Diploma Programme
Programme du diplôme
Programa del Diploma

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 3 – sets, relations and groups

Wednesday 15 May 2019 (morning)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

2219 – 7209

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 9]

The relation R is defined on $\mathbb{N}$ by $xRy \Leftrightarrow x^2 + y^2 \equiv 0 \pmod 2$.

(a) Show that R is an equivalence relation. [7]

(b) Determine the equivalence classes. [2]

2. [Maximum mark: 14]

Binary operations $\circ$ and $*$ are defined on the set of complex numbers such that $z_1 \circ z_2 = A(z_1 + z_2)$ and $z_1 * z_2 = Bz_1z_2$ where A and B are real non-zero constants.

(a) Determine the identity with respect to $*$. [2]

(b) Show that every element of the set of complex numbers, apart from $z = 0$, has an inverse with respect to $*$. [3]

(c) Find the value of A for which the operation $\circ$ is associative. [4]

(d) Show that $*$ is distributive over $\circ$ for all values of A and B. [5]

3. [Maximum mark: 9]

A non-Abelian group, $\{G, \circ\}$, has eight elements $e, a, a^2, a^3, b, ab, a^2b, a^3b$, where e is the identity. The binary operation is $\circ$ and, for example, $a \circ b$ is denoted by ab.
The element a has order 4, the element b has order 2 and $ba = a^3b$.

(a) Prove that

(i) $ba^2 = a^2b$;

(ii) $ba^3 = ab$. [4]

(b) The group $\{G, \circ\}$ has the following Cayley table. Find the elements $\boldsymbol{P, Q, R, S, T, U, V, W}$ of the group.

$\circ$	e	a	a^2	a^3	b	ab	a^2b	a^3b
e	e	a	a^2	a^3	b	ab	a^2b	a^3b
a	a	a^2	a^3	e	ab	a^2b	a^3b	b
a^2	a^2	a^3	e	a	a^2b	a^3b	b	ab
a^3	a^3	e	a	a^2	a^3b	b	ab	a^2b
b	b	a^3b	a^2b	ab	e	a^3	a^2	a
ab	ab	b	a^3b	a^2b	a	e	$\boldsymbol{R}$	$\boldsymbol{U}$
a^2b	a^2b	ab	b	$\boldsymbol{P}$	a^2	a	$\boldsymbol{S}$	$\boldsymbol{V}$
a^3b	a^3b	a^2b	ab	$\boldsymbol{Q}$	a^3	a^2	$\boldsymbol{T}$	$\boldsymbol{W}$

[4]

(c) State why $\{G, \circ\}$ is not isomorphic to the group $\{\{0, 1, 2, 3, 4, 5, 6, 7\}, +_8\}$, where $+_8$ denotes addition modulo 8. [1]

4. [Maximum mark: 7]

Let S be the set of real numbers of the form $a + b\sqrt{5}$ where $a, b \in \mathbb{Q}$, a^2 cannot equal $5b^2$, and a, b are not simultaneously zero. Show that S is a group under the operation of multiplication. Associativity may be assumed. [7]

5. [Maximum mark: 11]

Consider the function $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ defined by $f(x, y) = (x + y, x - y)$.

(a) Prove that the function f is a bijection. [9]

Consider the function $g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined by $g(x, y) = (x + y, x - y)$.

(b) Explain why the function g is not a bijection. [2]