



Oxford Cambridge and RSA

# Monday 19 October 2020 – Afternoon

## A Level Mathematics A

### H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours



**You must have:**

- the Printed Answer Booklet
- a scientific or graphical calculator

### INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

### INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [ ].
- This document has **12** pages.

### ADVICE

- Read each question carefully before you start your answer.

**Formulae**  
**A Level Mathematics A (H240)**

**Arithmetic series**

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$$

**Geometric series**

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

**Binomial series**

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

**Differentiation**

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

**Differentiation from first principles**

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

**Integration**

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

**Small angle approximations**

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

**Trigonometric identities**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

**Numerical methods**

Trapezium rule:  $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$ , where  $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving  $f(x) = 0$ :  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

**Probability**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

**Standard deviation**

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

**The binomial distribution**

If  $X \sim B(n, p)$  then  $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ , mean of  $X$  is  $np$ , variance of  $X$  is  $np(1-p)$

**Hypothesis test for the mean of a normal distribution**

If  $X \sim N(\mu, \sigma^2)$  then  $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$  and  $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

**Percentage points of the normal distribution**

If  $Z$  has a normal distribution with mean 0 and variance 1 then, for each value of  $p$ , the table gives the value of  $z$  such that  $P(Z \leq z) = p$ .

$p$	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$z$	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

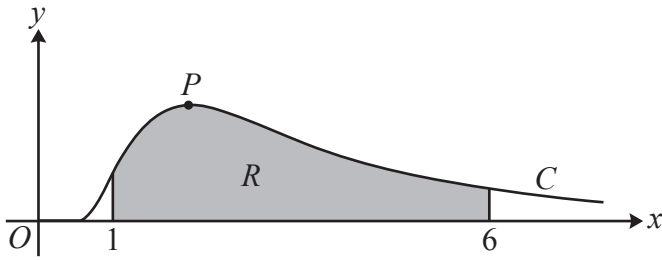
$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

## Section A: Pure Mathematics

Answer **all** the questions.

- 1 Triangle  $ABC$  has  $AB = 8.5$  cm,  $BC = 6.2$  cm and angle  $B = 35^\circ$ .  
Calculate the area of the triangle. [2]
- 2 A sequence of transformations maps the curve  $y = e^x$  to the curve  $y = e^{2x+3}$ .  
Give details of these transformations. [3]
- 3 The functions  $f$  and  $g$  are defined for all real values of  $x$  by  
 $f(x) = 2x^2 + 6x$  and  $g(x) = 3x + 2$ .
- (a) Find the range of  $f$ . [3]
- (b) Give a reason why  $f$  has no inverse. [1]
- (c) Given that  $fg(-2) = g^{-1}(a)$ , where  $a$  is a constant, determine the value of  $a$ . [4]
- (d) Determine the set of values of  $x$  for which  $f(x) > g(x)$ . Give your answer in set notation. [3]
- 4 A curve has equation  $y = 2 \ln(k - 3x) + x^2 - 3x$ , where  $k$  is a positive constant.
- (a) Given that the curve has a point of inflection where  $x = 1$ , show that  $k = 6$ . [5]
- It is also given that the curve intersects the  $x$ -axis at exactly one point.
- (b) Show by calculation that the  $x$ -coordinate of this point lies between 0.5 and 1.5. [2]
- (c) Use the Newton-Raphson method, with initial value  $x_0 = 1$ , to find the  $x$ -coordinate of the point where the curve intersects the  $x$ -axis, giving your answer correct to 5 decimal places. Show the result of each iteration to 6 decimal places. [3]
- (d) By choosing suitable bounds, verify that your answer to part (c) is correct to 5 decimal places. [1]

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The diagram shows the curve  $C$  with parametric equations

$$x = \frac{3}{t}, \quad y = t^3 e^{-2t}, \quad \text{where } t > 0.$$

The maximum point on  $C$  is denoted by  $P$ .

(a) Determine the exact coordinates of  $P$ . [4]

The shaded region  $R$  is enclosed by the curve, the  $x$ -axis and the lines  $x = 1$  and  $x = 6$ .

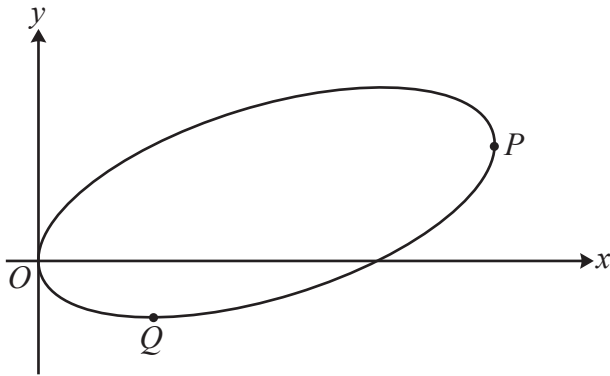
(b) Show that the area of  $R$  is given by

$$\int_a^b 3te^{-2t} dt,$$

where  $a$  and  $b$  are constants to be determined. [3]

(c) Hence determine the exact area of  $R$ . [5]

6 In this question you must show detailed reasoning.



The diagram shows the curve with equation  $4xy = 2(x^2 + 4y^2) - 9x$ .

(a) Show that  $\frac{dy}{dx} = \frac{4x - 4y - 9}{4x - 16y}$ . [3]

At the point  $P$  on the curve the tangent to the curve is parallel to the  $y$ -axis and at the point  $Q$  on the curve the tangent to the curve is parallel to the  $x$ -axis.

(b) Show that the distance  $PQ$  is  $k\sqrt{5}$ , where  $k$  is a rational number to be determined. [8]

**Section B: Mechanics**Answer **all** the questions.

- 7 A particle  $P$  moves with constant acceleration  $(-4\mathbf{i} + 2\mathbf{j})\text{ms}^{-2}$ . At time  $t = 0$  seconds,  $P$  is moving with velocity  $(7\mathbf{i} + 6\mathbf{j})\text{ms}^{-1}$ .
- (a) Determine the speed of  $P$  when  $t = 3$ . [4]
- (b) Determine the change in displacement of  $P$  between  $t = 0$  and  $t = 3$ . [2]

- 8 A car is travelling on a straight horizontal road. The velocity of the car,  $v\text{ms}^{-1}$ , at time  $t$  seconds as it travels past three points,  $P$ ,  $Q$  and  $R$ , is modelled by the equation

$$v = at^2 + bt + c,$$

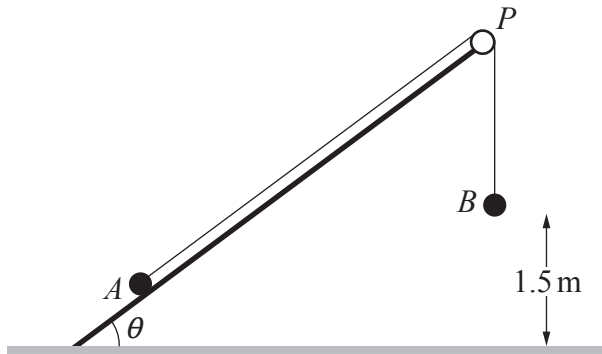
where  $a$ ,  $b$  and  $c$  are constants.

The car passes  $P$  at time  $t = 0$  with velocity  $8\text{ms}^{-1}$ .

- (a) State the value of  $c$ . [1]

The car passes  $Q$  at time  $t = 5$  and at that instant its deceleration is  $0.12\text{ms}^{-2}$ . The car passes  $R$  at time  $t = 18$  with velocity  $2.96\text{ms}^{-1}$ .

- (b) Determine the values of  $a$  and  $b$ . [4]
- (c) Find, to the nearest metre, the distance between points  $P$  and  $R$ . [2]



One end of a light inextensible string is attached to a particle  $A$  of mass  $2\text{ kg}$ . The other end of the string is attached to a second particle  $B$  of mass  $2.5\text{ kg}$ . Particle  $A$  is in contact with a rough plane inclined at  $\theta$  to the horizontal, where  $\cos\theta = \frac{4}{5}$ . The string is taut and passes over a small smooth pulley  $P$  at the top of the plane. The part of the string from  $A$  to  $P$  is parallel to a line of greatest slope of the plane. Particle  $B$  hangs freely below  $P$  at a distance  $1.5\text{ m}$  above horizontal ground, as shown in the diagram.

The coefficient of friction between  $A$  and the plane is  $\mu$ . The system is released from rest and in the subsequent motion  $B$  hits the ground before  $A$  reaches  $P$ . The speed of  $B$  at the instant that it hits the ground is  $1.2\text{ ms}^{-1}$ .

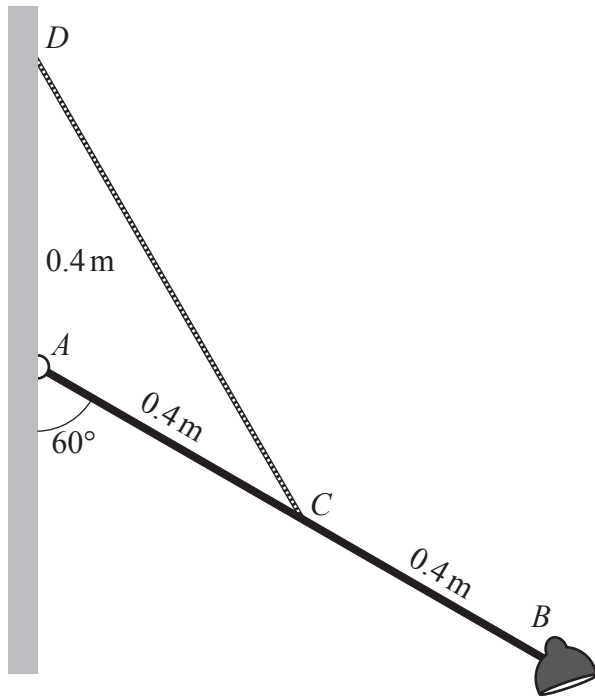
- (a) For the motion before  $B$  hits the ground, show that the acceleration of  $B$  is  $0.48\text{ ms}^{-2}$ . [1]
- (b) For the motion before  $B$  hits the ground, show that the tension in the string is  $23.3\text{ N}$ . [3]
- (c) Determine the value of  $\mu$ . [5]

After  $B$  hits the ground,  $A$  continues to travel up the plane before coming to instantaneous rest before it reaches  $P$ .

- (d) Determine the distance that  $A$  travels from the instant that  $B$  hits the ground until  $A$  comes to instantaneous rest. [4]



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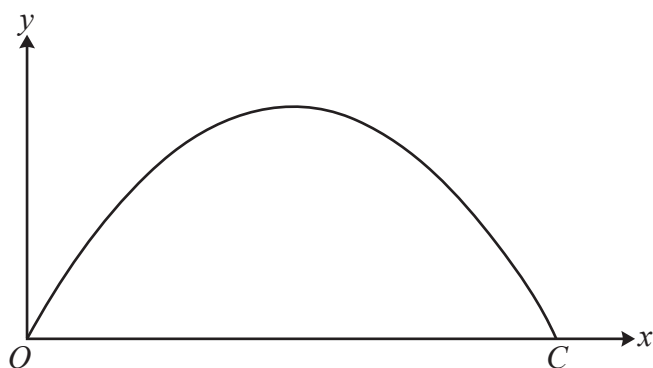


The diagram shows a wall-mounted light. It consists of a rod  $AB$  of mass  $0.25\text{ kg}$  and length  $0.8\text{ m}$  which is freely hinged to a vertical wall at  $A$ , and a lamp of mass  $0.5\text{ kg}$  fixed at  $B$ . The system is held in equilibrium by a chain  $CD$  whose end  $C$  is attached to the midpoint of  $AB$ . The end  $D$  is fixed to the wall a distance  $0.4\text{ m}$  vertically above  $A$ . The rod  $AB$  makes an angle of  $60^\circ$  with the downward vertical.

The chain is modelled as a light inextensible string, the rod is modelled as uniform and the lamp is modelled as a particle.

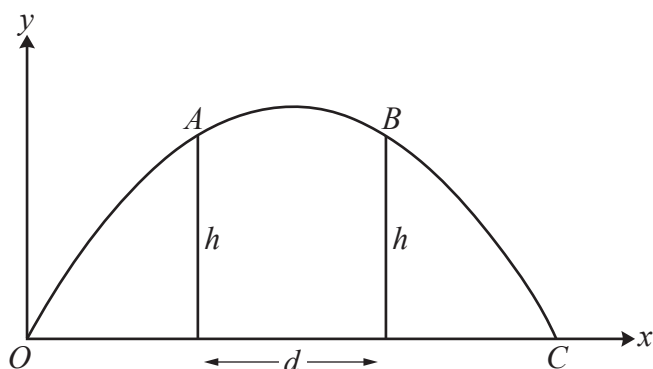
- (a) By taking moments about  $A$ , determine the tension in the chain. [4]
- (b) (i) Determine the magnitude of the force exerted on the rod at  $A$ . [4]
- (ii) Calculate the direction of the force exerted on the rod at  $A$ . [2]
- (c) Suggest one improvement that could be made to the model to make it more realistic. [1]

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A particle  $P$  moves freely under gravity in the plane of a fixed horizontal axis  $Ox$ , which lies on horizontal ground, and a fixed vertical axis  $Oy$ .  $P$  is projected from  $O$  with a velocity whose components along  $Ox$  and  $Oy$  are  $U$  and  $V$ , respectively.  $P$  returns to the ground at a point  $C$ .

- (a) Determine, in terms of  $U$ ,  $V$  and  $g$ , the distance  $OC$ . [4]



$P$  passes through two points  $A$  and  $B$ , each at a height  $h$  above the ground and a distance  $d$  apart, as shown in the diagram.

- (b) Write down the horizontal and vertical components of the velocity of  $P$  at  $A$ . [2]
- (c) Hence determine an expression for  $d$  in terms of  $U$ ,  $V$ ,  $g$  and  $h$ . [3]
- (d) Given that the direction of motion of  $P$  as it passes through  $A$  is inclined to the horizontal at an angle  $\theta$ , where  $\tan \theta = \frac{1}{2}$ , determine an expression for  $V$  in terms of  $g$ ,  $d$  and  $h$ . [4]

**END OF QUESTION PAPER**



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