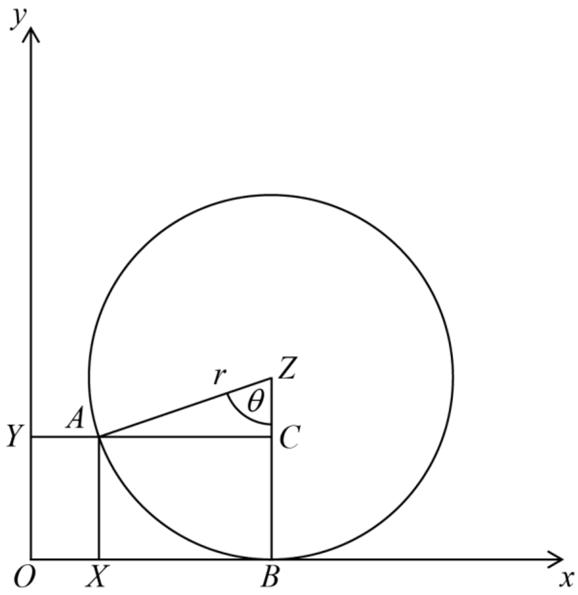
$=(r+1)^2$ ***A1***

which is a square for all r ***AG***

[3 marks]

Total [10 marks]

10. (a)



$\text{OX} = \text{OB} - \text{XB} = r\theta - r\sin\theta = x$ ***(M1)A1***

$\text{OY} = \text{ZB} - \text{ZC} = r - r\cos\theta = y$ ***A1***

[3 marks]

(b) $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = r - r\cos\theta$ ***A1***

$\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = r\sin\theta$ ***A1***

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{r\sin\theta}{r - r\cos\theta}$ ***M1***

$= \dfrac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$ ***M1A1A1***

$= \dfrac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$

$= \cot\dfrac{\theta}{2}$ ***AG***

[6 marks]

continued…

Question 10 continued

(c) when $\theta = \frac{\pi}{3}$, gradient $= \sqrt{3}$ ***A1***

$x = \frac{\pi}{3}r - r\frac{\sqrt{3}}{2}$, $y = r - \frac{r}{2} = \frac{r}{2}$ ***A1***

$y - \frac{r}{2} = \sqrt{3}\left(x - \frac{\pi}{3}r + r\frac{\sqrt{3}}{2}\right)$ or $y = \sqrt{3}x + 2r - \frac{\pi r}{\sqrt{3}}$ ***A1***

[3 marks]

Total [12 marks]

11. to be a bijection it must be injective and surjective ***R1***

Note: This ***R1*** may be awarded at any stage

suppose $f(x, y) = f(u, v)$ ***M1***
$2x + y = 2u + v$ (-i)
$x + y = u + v$ (-ii)
i-ii $\Rightarrow x = u$
i-2(ii) $\Rightarrow -y = -v$
$\Rightarrow x = u, y = v$ ***A1***
thus $(x, y) = (u, v)$ hence injective ***A1***
let $2x + y = s$ (-i)
$x + y = t$ (-ii) ***M1***
i-ii $x = s - t$
$\Rightarrow y = 2t - s$
both x and y are integer if s and t are integer ***R1***
hence it is surjective ***A1***
hence f is a bijection ***AG***

Note: Accept a valid argument based on matrices

Total [7 marks]

12. (a) (i) row reduction gives $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 3 \\ 0 & 7 & 7 \\ 0 & 3 & 3 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \left(\to \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)$ ***(M1)A1***

hence row rank is 2 ***A1***

Note: Accept the argument that Column 2 = Column 1 + Column 3

continued…

Question 12 continued

(ii) to find the kernel $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ ***M1***

Note: Allow the use of the original matrix

$x + 2y + z = 0$

$3y + 3z = 0$ ***A1***

let $z = \lambda$ ***M1***

hence $y = -\lambda$, $x = \lambda$ ***A1***

the kernel is therefore $\begin{bmatrix} \lambda \\ -\lambda \\ \lambda \end{bmatrix}$ ***A1***

[8 marks]

(b) (i) column rank is 2 ***A1***

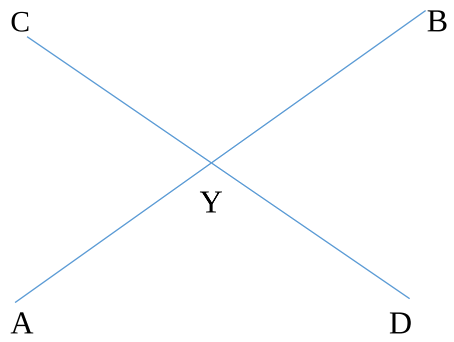
(ii) a basis for the range is defined by two independent vectors ***(M1)***

therefore a basis for the range is for example, $\begin{bmatrix} 1 \\ 2 \\ -3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 7 \\ 1 \\ 5 \end{bmatrix}$ ***A2***

[4 marks]

Total [12 marks]

13. (a) **METHOD 1**



Consider the triangles ACY and DBY *M1*

Then $YA \times YB = YC \times YD$

It follows that $\frac{YA}{YD} = \frac{YC}{YB}$ *A1*

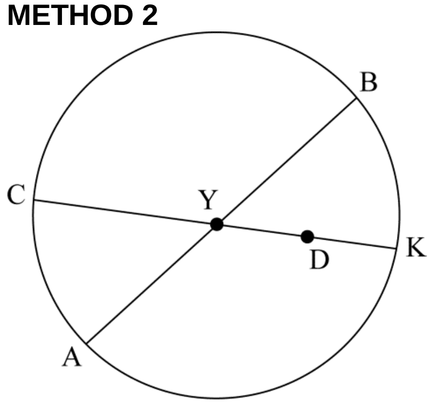
Also $A\hat{Y}C = D\hat{Y}B$ *A1*

The triangles ACY and DBY are therefore similar *A1*

So $A\hat{C}Y = D\hat{B}Y$ *A1*

Therefore by the converse to the angles subtended by a chord theorem, the points A, B, C, D lie on a circle. *R1*

METHOD 2



consider the circle passing through ABC *M1*

the circle then cuts the line (CD) at K *M1*

Note: May be seen on diagram

since Y lies inside the circle, Y divides the chord CK internally

hence K and D are on the same side of Y *(R1)*

$YA \times YB = YC \times YK$ since A, B, C and K are concyclic *M1*

$YA \times YB = YC \times YD$ given

$\Rightarrow YC \times YK = YC \times YD$ *A1*

hence K and D are the same point *R1*

the circle passes through D

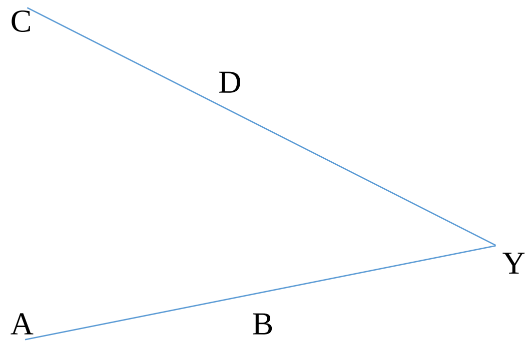
continued…

Question 13 continued

Note: Allow an argument based on similar triangles and angles in the segment Do not allow the use of the converse of the intersecting chords theorem in either (a) or (b)

[6 marks]

(b) **METHOD 1**



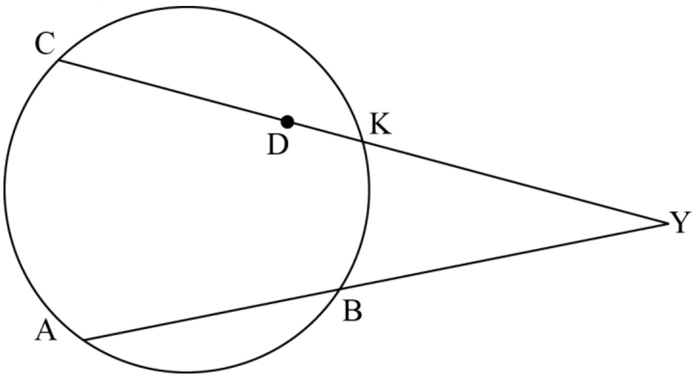
Since the triangles ACY and DBY are still similar $\mathrm{A\hat{C}Y=D\hat{B}Y}$ ***A1***

Therefore $\mathrm{A\hat{C}Y+D\hat{B}A=A\hat{C}Y+180^\circ-D\hat{B}Y}$

$=180^\circ$ ***A1***

ACDB is therefore a cyclic quadrilateral so the points A, B, C, D lie on a circle. ***R1***

METHOD 2



again consider the circle passing through ABC and again let it cut the line CD at K. ***M1***

in this case Y lies outside the circle ABC and therefore Y divides the chord CK externally. ***M1***

by the secant-secant theorem the same working applies as in part (a) ***R1***

and the proof follows identically. ***AG***

[3 marks]

Total [9 marks]

14. $\bar{X} \sim N\left(430, \frac{35^2}{n}\right)$ ***(M1)(A1)***

> **Note:** The M1 is for considering the distribution of $\bar{X}$

type I error gives $P(\bar{X} \le k / \mu = 430) = 0.0851$

$\frac{k-430}{\frac{35}{\sqrt{n}}} = -1.37156\ldots$ ***M1A1***

type II error gives $P(\bar{X} > k / \mu = 400) = 0.115$

$\frac{k-400}{\frac{35}{\sqrt{n}}} = 1.20035\ldots$ ***M1A1***

> **Note:** The two ***M1*** marks above are for attempting to standardize $\bar{X}$ and obtain the corresponding equations with inverse normal values

solving simultaneously ***(M1)***

$k = 414$ ***A1***

$n = 9$ ***A1***

Total [9 marks]

15. (a) ρ_1

$(x_1, y_1)\ \rho_1\ (x_1, y_1) \Rightarrow 0 = 0$ hence reflexive. ***R1***

$(x_1, y_1)\ \rho_1\ (x_2, y_2) \Rightarrow x_1^2 - x_2^2 = y_1^2 - y_2^2$

$\Rightarrow -(x_1^2 - x_2^2) = -(y_1^2 - y_2^2)$

$\Rightarrow x_2^2 - x_1^2 = y_2^2 - y_1^2 \Rightarrow (x_2, y_2)\ \rho_1\ (x_1, y_1)$ hence symmetric ***M1A1***

$(x_1, y_1)\ \rho_1\ (x_2, y_2) \Rightarrow x_1^2 - x_2^2 = y_1^2 - y_2^2$ - i

$(x_2, y_2)\ \rho_1\ (x_3, y_3) \Rightarrow x_2^2 - x_3^2 = y_2^2 - y_3^2$ - ii ***M1***

i+ii $\Rightarrow x_1^2 - x_3^2 = y_1^2 - y_3^2 \Rightarrow (x_1, y_1)\ \rho_1\ (x_3, y_3)$ hence transitive ***A1***

ρ_2

$(x_1, y_1)\ \rho_2\ (x_1, y_1) \Rightarrow \sqrt{2x_1^2} \le \sqrt{2y_1^2}$ This is not true in the case of $(3,1)$

hence not reflexive. ***R1***

$(x_1, y_1)\ \rho_2\ (x_2, y_2) \Rightarrow \sqrt{x_1^2 + x_2^2} \le \sqrt{y_1^2 + y_2^2}$

$\Rightarrow \sqrt{x_2^2 + x_1^2} \le \sqrt{y_2^2 + y_1^2} \Rightarrow (x_2, y_2)\ \rho_2\ (x_1, y_1)$ hence symmetric. ***A1***

it is not transitive. ***A1***

attempt to find a counterexample ***(M1)***

for example $(1,0)\rho_2(0,1)$ and $(0,1)\rho_2(1,0)$ ***A1***

however, it is not true that $(1,0)\rho_2(1,0)$ ***A1***

[11 marks]

(b) ρ_1 is an equivalence relation ***A1***

the equivalence classes for ρ_1 form a family of curves of the form

$y^2 - x^2 = k$ ***A1***

[2 marks]

Total [13 marks]

16. METHOD 1

$x^2+y^2+dx+ey+c+\lambda(lx+my+n)=0$ ***M1 A1***

ie $x^2+y^2+x(d+\lambda l)+y(e+\lambda m)+c+\lambda n=0$ ***A1***

since x^2 and y^2 have the same coefficients and there is no xy term,
this is a circle ***R1***
we know the pair of points fit the equation. ***R1***
hence this is the required equation.

METHOD 2

Let the general equation be
$x^2+y^2+ax+by+q=0$ ***M1***
The intersection with the given circle satisfies
$(a-d)x+(b-e)y+(q-c)=0$ ***M1A1***
This must be the same line as $lx+my+n=0$ ***R1***
Therefore

$a-d=\lambda l$ giving $a=d+\lambda l$

$b-e=\lambda m$ giving $b=e+\lambda m$ ***A1***

$q-c=\lambda n$ giving $q=c+\lambda n$

leading to the required general equation

> **Note:** Award ***M1*** to candidates who only attempt to find the points of intersection of the line and circle

Total [5 marks]