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**Mathematics**  
**Higher level**  
**Paper 3 – calculus**

Wednesday 15 May 2019 (morning)

1 hour

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**Instructions to candidates**

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 7]

A simple model to predict the population of the world is set up as follows. At time  $t$  years the population of the world is  $x$ , which can be assumed to be a continuous variable. The rate of increase of  $x$  due to births is  $0.056x$  and the rate of decrease of  $x$  due to deaths is  $0.035x$ .

(a) Show that  $\frac{dx}{dt} = 0.021x$ . [1]

(b) Find a prediction for the number of years it will take for the population of the world to double. [6]

2. [Maximum mark: 9]

(a) Show that  $1 - x^2 + x^4 - x^6 + \dots = \frac{1}{1+x^2}$ , where  $|x| < 1$ . [1]

(b) Hence write down the first four non-zero terms of the power series for  $f(x) = \frac{1}{1+4x^2}$ . [2]

(c) Using the result in (b), find the first four non-zero terms of the power series for  $g(x) = \arctan 2x$ . [6]

3. [Maximum mark: 9]

Consider the series  $\sum_{n=1}^{\infty} \frac{A \times 8^n}{3^{2n+1}}$ .

(a) Given that  $A = \frac{1}{n}$ , use the comparison test to show that the series converges. [4]

(b) Given that  $A = n$ , determine whether the series diverges or converges. [5]

4. [Maximum mark: 9]

Using L'Hôpital's rule, find  $\lim_{x \rightarrow 0} \left( \frac{\tan 3x - 3 \tan x}{\sin 3x - 3 \sin x} \right)$ . [9]

5. [Maximum mark: 16]

Consider the differential equation  $2xy \frac{dy}{dx} = y^2 - x^2$ , where  $x > 0$ .

- (a) Solve the differential equation and show that a general solution is  $x^2 + y^2 = cx$  where  $c$  is a positive constant. [11]
- (b) Prove that there are two horizontal tangents to the general solution curve and state their equations, in terms of  $c$ . [5]
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