M13/5/FURMA/SP1/ENG/TZ0/XX

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FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 1

Monday 20 May 2013 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[60 marks]*.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 12]*

(a) (i) Use the Euclidean algorithm to find $\gcd(6750, 144)$.

(ii) Express your answer in the form $6750r + 144s$ where $r, s \in \mathbb{Z}$. *[6 marks]*

(b) Consider the base 15 number CBA, where A, B, C represent respectively the digits ten, eleven, twelve.

(i) Write this number in base 10.

(ii) Hence express this number as a product of prime factors, writing the factors in base 4. *[6 marks]*

2. *[Maximum mark: 12]*

G is a group. The elements $a, b \in G$, satisfy $a^3 = b^2 = e$ and $ba = a^2 b$, where e is the identity element of G.

(a) Show that $(ba)^2 = e$. *[3 marks]*

(b) Express $(bab)^{-1}$ in its simplest form. *[3 marks]*

Given that $a \neq e$,

(c) (i) show that $b \neq e$;

(ii) show that G is not Abelian. *[6 marks]*

3. *[Maximum mark: 12]*

(a) A triangle T has sides of length 3, 4 and 5.

(i) Find the radius of the circumscribed circle of T.

(ii) Find the radius of the inscribed circle of T. *[6 marks]*

(b) A triangle U has sides of length 4, 5 and 7.

(i) Show that the orthocentre, H, of U lies outside the triangle.

(ii) Show that the foot of the perpendicular from H to the longest side divides it in the ratio 29:20. *[6 marks]*

4. *[Maximum mark: 13]*

(a) Find the general solution of the differential equation $(1-x^2)\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + xy$, for $|x| < 1$. *[7 marks]*

(b) (i) Show that the solution $y = f(x)$ that satisfies the condition $f(0) = \frac{\pi}{2}$ is

$$f(x) = \frac{\arcsin x + \frac{\pi}{2}}{\sqrt{1-x^2}}.$$

(ii) Find $\lim_{x \to -1} f(x)$. *[6 marks]*

5. *[Maximum mark: 11]*

Let X_k be independent normal random variables, where $\mathrm{E}(X_k) = \mu$ and $\mathrm{Var}(X_k) = \sqrt{k}$, for $k = 1, 2, \ldots$

The random variable Y is defined by $Y = \sum_{k=1}^{6} \frac{(-1)^{k+1}}{\sqrt{k}} X_k$.

(a) (i) Find $\mathrm{E}(Y)$ in the form $p\mu$, where $p \in \mathbb{R}$.

(ii) Find k if $\mathrm{Var}(X_k) < \mathrm{Var}(Y) < \mathrm{Var}(X_{k+1})$. *[5 marks]*

(b) A random sample of n values of Y was found to have a mean of 8.76.

(i) Given that $n = 10$, determine a 95 % confidence interval for μ.

(ii) The width of the confidence interval needs to be halved. Find the appropriate value of n. *[6 marks]*