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**Mathematics**  
**Higher level**  
**Paper 3 – sets, relations and groups**

Wednesday 15 May 2019 (morning)

1 hour

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**Instructions to candidates**

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 9]

The relation  $R$  is defined on  $\mathbb{N}$  by  $xRy \Leftrightarrow x^2 + y^2 \equiv 0 \pmod{2}$ .

(a) Show that  $R$  is an equivalence relation. [7]

(b) Determine the equivalence classes. [2]

2. [Maximum mark: 14]

Binary operations  $\circ$  and  $*$  are defined on the set of complex numbers such that  $z_1 \circ z_2 = A(z_1 + z_2)$  and  $z_1 * z_2 = Bz_1z_2$  where  $A$  and  $B$  are real non-zero constants.

(a) Determine the identity with respect to  $*$ . [2]

(b) Show that every element of the set of complex numbers, apart from  $z = 0$ , has an inverse with respect to  $*$ . [3]

(c) Find the value of  $A$  for which the operation  $\circ$  is associative. [4]

(d) Show that  $*$  is distributive over  $\circ$  for all values of  $A$  and  $B$ . [5]

## 3. [Maximum mark: 9]

A non-Abelian group,  $\{G, \circ\}$ , has eight elements  $e, a, a^2, a^3, b, ab, a^2b, a^3b$ , where  $e$  is the identity. The binary operation is  $\circ$  and, for example,  $a \circ b$  is denoted by  $ab$ .

The element  $a$  has order 4, the element  $b$  has order 2 and  $ba = a^3b$ .

(a) Prove that

(i)  $ba^2 = a^2b$ ;

(ii)  $ba^3 = ab$ .

[4]

(b) The group  $\{G, \circ\}$  has the following Cayley table. Find the elements  $P, Q, R, S, T, U, V, W$  of the group.

$\circ$	$e$	$a$	$a^2$	$a^3$	$b$	$ab$	$a^2b$	$a^3b$
$e$	$e$	$a$	$a^2$	$a^3$	$b$	$ab$	$a^2b$	$a^3b$
$a$	$a$	$a^2$	$a^3$	$e$	$ab$	$a^2b$	$a^3b$	$b$
$a^2$	$a^2$	$a^3$	$e$	$a$	$a^2b$	$a^3b$	$b$	$ab$
$a^3$	$a^3$	$e$	$a$	$a^2$	$a^3b$	$b$	$ab$	$a^2b$
$b$	$b$	$a^3b$	$a^2b$	$ab$	$e$	$a^3$	$a^2$	$a$
$ab$	$ab$	$b$	$a^3b$	$a^2b$	$a$	$e$	$R$	$U$
$a^2b$	$a^2b$	$ab$	$b$	$P$	$a^2$	$a$	$S$	$V$
$a^3b$	$a^3b$	$a^2b$	$ab$	$Q$	$a^3$	$a^2$	$T$	$W$

[4]

(c) State why  $\{G, \circ\}$  is not isomorphic to the group  $\{0, 1, 2, 3, 4, 5, 6, 7, +_8\}$ , where  $+_8$  denotes addition modulo 8.

[1]

## 4. [Maximum mark: 7]

Let  $S$  be the set of real numbers of the form  $a + b\sqrt{5}$  where  $a, b \in \mathbb{Q}$ ,  $a^2$  cannot equal  $5b^2$ , and  $a, b$  are not simultaneously zero. Show that  $S$  is a group under the operation of multiplication. Associativity may be assumed.

[7]

## 5. [Maximum mark: 11]

Consider the function  $f: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \times \mathbb{R}$  defined by  $f(x, y) = (x + y, x - y)$ .

(a) Prove that the function  $f$  is a bijection.

[9]

Consider the function  $g: \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$  defined by  $g(x, y) = (x + y, x - y)$ .

(b) Explain why the function  $g$  is not a bijection.

[2]