

**Mathematical studies**
Standard level
Paper 1

Tuesday 12 May 2015 (morning)

Candidate session number

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1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all questions.
- Write your answers in the boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.



Please **do not** write on this page.

Answers written on this page
will not be marked.



Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Write your answers in the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. The distance d from a point $P(x, y)$ to the point $A(1, -2)$ is given by

$$d = \sqrt{(x-1)^2 + (y+2)^2}.$$

- (a) Find the distance from $P(100, 200)$ to A . Give your answer correct to two decimal places. [3]
- (b) Write down your answer to **part (a)** correct to three significant figures. [1]
- (c) Write down your answer to **part (b)** in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$. [2]

Working:

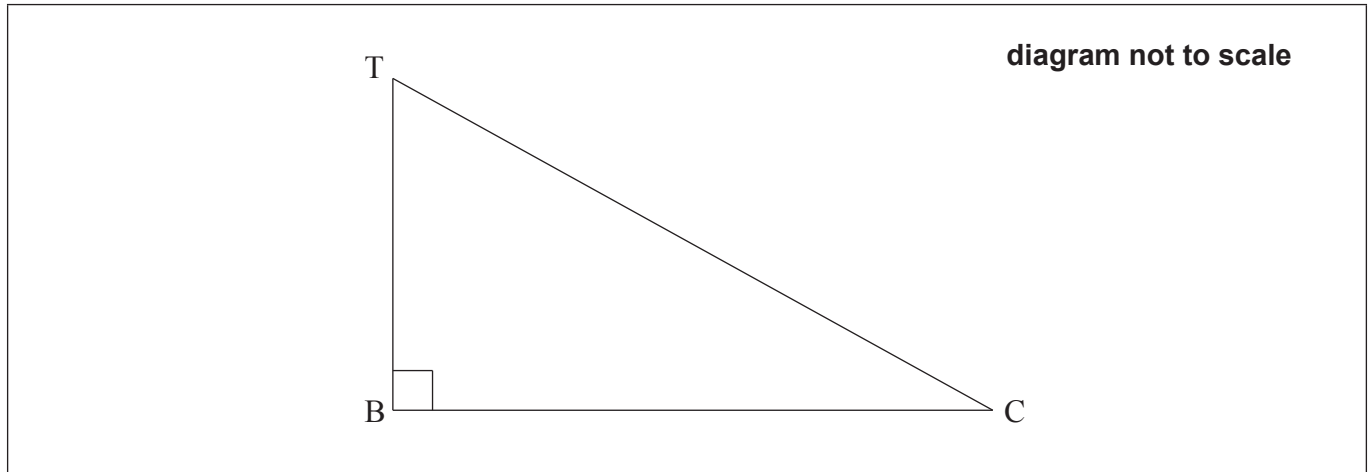
Answers:

- (a)
- (b)
- (c)



2. Fabián stands on top of a building, T, which is on a horizontal street. He observes a car, C, on the street, at an angle of depression of 30° . The base of the building is at B. The height of the building is 80 metres.

The following diagram indicates the positions of T, B and C.



- (a) Show, in the appropriate place on the diagram, **the values of**
- (i) the height of the building;
 - (ii) the angle of depression. [2]
- (b) Find the distance, BC, from the base of the building to the car. [2]
- (c) Fabián estimates that the distance from the base of the building to the car is 150 metres. Calculate the percentage error of Fabián's estimate. [2]

Working:

Answers:

(b)

(c)



3. The equation of the line L_1 is $2x + y = 10$.

(a) Write down

(i) the gradient of L_1 ;

(ii) the y -intercept of L_1 .

[2]

The line L_2 is parallel to L_1 and passes through the point $P(0, 3)$.

(b) Write down the equation of L_2 .

[2]

(c) Find the x -coordinate of the point where L_2 crosses the x -axis.

[2]

Working:

Answers:

(a) (i)

(ii)

(b)

(c)

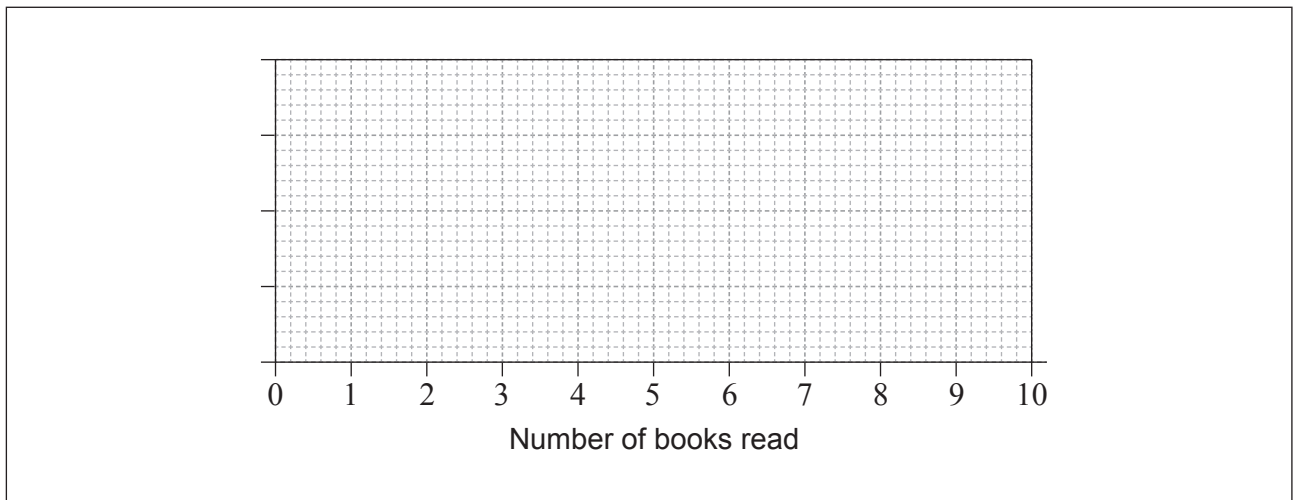


4. Two groups of 40 students were asked how many books they have read in the last two months. The results for **the first group** are shown in the following table.

| Number of books read | Frequency |
|----------------------|-----------|
| 2 | 5 |
| 3 | 8 |
| 4 | 13 |
| 5 | 7 |
| 6 | 4 |
| 7 | 2 |
| 8 | 1 |

The quartiles for these results are 3 and 5.

- (a) Write down the value of the median for these results. [1]
- (b) Draw a box-and-whisker diagram for these results on the following grid. [3]

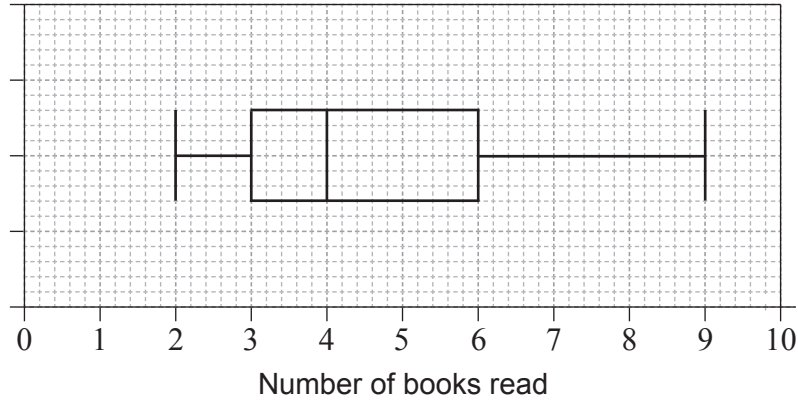


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(Question 4 continued)

The results for **the second group** of 40 students are shown in the following box-and-whisker diagram.



- (c) Estimate the number of students **in the second group** who have read at least 6 books. [2]

Working:

Answers:

- (a)
- (c)



5. Consider the propositions r , p and q .

(a) Complete the following truth table.

[4]

| r | p | q | $r \wedge p$ | $\neg q$ | $(r \wedge p) \vee \neg q$ | $\neg((r \wedge p) \vee \neg q)$ | $\neg(r \wedge p)$ | $\neg(r \wedge p) \wedge q$ |
|-----|-----|-----|--------------|----------|----------------------------|----------------------------------|--------------------|-----------------------------|
| T | T | T | | F | | | F | |
| T | T | F | | T | | | F | |
| T | F | T | | F | | | T | |
| T | F | F | | T | | | T | |
| F | T | T | | F | | | T | |
| F | T | F | | T | | | T | |
| F | F | T | | F | | | T | |
| F | F | F | | T | | | T | |

(b) Determine whether the compound proposition $\neg((r \wedge p) \vee \neg q) \Leftrightarrow \neg(r \wedge p) \wedge q$ is a tautology, a contradiction or neither.

Give a reason.

[2]

Working:

Answers:

(b)



6. Consider the following Venn diagrams. Each diagram is shaded differently.

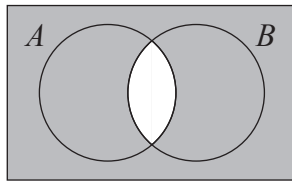


Diagram 1

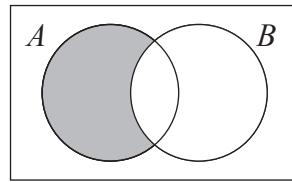


Diagram 2

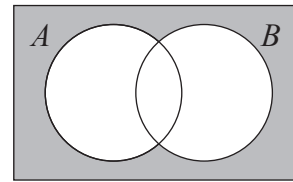


Diagram 3

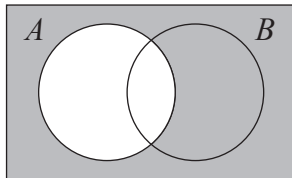


Diagram 4

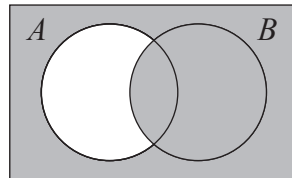


Diagram 5

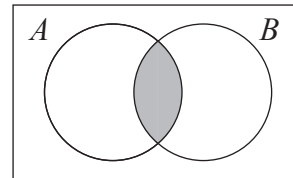


Diagram 6

In the following table there are six sets. Each of these sets corresponds to the shaded region of one of the Venn diagrams. In the correct space, write the number of the diagram that corresponds to that set.

| Set | Diagram |
|---------------|---------|
| $(A \cup B)'$ | |
| $A' \cup B'$ | |
| $A \cap B'$ | |
| $A \cap B$ | |
| $A' \cup B$ | |
| A' | |

[6]



7. The producer of a TV dancing show asked a group of 150 viewers their age and the type of Latin dance they preferred. The types of Latin dances in the show were Argentine tango, Samba, Rumba and Cha-cha-cha. The data obtained were organized in the following table.

| | Dance | | | |
|--------------------------|-----------------|-------|-------|-------------|
| | Argentine tango | Samba | Rumba | Cha-cha-cha |
| 20 years old and younger | 35 | 23 | 12 | 10 |
| Older than 20 years old | 20 | 17 | 18 | 15 |

A χ^2 test was carried out, at the 5% significance level.

- (a) Write down the null hypothesis for this test. [1]
- (b) Write down the observed number of viewers who preferred Rumba **and** were older than 20 years old. [1]
- (c) Use your graphic display calculator to find the p -value for this test. [2]

The producer claims that the type of Latin dance a viewer preferred is independent of their age.

- (d) Decide whether this claim is justified. Give a reason for your decision. [2]

(This question continues on the following page)



(Question 7 continued)

Working:

Answers:

- (a)
-
- (b)
- (c)
- (d)
-



8. The diagram shows a triangle ABC . The size of angle \hat{CAB} is 55° and the length of AM is 10 m , where M is the midpoint of AB . Triangle CMB is isosceles with $CM = MB$.

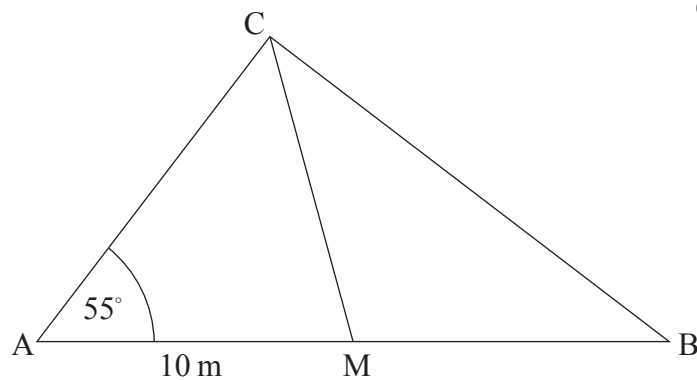


diagram not to scale

- (a) Write down the length of MB . [1]
- (b) Find the size of angle \hat{CMB} . [2]
- (c) Find the length of CB . [3]

Working:

Answers:

- (a)
- (b)
- (c)



9. Only one of the following four sequences is arithmetic and only one of them is geometric.

$$a_n = 1, 2, 3, 5, \dots$$

$$b_n = 1, \frac{3}{2}, \frac{9}{4}, \frac{27}{8}, \dots$$

$$c_n = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$$

$$d_n = 1, 0.95, 0.90, 0.85, \dots$$

- (a) State which sequence is

(i) arithmetic;

(ii) geometric.

[2]

- (b) For **another** geometric sequence $e_n = -6, -3, -\frac{3}{2}, -\frac{3}{4}, \dots$

(i) write down the common ratio;

[1]

(ii) find the **exact** value of the tenth term. Give your answer as a fraction.

[3]

Working:

Answers:

(a) (i)

(ii)

(b) (i)

(ii)



10. Minta deposits 1000 euros in a bank account. The bank pays a nominal annual interest rate of 5%, **compounded quarterly**.

(a) Find the amount of money that Minta will have in the bank after 3 years. Give your answer correct to two decimal places. [3]

Minta will withdraw the money from her bank account when the interest earned is 300 euros.

(b) Find the time, in years, until Minta withdraws the money from her bank account. [3]

Working:

Answers:

(a)

(b)



11. Consider the propositions

p : x is a multiple of 12
 q : x is a multiple of 6.

(a) Write down in words $\neg p$. [1]

(b) Write down in symbolic form the compound statement

r : If x is a multiple of 12, then x is a multiple of 6. [2]

Consider the compound statement

s : If x is a multiple of 6, then x is a multiple of 12.

(c) Identify whether s is the inverse, the converse or the contrapositive of r . [1]

(d) Determine the validity of s . Justify your decision. [2]

Working:

Answers:

(a)

.....

(b)

(c)

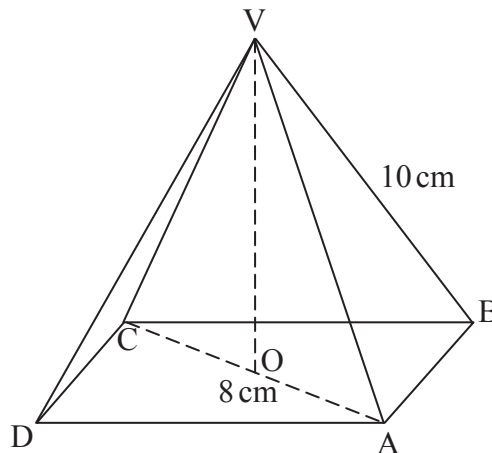
(d)

.....



12. In the following diagram, ABCD is the square base of a right pyramid with vertex V. The centre of the base is O. The diagonal of the base, AC, is 8 cm long. The sloping edges are 10 cm long.

diagram not to scale



- (a) Write down the length of AO. [1]
- (b) Find the size of the angle that the sloping edge VA makes with the base of the pyramid. [2]
- (c) Hence, or otherwise, find the area of the triangle CAV. [3]

Working:

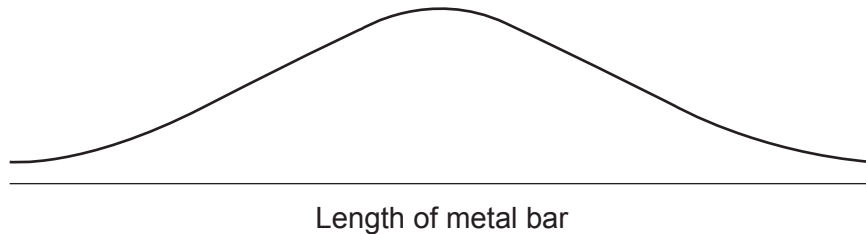
Answers:

- (a)
- (b)
- (c)



13. A factory makes metal bars. Their lengths are assumed to be normally distributed with a mean of 180 cm and a standard deviation of 5 cm.

- (a) On the following diagram, shade the region representing the probability that a metal bar, chosen at random, will have a length less than 175 cm. [2]



A metal bar is chosen at random.

- (b) (i) The probability that the length of the metal bar is less than 175 cm is equal to the probability that the length is greater than h cm. Write down the value of h .
- (ii) Find the probability that the length of the metal bar is greater than one standard deviation above the mean. [4]

Working:

Answers:

(b) (i)

(ii)



14. The number of fish, N , in a pond is decreasing according to the model

$$N(t) = ab^{-t} + 40, t \geq 0$$

where a and b are positive constants, and t is the time in months since the number of fish in the pond was first counted.

At the beginning 840 fish were counted.

- (a) Find the value of a . [2]

After 4 months 90 fish were counted.

- (b) Find the value of b . [3]

The number of fish in the pond will **not** decrease below p .

- (c) Write down the value of p . [1]

Working:

Answers:

(a)

(b)

(c)



15. A building company has many rectangular construction sites, of varying widths, along a road. The area, A , of each site is given by the function

$$A(x) = x(200 - x)$$

where x is the **width** of the site in metres and $20 \leq x \leq 180$.

- (a) Site S has a width of 20 m. Write down the area of S. [1]
- (b) Site T has the same area as site S, but a different width. Find the width of T. [2]

When the width of the construction site is b metres, the site has a maximum area.

- (c) (i) Write down the value of b .
- (ii) Write down the maximum area. [2]

The range of $A(x)$ is $m \leq A(x) \leq n$.

- (d) Hence write down the value of m and of n . [1]

Working:

Answers:

- (a)
- (b)
- (c) (i)
- (ii)
- (d)



Please **do not** write on this page.

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will not be marked.

