

## Mathematics

### Higher level

### Paper 1

Monday 12 November 2018 (afternoon)

Candidate session number

2 hours

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#### Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.







3. [Maximum mark: 7]

Consider the function  $g(x) = 4\cos x + 1$ ,  $a \leq x \leq \frac{\pi}{2}$  where  $a < \frac{\pi}{2}$ .

(a) For  $a = -\frac{\pi}{2}$ , sketch the graph of  $y = g(x)$ . Indicate clearly the maximum and minimum values of the function. [3]

(b) Write down the least value of  $a$  such that  $g$  has an inverse. [1]

(c) For the value of  $a$  found in part (b),

(i) write down the domain of  $g^{-1}$ ;

(ii) find an expression for  $g^{-1}(x)$ . [3]

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5. [Maximum mark: 6]

The vectors  $\mathbf{a}$  and  $\mathbf{b}$  are defined by  $\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ t \end{pmatrix}$ ,  $\mathbf{b} = \begin{pmatrix} 0 \\ -t \\ 4t \end{pmatrix}$ , where  $t \in \mathbb{R}$ .

- (a) Find and simplify an expression for  $\mathbf{a} \cdot \mathbf{b}$  in terms of  $t$ . [2]
- (b) Hence or otherwise, find the values of  $t$  for which the angle between  $\mathbf{a}$  and  $\mathbf{b}$  is obtuse. [4]

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6. [Maximum mark: 6]

Use mathematical induction to prove that  $\sum_{r=1}^n r(r!) = (n+1)! - 1$ , for  $n \in \mathbb{Z}^+$ .

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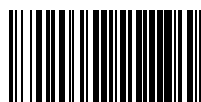
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7. [Maximum mark: 6]

Consider the curves  $C_1$  and  $C_2$  defined as follows

$$C_1: xy = 4, x > 0$$
$$C_2: y^2 - x^2 = 2, x > 0$$

(a) Using implicit differentiation, or otherwise, find  $\frac{dy}{dx}$  for each curve in terms of  $x$  and  $y$ . [4]

Let  $P(a, b)$  be the unique point where the curves  $C_1$  and  $C_2$  intersect.

(b) Show that the tangent to  $C_1$  at  $P$  is perpendicular to the tangent to  $C_2$  at  $P$ . [2]

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Do **not** write solutions on this page.

### Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

Consider a triangle  $OAB$  such that  $O$  has coordinates  $(0, 0, 0)$ ,  $A$  has coordinates  $(0, 1, 2)$  and  $B$  has coordinates  $(2b, 0, b - 1)$  where  $b < 0$ .

(a) Find, in terms of  $b$ , a Cartesian equation of the plane  $\Pi$  containing this triangle. [5]

Let  $M$  be the midpoint of the line segment  $[OB]$ .

(b) Find, in terms of  $b$ , the equation of the line  $L$  which passes through  $M$  and is perpendicular to the plane  $\Pi$ . [3]

(c) Show that  $L$  does not intersect the  $y$ -axis for any negative value of  $b$ . [7]



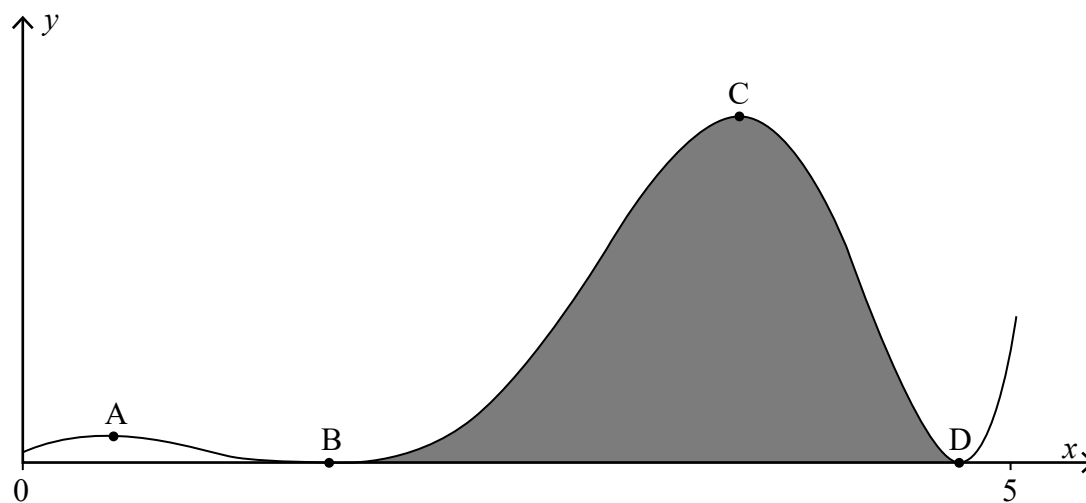
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10. [Maximum mark: 19]

(a) Use integration by parts to show that  $\int e^x \cos 2x dx = \frac{2e^x}{5} \sin 2x + \frac{e^x}{5} \cos 2x + c, c \in \mathbb{R}$ . [5]

(b) Hence, show that  $\int e^x \cos^2 x dx = \frac{e^x}{5} \sin 2x + \frac{e^x}{10} \cos 2x + \frac{e^x}{2} + c, c \in \mathbb{R}$ . [3]

The function  $f$  is defined by  $f(x) = e^x \cos^2 x$ , where  $0 \leq x \leq 5$ . The curve  $y = f(x)$  is shown on the following graph which has local maximum points at A and C and touches the  $x$ -axis at B and D.



(c) Find the  $x$ -coordinates of A and of C, giving your answers in the form  $a + \arctan b$ , where  $a, b \in \mathbb{R}$ . [6]

(d) Find the area enclosed by the curve and the  $x$ -axis between B and D, as shaded on the diagram. [5]



Do **not** write solutions on this page.

11. [Maximum mark: 16]

- (a) Find the roots of  $z^{24} = 1$  which satisfy the condition  $0 < \arg(z) < \frac{\pi}{2}$ , expressing your answers in the form  $re^{i\theta}$ , where  $r, \theta \in \mathbb{R}^+$ . [5]
- (b) Let  $S$  be the sum of the roots found in part (a).
- (i) Show that  $\operatorname{Re} S = \operatorname{Im} S$ .
- (ii) By writing  $\frac{\pi}{12}$  as  $\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$ , find the value of  $\cos \frac{\pi}{12}$  in the form  $\frac{\sqrt{a} + \sqrt{b}}{c}$ , where  $a, b$  and  $c$  are integers to be determined.
- (iii) Hence, or otherwise, show that  $S = \frac{1}{2} (1 + \sqrt{2})(1 + \sqrt{3})(1 + i)$ . [11]
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