Diploma Programme
Programme du diplôme
Programa del Diploma

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Mathematics
Standard level
Paper 1

Monday 13 May 2019 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

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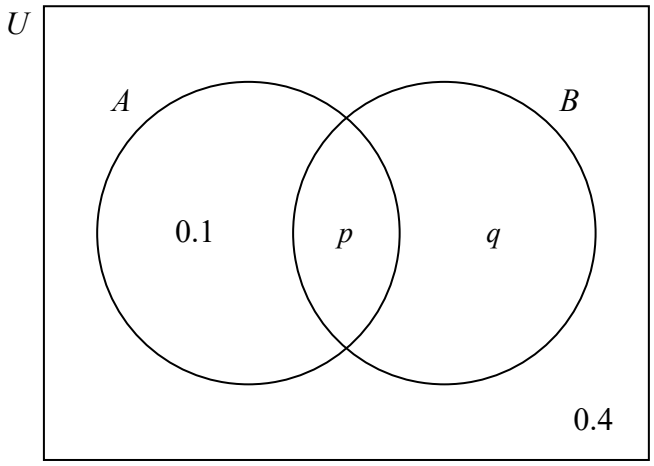
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following Venn diagram shows the events A and B, where $\mathrm{P}(A) = 0.3$. The values shown are probabilities.



(a) Find the value of p. [2]

(b) Find the value of q. [2]

(c) Find $\mathrm{P}(A' \cup B)$. [2]

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(Question 1 continued)

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2. [Maximum mark: 6]

A line, L_1, has equation $\boldsymbol{r} = \begin{pmatrix} -3 \\ 9 \\ 10 \end{pmatrix} + s \begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix}$. Point $\mathrm{P}(15, 9, c)$ lies on L_1.

(a) Find c. [4]

A second line, L_2, is parallel to L_1 and passes through $(1, 2, 3)$.

(b) Write down a vector equation for L_2. [2]

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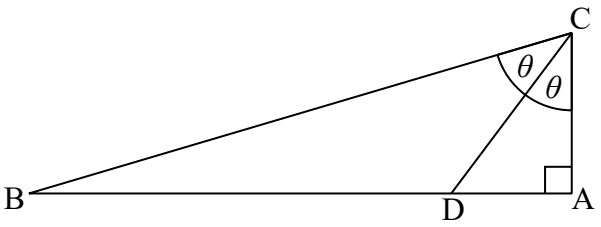
3. [Maximum mark: 7]

The following diagram shows a right triangle ABC. Point D lies on AB such that CD bisects $A\hat{C}B$.

diagram not to scale



$A\hat{C}D = \theta$ and $AC = 14\,\text{cm}$

(a) Given that $\sin\theta = \frac{3}{5}$, find the value of $\cos\theta$. [3]

(b) Find the value of $\cos 2\theta$. [2]

(c) Hence or otherwise, find BC. [2]

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4. [Maximum mark: 6]

Let $f(x) = \frac{2x-1}{x+3}$, $x \neq -3$.

(a) Write down the equation of the vertical asymptote of the graph of f. [1]

(b) Find $f^{-1}(x)$. [3]

(c) Find the equation of the horizontal asymptote of the graph of f^{-1}. [2]

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5. [Maximum mark: 5]

The derivative of a function f is given by $f'(x) = 2e^{-3x}$. The graph of f passes through $\left(\frac{1}{3}, 5\right)$.

Find $f(x)$.

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will not be marked.

6. [Maximum mark: 7]

The magnitudes of two vectors, $\boldsymbol{u}$ and $\boldsymbol{v}$, are 4 and $\sqrt{3}$ respectively. The angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is $\frac{\pi}{6}$.

Let $\boldsymbol{w} = \boldsymbol{u} - \boldsymbol{v}$. Find the magnitude of $\boldsymbol{w}$.

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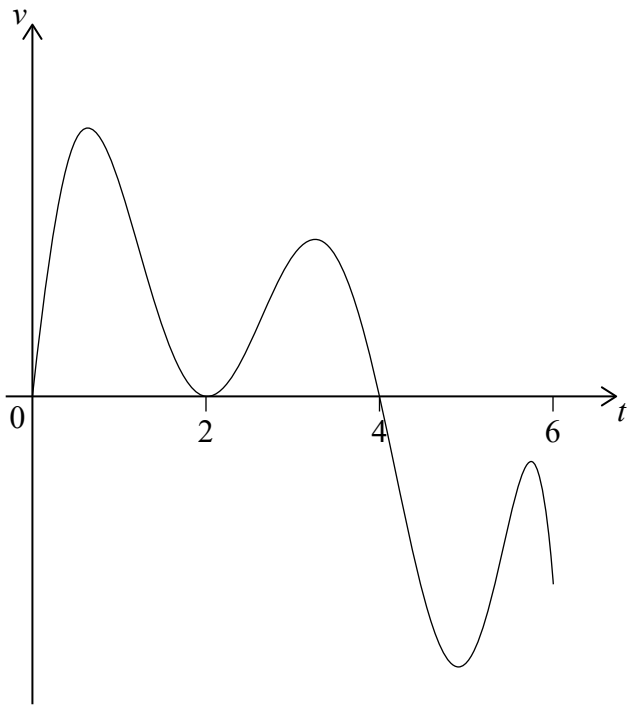
7. [Maximum mark: 7]

A particle P starts from point O and moves along a straight line. The graph of its velocity, $v\,\text{ms}^{-1}$ after t seconds, for $0 \le t \le 6$, is shown in the following diagram.



The graph of v has t-intercepts when $t = 0, 2$ and 4.

The function $s(t)$ represents the displacement of P from O after t seconds.

It is known that P travels a distance of 15 metres in the first 2 seconds. It is also known that $s(2) = s(5)$ and $\int_2^4 v\,\mathrm{d}t = 9$.

(a) Find the value of $s(4) - s(2)$. [2]

(b) Find the total distance travelled in the first 5 seconds. [5]

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(Question 7 continued)

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Section B

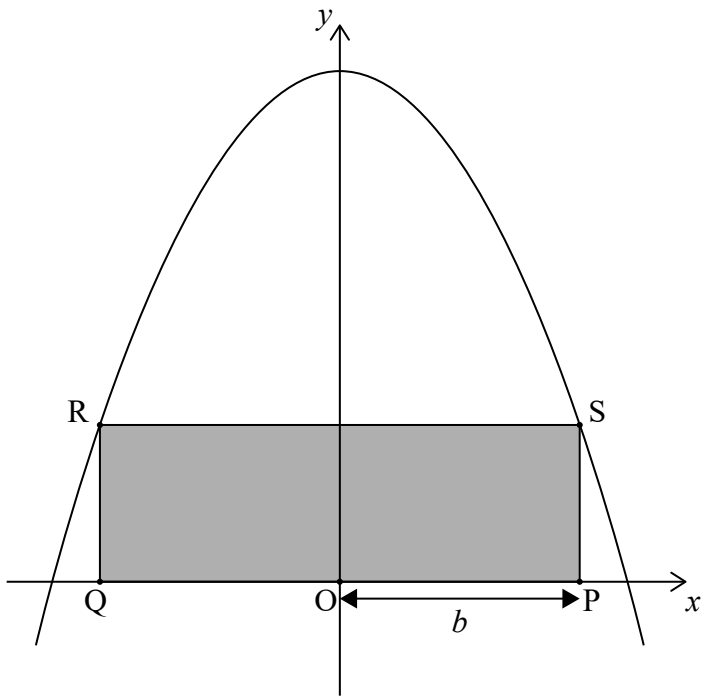
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

Let $f(x) = 9 - x^2$, $x \in \mathbb{R}$.

(a) Find the x-intercepts of the graph of f. [2]

The following diagram shows part of the graph of f.



Rectangle PQRS is drawn with P and Q on the x-axis and R and S on the graph of f.

Let $\mathrm{OP} = b$.

(b) Show that the area of PQRS is $18b - 2b^3$. [2]

(c) Hence find the value of b such that the area of PQRS is a maximum. [5]

Consider another function $g(x) = (x-3)^2 + k$, $x \in \mathbb{R}$.

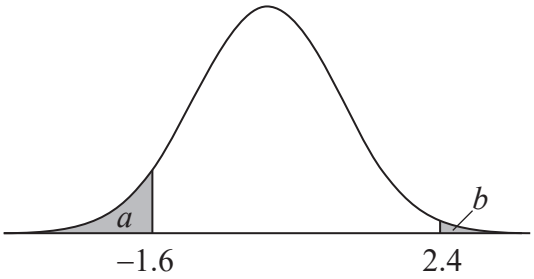
(d) Show that when the graphs of f and g intersect, $2x^2 - 6x + k = 0$. [2]

(e) Given that the graphs of f and g intersect only once, find the value of k. [5]

Do **not** write solutions on this page.

9. [Maximum mark: 13]

A random variable Z is normally distributed with mean 0 and standard deviation 1. It is known that $\mathrm{P}(z < -1.6) = a$ and $\mathrm{P}(z > 2.4) = b$. This is shown in the following diagram.



(a) Find $\mathrm{P}(-1.6 < z < 2.4)$. Write your answer in terms of a and b. [2]

(b) Given that $z > -1.6$, find the probability that $z < 2.4$. Write your answer in terms of a and b. [4]

A second random variable X is normally distributed with mean m and standard deviation s.

It is known that $\mathrm{P}(x < 1) = a$.

(c) Write down the standardized value for $x = 1$. [1]

It is also known that $\mathrm{P}(x > 2) = b$.

(d) Find s. [6]

Do **not** write solutions on this page.

10. [Maximum mark: 17]

Consider $f(x) = \sqrt{x}\sin\left(\frac{\pi}{4}x\right)$ and $g(x) = \sqrt{x}$ for $x \geq 0$. The first time the graphs of f and g intersect is at $x = 0$.

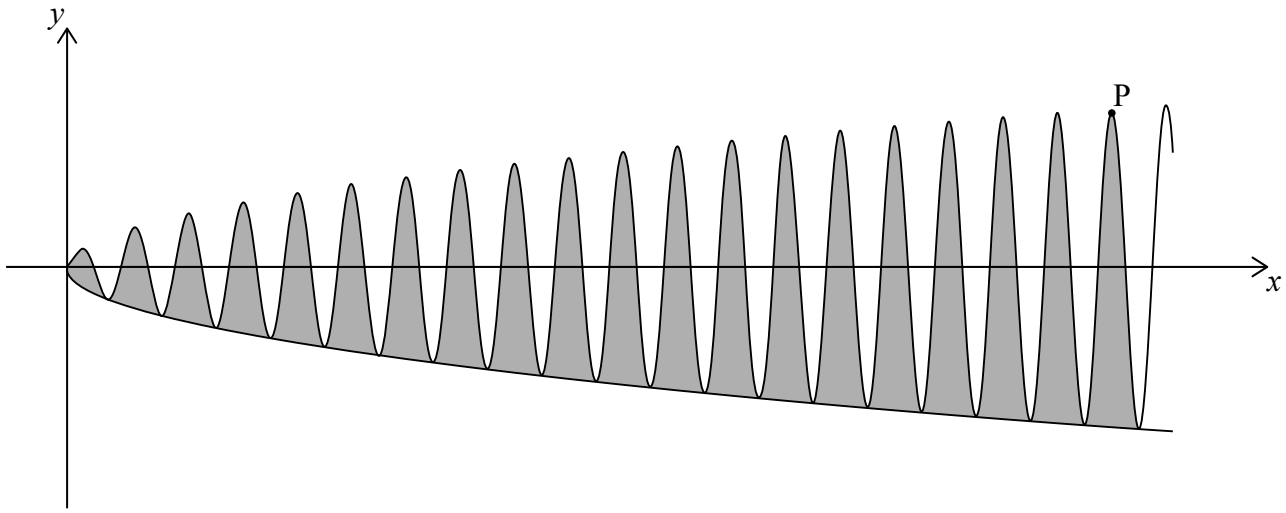
(a) Find the **two** smallest non-zero values of x for which $f(x) = g(x)$. [5]

The set of all non-zero values that satisfy $f(x) = g(x)$ can be described as an arithmetic sequence, $u_n = a + bn$ where $n \geq 1$.

(b) Find the value of a and of b. [4]

(c) At point P, the graphs of f and g intersect for the 21st time. Find the coordinates of P. [4]

The following diagram shows part of the graph of g **reflected** in the x-axis. It also shows part of the graph of f and the point P.



(d) Find an expression for the area of the shaded region. Do not calculate the value of the expression. [4]

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