



No part of this product may be reproduced in any form or by any electronic or mechanical means, including information storage and retrieval systems, without written permission from the IB.

Additionally, the license tied with this product prohibits commercial use of any selected files or extracts from this product. Use by third parties, including but not limited to publishers, private teachers, tutoring or study services, preparatory schools, vendors operating curriculum mapping services or teacher resource digital platforms and app developers, is not permitted and is subject to the IB's prior written consent via a license. More information on how to request a license can be obtained from <http://www.ibo.org/contact-the-ib/media-inquiries/for-publishers/guidance-for-third-party-publishers-and-providers/how-to-apply-for-a-license>.

Aucune partie de ce produit ne peut être reproduite sous quelque forme ni par quelque moyen que ce soit, électronique ou mécanique, y compris des systèmes de stockage et de récupération d'informations, sans l'autorisation écrite de l'IB.

De plus, la licence associée à ce produit interdit toute utilisation commerciale de tout fichier ou extrait sélectionné dans ce produit. L'utilisation par des tiers, y compris, sans toutefois s'y limiter, des éditeurs, des professeurs particuliers, des services de tutorat ou d'aide aux études, des établissements de préparation à l'enseignement supérieur, des fournisseurs de services de planification des programmes d'études, des gestionnaires de plateformes pédagogiques en ligne, et des développeurs d'applications, n'est pas autorisée et est soumise au consentement écrit préalable de l'IB par l'intermédiaire d'une licence. Pour plus d'informations sur la procédure à suivre pour demander une licence, rendez-vous à l'adresse <http://www.ibo.org/fr/contact-the-ib/media-inquiries/for-publishers/guidance-for-third-party-publishers-and-providers/how-to-apply-for-a-license>.

No se podrá reproducir ninguna parte de este producto de ninguna forma ni por ningún medio electrónico o mecánico, incluidos los sistemas de almacenamiento y recuperación de información, sin que medie la autorización escrita del IB.

Además, la licencia vinculada a este producto prohíbe el uso con fines comerciales de todo archivo o fragmento seleccionado de este producto. El uso por parte de terceros —lo que incluye, a título enunciativo, editoriales, profesores particulares, servicios de apoyo académico o ayuda para el estudio, colegios preparatorios, desarrolladores de aplicaciones y entidades que presten servicios de planificación curricular u ofrezcan recursos para docentes mediante plataformas digitales— no está permitido y estará sujeto al otorgamiento previo de una licencia escrita por parte del IB. En este enlace encontrará más información sobre cómo solicitar una licencia: <http://www.ibo.org/es/contact-the-ib/media-inquiries/for-publishers/guidance-for-third-party-publishers-and-providers/how-to-apply-for-a-license>.

Mathematics

Higher level

Paper 1

Monday 13 May 2019 (afternoon)

Candidate session number

2 hours

--	--	--	--	--	--	--	--	--	--

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

$$\text{Let } \mathbf{a} = \begin{pmatrix} 2 \\ k \\ -1 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} -3 \\ k+2 \\ k \end{pmatrix}, k \in \mathbb{R}.$$

Given that \mathbf{a} and \mathbf{b} are perpendicular, find the possible values of k .

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



2. [Maximum mark: 4]

Determine the first three terms of $(1 - 2x)^{11}$ in ascending powers of x , giving each term in its simplest form.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

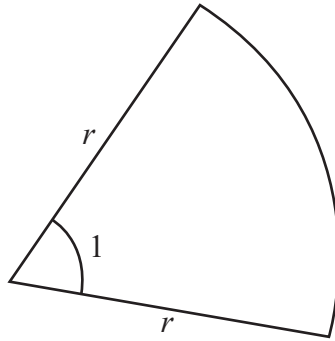


16EP03

Turn over

3. [Maximum mark: 4]

A sector of a circle with radius r cm, where $r > 0$, is shown on the following diagram.
The sector has an angle of 1 radian at the centre.



Let the area of the sector be A cm² and the perimeter be P cm. Given that $A = P$, find the value of r .

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



4. [Maximum mark: 7]

The lengths of two of the sides in a triangle are 4 cm and 5 cm. Let θ be the angle between the two given sides. The triangle has an area of $\frac{5\sqrt{15}}{2}$ cm².

(a) Show that $\sin \theta = \frac{\sqrt{15}}{4}$. [1]

(b) Find the two possible values for the length of the third side. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

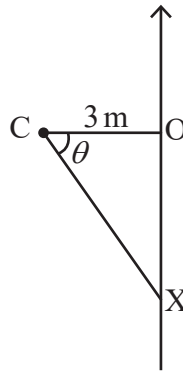
.....



Turn over

5. [Maximum mark: 6]

A camera at point C is 3 m from the edge of a straight section of road as shown in the following diagram. The camera detects a car travelling along the road at $t = 0$. It then rotates, always pointing at the car, until the car passes O , the point on the edge of the road closest to the camera.



A car travels along the road at a speed of 24 ms^{-1} . Let the position of the car be X and let $\widehat{OCX} = \theta$.

Find $\frac{d\theta}{dt}$, the rate of rotation of the camera, in radians per second, at the instant the car passes the point O .

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



6. [Maximum mark: 7]

Let X be a random variable which follows a normal distribution with mean μ . Given that $P(X < \mu - 5) = 0.2$, find

(a) $P(X > \mu + 5)$; [2]

(b) $P(X < \mu + 5 \mid X > \mu - 5)$. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



Please **do not** write on this page.

Answers written on this page
will not be marked.



7. [Maximum mark: 9]

Find the coordinates of the points on the curve $y^3 + 3xy^2 - x^3 = 27$ at which $\frac{dy}{dx} = 0$.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

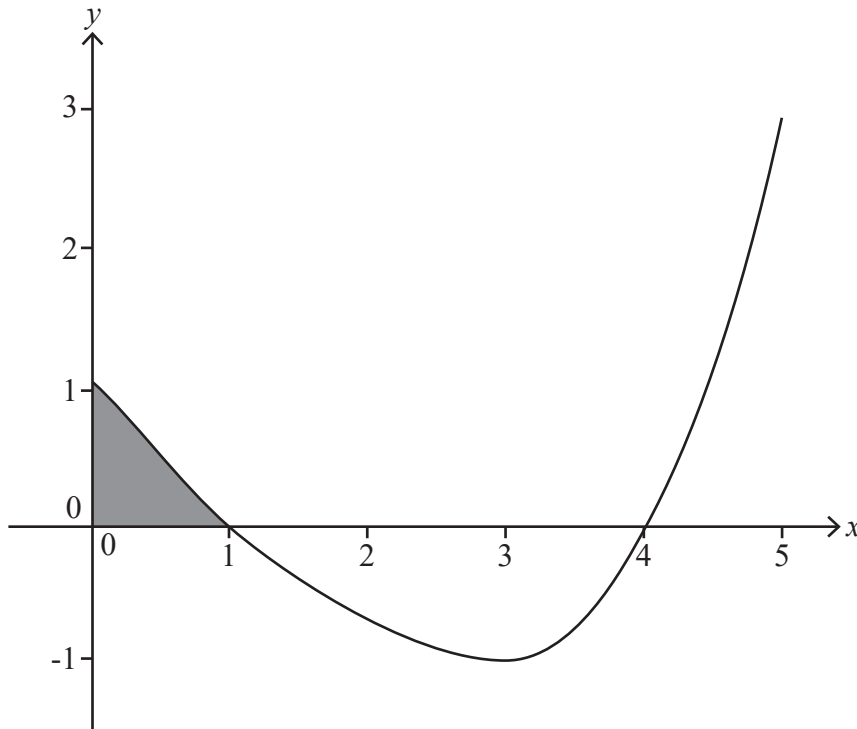
.....

.....



8. [Maximum mark: 9]

The graph of $y = f'(x)$, $0 \leq x \leq 5$ is shown in the following diagram. The curve intercepts the x -axis at $(1, 0)$ and $(4, 0)$ and has a local minimum at $(3, -1)$.



(a) Write down the x -coordinate of the point of inflexion on the graph of $y = f'(x)$. [1]

The shaded area enclosed by the curve $y = f'(x)$, the x -axis and the y -axis is 0.5 .
Given that $f(0) = 3$,

(b) find the value of $f(1)$. [3]

The area enclosed by the curve $y = f'(x)$ and the x -axis between $x = 1$ and $x = 4$ is 2.5 .

(c) Find the value of $f(4)$. [2]

(d) Sketch the curve $y = f(x)$, $0 \leq x \leq 5$ indicating clearly the coordinates of the maximum and minimum points and any intercepts with the coordinate axes. [3]

.....

.....

.....

.....

.....

.....

(This question continues on the following page)



(Question 8 continued)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



16EP11

Turn over

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

(a) Show that $(\sin x + \cos x)^2 = 1 + \sin 2x$. [2]

(b) Show that $\sec 2x + \tan 2x = \frac{\cos x + \sin x}{\cos x - \sin x}$. [4]

(c) Hence or otherwise find $\int_0^{\frac{\pi}{6}} (\sec 2x + \tan 2x) dx$ in the form $\ln(a + \sqrt{b})$ where $a, b \in \mathbb{Z}$. [9]

10. [Maximum mark: 15]

The function $p(x)$ is defined by $p(x) = x^3 - 3x^2 + 8x - 24$ where $x \in \mathbb{R}$.

(a) Find the remainder when $p(x)$ is divided by

(i) $(x - 2)$

(ii) $(x - 3)$. [3]

(b) Prove that $p(x)$ has only one real zero. [4]

(c) Write down the transformation that will transform the graph of $y = p(x)$ onto the graph of $y = 8x^3 - 12x^2 + 16x - 24$. [2]

The random variable X follows a Poisson distribution with a mean of λ and $6P(X = 3) = 3P(X = 2) - 2P(X = 1) + 3P(X = 0)$.

(d) Find the value of λ . [6]



Do **not** write solutions on this page.

11. [Maximum mark: 20]

Two distinct lines, l_1 and l_2 , intersect at a point P. In addition to P, four distinct points are marked out on l_1 and three distinct points on l_2 . A mathematician decides to join some of these eight points to form polygons.

- (a) (i) Find how many sets of four points can be selected which can form the vertices of a quadrilateral.
- (ii) Find how many sets of three points can be selected which can form the vertices of a triangle. [6]

The line l_1 has vector equation $\mathbf{r}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\lambda \in \mathbb{R}$ and the line l_2 has vector equation

$$\mathbf{r}_2 = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 6 \\ 2 \end{pmatrix}, \mu \in \mathbb{R}.$$

The point P has coordinates (4, 6, 4).

- (b) Verify that P is the point of intersection of the two lines. [3]

The point A has coordinates (3, 4, 3) and lies on l_1 .

- (c) Write down the value of λ corresponding to the point A. [1]

The point B has coordinates (-1, 0, 2) and lies on l_2 .

- (d) Write down \vec{PA} and \vec{PB} . [2]

Let C be the point on l_1 with coordinates (1, 0, 1) and D be the point on l_2 with parameter $\mu = -2$.

- (e) Find the area of the quadrilateral CDBA. [8]



Please **do not** write on this page.

Answers written on this page
will not be marked.



Please **do not** write on this page.

Answers written on this page
will not be marked.



Please **do not** write on this page.

Answers written on this page
will not be marked.

