Diploma Programme
Programme du diplôme
Programa del Diploma

# Mathematics
# Standard level
# Paper 1

Tuesday 12 May 2015 (morning)

Candidate session number

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1 hour 30 minutes

## Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

11 pages

2215 – 7303

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

## Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

**1.** [Maximum mark: 6]

A discrete random variable $X$ has the following probability distribution.

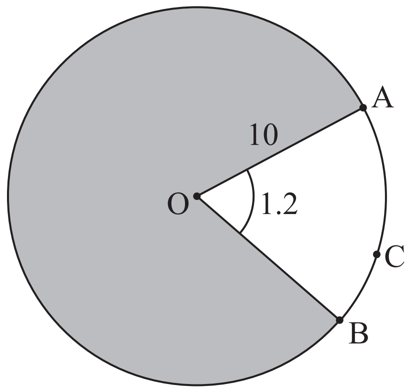
| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\mathrm{P}(X = x)$ | $\frac{3}{10}$ | $\frac{4}{10}$ | $\frac{2}{10}$ | $p$ |

(a) Find $p$. [3]

(b) Find $\mathrm{E}(X)$. [3]

**2.** [Maximum mark: 5]

The following diagram shows a circle with centre O and a radius of $10$ cm. Points A, B and C lie on the circle.




Angle AOB is $1.2$ radians.

(a) Find the length of arc ACB. [2]

(b) Find the perimeter of the shaded region. [3]

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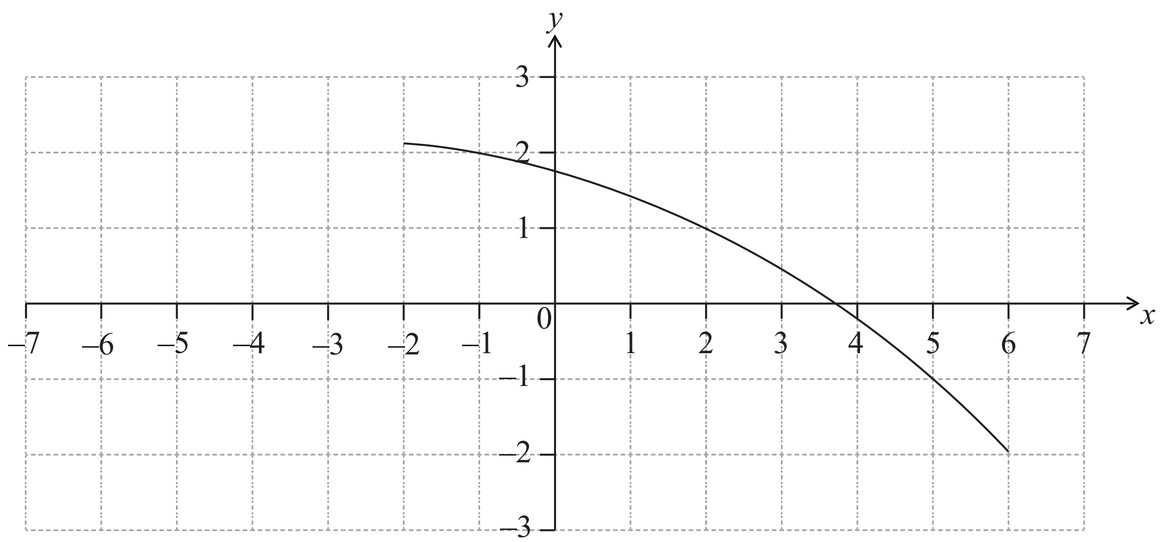
**3.** [Maximum mark: 6]

(a) Given that $2^m = 8$ and $2^n = 16$, write down the value of $m$ and of $n$. [2]

(b) Hence or otherwise solve $8^{2x+1} = 16^{2x-3}$. [4]

**4.** [Maximum mark: 7]

The following diagram shows the graph of a function $f$.




(a) Find $f^{-1}(-1)$. [2]

(b) Find $(f \circ f)(-1)$. [3]

(c) On the same diagram, sketch the graph of $y = f(-x)$. [2]

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**5.** [Maximum mark: 7]

Given that $\sin x = \frac{3}{4}$, where $x$ is an obtuse angle, find the value of

(a) $\cos x$; [4]

(b) $\cos 2x$. [3]

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**6.** [Maximum mark: 6]

Let $f(x) = px^2 + (10 - p)x + \frac{5}{4}p - 5$.

(a) Show that the discriminant of $f(x)$ is $100 - 4p^2$. [3]

(b) Find the values of $p$ so that $f(x) = 0$ has two **equal** roots. [3]

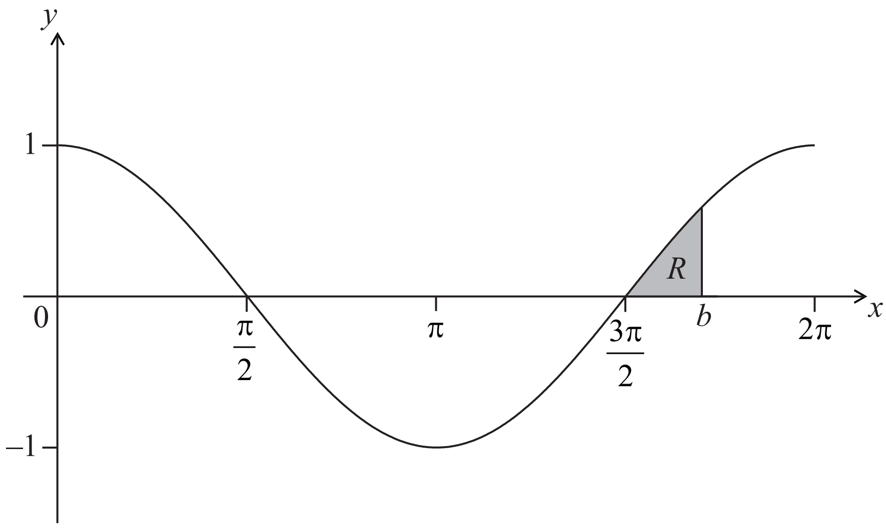


Turn over

**7.** [Maximum mark: 8]

Let $f(x) = \cos x$, for $0 \le x \le 2\pi$. The following diagram shows the graph of $f$.

There are $x$-intercepts at $x = \frac{\pi}{2}, \frac{3\pi}{2}$.




The shaded region $R$ is enclosed by the graph of $f$, the line $x = b$, where $b > \frac{3\pi}{2}$, and the $x$-axis. The area of $R$ is $\left(1 - \frac{\sqrt{3}}{2}\right)$. Find the value of $b$.

Do **not** write solutions on this page.

## Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

**8.** [Maximum mark: 16]

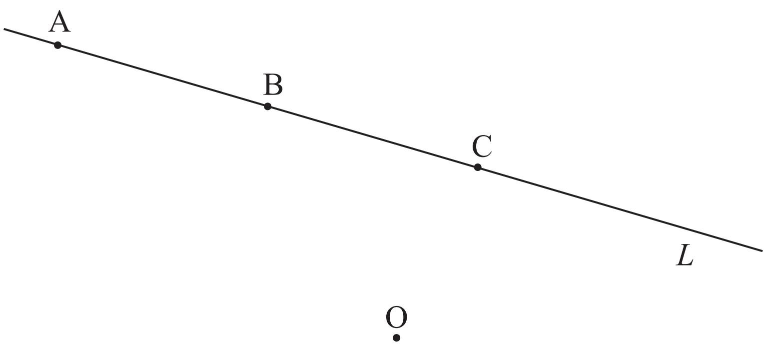
A line $L$ passes through points $\mathrm{A}(-2, 4, 3)$ and $\mathrm{B}(-1, 3, 1)$.

(a) (i) Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$.

(ii) Find $\left|\overrightarrow{\mathrm{AB}}\right|$. [3]

(b) Find a vector equation for $L$. [2]

The following diagram shows the line $L$ and the origin $\mathrm{O}$. The point $\mathrm{C}$ also lies on $L$.




Point $\mathrm{C}$ has position vector $\begin{pmatrix} 0 \\ y \\ -1 \end{pmatrix}$.

(c) Show that $y = 2$. [4]

(d) (i) Find $\overrightarrow{\mathrm{OC}} \bullet \overrightarrow{\mathrm{AB}}$.

(ii) Hence, write down the size of the angle between $\mathrm{OC}$ and $L$. [3]

(e) Hence or otherwise, find the area of triangle $\mathrm{OAB}$. [4]



Turn over

Do **not** write solutions on this page.

**9.** [Maximum mark: 14]

A function $f$ has its derivative given by $f'(x) = 3x^2 - 2kx - 9$, where $k$ is a constant.

(a) Find $f''(x)$. [2]

The graph of $f$ has a point of inflexion when $x = 1$.

(b) Show that $k = 3$. [3]

(c) Find $f'(-2)$. [2]

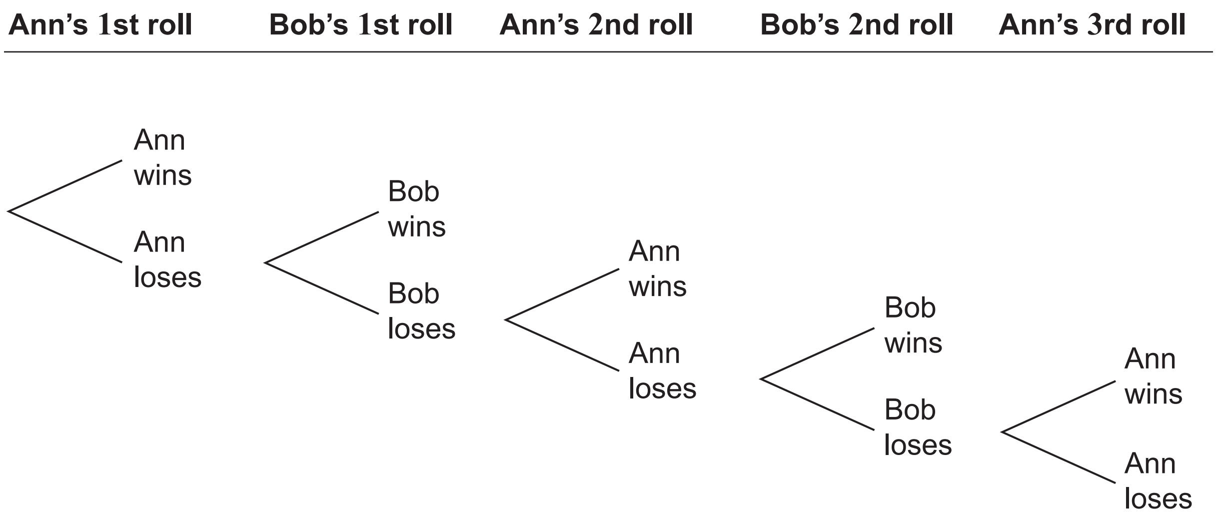
(d) Find the equation of the tangent to the curve of $f$ at $(-2, 1)$, giving your answer in the form $y = ax + b$. [4]

(e) Given that $f'(-1) = 0$, explain why the graph of $f$ has a local maximum when $x = -1$. [3]

Do **not** write solutions on this page.

**10.** [Maximum mark: 15]

Ann and Bob play a game where they each have an eight-sided die. Ann's die has three green faces and five red faces; Bob's die has four green faces and four red faces. They take turns rolling their own die and note what colour faces up. The first player to roll green wins. Ann rolls first. Part of a tree diagram of the game is shown below.




(a) Find the probability that Ann wins on her first roll. [2]

(b) (i) The probability that Ann wins on her third roll is $\frac{5}{8} \times \frac{4}{8} \times p \times q \times \frac{3}{8}$.

Write down the value of $p$ and of $q$.

(ii) The probability that Ann wins on her tenth roll is $\frac{3}{8} r^k$ where $r \in \mathbb{Q}$, $k \in \mathbb{Z}$.

Find the value of $r$ and of $k$. [6]

(c) Find the probability that Ann wins the game. [7]

Please **do not** write on this page.

Answers written on this page
will not be marked.