



No part of this product may be reproduced in any form or by any electronic or mechanical means, including information storage and retrieval systems, without written permission from the IB.

Additionally, the license tied with this product prohibits commercial use of any selected files or extracts from this product. Use by third parties, including but not limited to publishers, private teachers, tutoring or study services, preparatory schools, vendors operating curriculum mapping services or teacher resource digital platforms and app developers, is not permitted and is subject to the IB's prior written consent via a license. More information on how to request a license can be obtained from <http://www.ibo.org/contact-the-ib/media-inquiries/for-publishers/guidance-for-third-party-publishers-and-providers/how-to-apply-for-a-license>.

Aucune partie de ce produit ne peut être reproduite sous quelque forme ni par quelque moyen que ce soit, électronique ou mécanique, y compris des systèmes de stockage et de récupération d'informations, sans l'autorisation écrite de l'IB.

De plus, la licence associée à ce produit interdit toute utilisation commerciale de tout fichier ou extrait sélectionné dans ce produit. L'utilisation par des tiers, y compris, sans toutefois s'y limiter, des éditeurs, des professeurs particuliers, des services de tutorat ou d'aide aux études, des établissements de préparation à l'enseignement supérieur, des fournisseurs de services de planification des programmes d'études, des gestionnaires de plateformes pédagogiques en ligne, et des développeurs d'applications, n'est pas autorisée et est soumise au consentement écrit préalable de l'IB par l'intermédiaire d'une licence. Pour plus d'informations sur la procédure à suivre pour demander une licence, rendez-vous à l'adresse <http://www.ibo.org/fr/contact-the-ib/media-inquiries/for-publishers/guidance-for-third-party-publishers-and-providers/how-to-apply-for-a-license>.

No se podrá reproducir ninguna parte de este producto de ninguna forma ni por ningún medio electrónico o mecánico, incluidos los sistemas de almacenamiento y recuperación de información, sin que medie la autorización escrita del IB.

Además, la licencia vinculada a este producto prohíbe el uso con fines comerciales de todo archivo o fragmento seleccionado de este producto. El uso por parte de terceros —lo que incluye, a título enunciativo, editoriales, profesores particulares, servicios de apoyo académico o ayuda para el estudio, colegios preparatorios, desarrolladores de aplicaciones y entidades que presten servicios de planificación curricular u ofrezcan recursos para docentes mediante plataformas digitales— no está permitido y estará sujeto al otorgamiento previo de una licencia escrita por parte del IB. En este enlace encontrará más información sobre cómo solicitar una licencia: <http://www.ibo.org/es/contact-the-ib/media-inquiries/for-publishers/guidance-for-third-party-publishers-and-providers/how-to-apply-for-a-license>.

**Mathematics**  
**Higher level**  
**Paper 3 – discrete mathematics**

Wednesday 15 May 2019 (morning)

1 hour

---

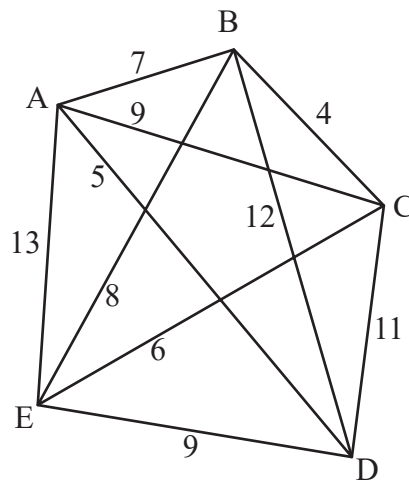
**Instructions to candidates**

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 14]

In a housing complex a TV company is installing cables to five houses A, B, C, D and E. The possible routes for the cables are given by the following graph  $G$  where each vertex represents a house and each edge represents a possible route for the cables. The weights of the edges represent the cost, in thousands of dollars, of installing the cables between the houses.



The company wants to find the lowest installation cost that includes each house and returns to the start.

- (a) Starting at A, use the nearest-neighbour algorithm to find an upper bound for the lowest installation cost. [3]
- (b) By removing A, use the deleted vertex algorithm to find a lower bound for the lowest installation cost. [5]
- (c) (i) State a route which produces the lower bound.
- (ii) State why this is a solution to finding the lowest installation cost. [2]

(This question continues on the following page)

**(Question 1 continued)**

In a second housing complex the design for installing cables between five houses P, Q, R, S and T is given in the following adjacency table.

	P	Q	R	S	T
P	–	1	0	0	1
Q	1	–	1	1	0
R	0	1	–	0	1
S	0	1	0	–	1
T	1	0	1	1	–

In the adjacency table 0 denotes no cable connecting the two houses and 1 denotes a cable connecting the two houses.

- (d) Draw a graph  $H$  to represent the second housing complex. [2]
- (e) Explain why the graph  $H$  is bipartite. [2]

**2.** [Maximum mark: 16]

- (a) Use the Euclidean algorithm to find  $\gcd(564, 254)$ . [5]
- (b) Find a general solution to the linear Diophantine equation  $564x + 254y = 94$ . [8]
- (c) Find the two solutions such that  $x, y \in [-300, 300]$ . [3]

**3.** [Maximum mark: 12]

On the 1st March in a country there are 5000 environmentally contaminated sites requiring clean-up. By the 1st April 80% of these 5000 contaminated sites are cleaned up but 200 new sites requiring clean-up are identified. This situation is assumed to recur every month. Jim sets up a first-degree recurrence relation that represents this information.

- (a) (i) State Jim's first-degree recurrence relation for the number of sites,  $u_n$ , requiring clean-up after  $n$  months in the form  $u_n = Au_{n-1} + B$ , where  $A$  and  $B$  are non-zero constants.
- (ii) State the value of  $u_0$ . [2]
- (b) Solve Jim's first-degree recurrence relation. [5]

Jim now sets up a second-degree recurrence relation that gives information regarding environmental clean-up in a different country.

The second model is  $d_n = 0.6d_{n-1} - 0.09d_{n-2}$  with initial conditions  $d_0 = d_1 = 4000$ .

- (c) Solve Jim's second-degree recurrence relation. [5]

4. [Maximum mark: 8]

(a) Using Fermat's little theorem, show that the congruence  $x^{22} + x^{11} \equiv 2 \pmod{11}$  can be expressed in the form  $(x + 6)^2 - 36 \equiv 2 \pmod{11}$ . [4]

(b) Hence solve  $x^{22} + x^{11} \equiv 2 \pmod{11}$ . [4]

---