Oxford Cambridge and RSA

GCE

Mathematics A

H240/01: Pure Mathematics

Advanced GCE

Mark Scheme for November 2020

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

1. Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.

- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(a)		$2\left(1-\frac{1}{2}\theta^2\right)+(1-\theta)^2$	B1	2.1	Correct statement	First term could possibly already be expanded
			$2-\theta^2+1-2\theta+\theta^2$	M1	2.1	Attempt to expand and simplify given expression	
			$=3-2\theta$ **A.G.**	A1	2.1	Obtain given answer	Max of **M1A1** if neither $(1-\theta)^2$ nor $1-2\tan\theta+\tan^2\theta$ seen
				[3]			
	(b)		$3-2\theta=28\theta$	M1	1.1a	Use $28\sin\theta\approx 28\theta$ and attempt to solve	
			$\theta = 0.1$	A1	1.1	Obtain 0.1 oe	BOD if 0.1^o ISW once 0.1 seen
				[2]			

Question			Answer	Marks	AO	Guidance	
2	(a)		$6a^3$	**B1** **B1** **[2]**	**1.1** **1.1**	Obtain 6 Obtain a^3	**B1** only for $\pm 6a^3$
	(b)		$(64b^3)^{\frac{1}{3}} = 4b$ or $(4b^4)^{-\frac{1}{2}} = \frac{1}{2b^2}$ $2b^{-1}$ or $\frac{2}{b}$	**B1** **B1** **[2]**	**1.1a** **1.1**	Correct simplification of either term Correct final answer	Allow $(2b^2)^{-1}$ for the second term
	(c)		$9^{3c} = 3^{6c}$ $27^{2c} = 3^{6c}$ $7 \times 3^{6c} - 4 \times 3^{6c} = 3 \times 3^{6c}$ $= 3^{6c+1}$	**B1** **M1** **A1** **A1** **[4]**	**1.1** **1.1a** **3.1a** **1.1**	Either 9^c or 27^{2c} correct as a power of 3 (or 729) Attempt to write the other one of 9^c and 27^{2c} with the same base Combine to obtain correct single term Obtain correct final answer	Ignore coefficient Index must be simplified Ignore coefficient Allow unsimplified index **B2** for $27^{2c} = 9^{3c}$ Allow equiv eg 3×729^c or 3×27^{2c} or 3×9^{3c} Must be single power of 3 **OR** **B1** $9^{2c}(7 \times 9^c - 4 \times 3^{2c})$ **M1** $9^{2c}(7 \times 3^{2c} - 4 \times 3^{2c})$ $9^{2c} \times 3 \times 3^{2c}$ **A1** 3×27^{2c} **A1** 3^{6c+1}

Question			Answer	Marks	AO	Guidance	
3	(a)		$\pi r^2 h = 16000\pi$	**B1**	**3.1b**	Correct equation for volume seen or used	*h* likely to be used, but could be other variable
			$A = 2\pi r^2 + 2\pi rh$	**B1**	**3.1b**	Correct expression for surface area seen	Two terms may be seen at separate stages of the proof If alternative formula used eg $2\pi r^2 + 2Vr^{-1}$ then this must be clearly derived Allow BOD for $2\pi r^2 + 2\pi r \times 16000r^{-2}$ as long as *h* seen explicitly in terms of *r* first
			$= 2\pi r^2 + 2\pi r \times 16000r^{-2}$	**M1**	**1.1a**	Eliminate *h* from expression for surface area	Allow if just attempt at curved surface area
			$= 2\pi r^2 + 32000\pi r^{-1}$ **A.G.**	**A1**	**1.1**	Obtain given answer	If $2\pi r^2$ is first seen in the final answer then it must be justified eg 'plus two ends', otherwise max B1B0M1A0
				[4]			

Question			Answer	Marks	AO	Guidance	
	(b)		$\frac{dA}{dr} = 4\pi r - 32000\pi r^{-2}$	**M1**	**1.1a**	Attempt differentiation	Both powers decrease by 1
			$4\pi r - 32000\pi r^{-2} = 0$ $r^3 = 8000$	**M1**	**3.1b**	Equate derivative to 0 and attempt to solve for r (or h)	$-32000\pi h^{-2} + \pi\sqrt{16000}h^{-\frac{1}{2}}$
			$r = 20$	**A1**	**1.1**	Obtain correct r, units not needed	$h^{\frac{3}{2}} = \sqrt{64000}$
			Surface area = 2400π cm^2 / 7540 cm^2	**A1**	**1.1**	Obtain correct A, units not needed	Allow exact or decimal (3sf or better)
			$\frac{d^2A}{dr^2} = 4\pi + 64000\pi r^{-3}$ when $r = 20$, $\frac{d^2A}{dr^2} = 12\pi$ (or 37.7)	**M1**	**2.1**	Attempt method to justify minimum, including substitution or consideration of sign	Could also test first derivative, or A, on both sides of $r = 20$
			$\frac{d^2A}{dr^2} > 0$, hence minimum	**A1**	**2.2a**	Correct conclusion, with justification, from correct working	If second derivative is evaluated, it must be correct (condone truncated decimal of 37.6)
				[6]			
4			Assume that there is a greatest multiple of 5 ie $N = 5k$	**B1***	**2.1**	Assumption for contradiction	Some indication that they are starting with the greatest multiple of 5
			$N + 5 = 5k + 5 = 5(k + 1)$	**M1**	**2.1**	Add on 5, or a multiple of 5	Or any equiv operation that would result in a larger multiple of 5 M0 if just numerical example
			This is a multiple of 5, and $N + 5 > N$ which contradicts the assumption Hence there is no greatest multiple of 5	**A1d***	**2.4**	Statement denying assumption	Need justification about why it is a multiple of 5, why it is greater, as well as 'contradiction' or clear equiv such as 'initial assumption is incorrect'
				[3]			

Question			Answer	Marks	AO	Guidance	
5	(a)		$\overrightarrow{BQ} = \frac{1}{2}(\mathbf{a} - \mathbf{b})$	B1	1.1a	Correct $\overrightarrow{BQ}$ or $\overrightarrow{QB}$	Or any correct vector involving Q, but must be clear which vector it is
			$\overrightarrow{PQ} = \frac{1}{4}\mathbf{b} + \frac{1}{2}(\mathbf{a} - \mathbf{b}) = \frac{1}{2}\mathbf{a} - \frac{1}{4}\mathbf{b}$	B1 [2]	1.1	Correct $\overrightarrow{PQ}$	Must be simplified to two terms **SC** Allow **B1** if correct unsimplified PQ is seen but individual vectors not explicit
	(b)		$\overrightarrow{PR}$ has the same direction as $\overrightarrow{PQ}$, so vector must be a multiple of $\overrightarrow{PQ}$ So $\overrightarrow{PR} = \lambda(\frac{1}{2}\mathbf{a} - \frac{1}{4}\mathbf{b}) = \frac{1}{4}\lambda(2\mathbf{a} - \mathbf{b})$ $= k(2\mathbf{a} - \mathbf{b})$ **A.G.**	B1 B1 [2]	2.4 2.1	Explain parallel (or collinear) vectors have direction vectors that are multiples of each other Show given answer convincingly	Allow 'gradient' for 'direction', or 'they are on the same straight line', but must state or use 'multiple' Clear detail of scaling factor
	(c)		$\overrightarrow{AR} = -\mathbf{a} + \frac{3}{4}\mathbf{b} + k(2\mathbf{a} - \mathbf{b})$ $\overrightarrow{AR}$ multiple of $\mathbf{a}$ only, $\frac{3}{4}\mathbf{b} - k\mathbf{b} = 0$ Obtain $k = \frac{3}{4}$ ratio $OA : AR = 2:1$	B1 M1 A1 A1 [4]	1.1 3.1a 1.1 1.1	Correct expression for $\overrightarrow{AR}$ (or $\overrightarrow{OR}$), in terms of k Use coefficient of $\mathbf{b} = 0$ Obtain correct value for k Correct ratio (allow 1: $\frac{1}{2}$) oe	Could use A to Q to R (condone if k still used) Must be used in $\overrightarrow{AR}$ or $\overrightarrow{OR}$ May get different value for their k Answer only is 0, as question says 'determine'

Question			Answer	Marks	AO	Guidance	
6	(a)		$\log_{10}S = \log_{10}(ab^t)$ $\log_{10}S = \log_{10}a + \log_{10}b^t$	**M1**	**2.1**	Attempt to show reduction to linear form	Introduce logs on both sides, and correctly split to the sum of two terms
			$\log_{10}S = t\log_{10}b + \log_{10}a$	**A1**	**2.1**	Obtain correct equation	Condone no base; any bases seen must be 10 A0 for $\log_{10}bt$ unless previously seen as $t\log_{10}b$
			which is of the form $Y = mX + c$	**A1** **[3]**	**2.4**	Link to equation of straight line	Base of 10 must now be explicit throughout Could instead refer to a linear relationship
	(b)		$\log_{10}a = 0.583 \Rightarrow a = 10^{0.583} = 3.8$	**B1**	**1.1**	Obtain $a = 3.8$, or better, from either eqn	Must clearly be value for a
			$\log_{10}b = 0.146 \Rightarrow b = 10^{0.146} = 1.4$	**B1** **[2]**	**1.1**	Obtain $b = 1.4$, or better, from either eqn	Must clearly be value for b
	(c)		$3.8 \times 1.4^t = 200$ $1.4^t = 52.63$	**M1**	**3.1a**	Link their model to 200 and attempt to solve for t	Or use linear equation and attempt to solve for t Must use correct solution method Allow M1 if using $S = 200{,}000{,}000$ Allow if their a and b transposed
			$t = 11.8$	**A1**	**1.1**	Obtain $t = 11.8$, or better, www (allow $t = 12$)	Condone 11.7 as truncated value for t
			so year is 2027	**A1FT** **[3]**	**3.2a**	FT their value for t	Answer in context, so not just '12 years later' FT on 2015 + integer number of years, from rounding up their t

Question			Answer	Marks	AO	Guidance	
	(d)		Unlikely that sales will continue at same rate Finite market	**B1** **[1]**	**3.5b**	Any sensible reason – eg pattern not necessarily continuing or the market being limited by no. of customers	Allow 'extrapolation unreliable' Reason needed not just eg 'other external factors'
7	**(a)**		Anna = $30 + 9 \times 15 = 165$ mins Ben = $30 \times 1.1^9 = 71$ mins $165 - 71 = 94$ minutes **A.G.**	**M1** **M1** **A1** **[3]**	**3.3** **3.3** **3.4**	Attempt u_{10} for AP, using correct eqn Attempt u_{10} for GP, using correct eqn Obtain given answer of 94 minutes www	Method must be seen Method must be seen AG so both terms need to be explicitly evaluated for the A1 Show subtraction, or give more accurate value before 94
	(b)		Anna: $u_X = 30 + 15(X - 1)$ Ben: $u_X = 30 \times 1.1^{X-1}$ $30 \times 1.1^{X-1} > 30 + 15(X - 1)$ $30 \times 1.1^{X-1} > 15X + 15$ $1.1^{X-1} > 0.5X + 0.5$ $X - 1 > \log_{1.1}(0.5X + 0.5)$ $X > \log_{1.1}(0.5X + 0.5) + 1$ **A.G.**	**B1** **M1** **A1** **[3]**	**3.4** **2.1** **2.1**	Both u_X terms correct Link correct expressions and attempt to rearrange Show given answer convincingly	Condone unknown other than X Condone an incorrect linking sign eg = not > Must be using a correct process, so allow sign errors only Must now be >, with justification if = used in proof, and with X
	(c)		18.9 25.1, 28.0, 29.0, 29.4, 29.6, 29.6,… $X = 30$	**B1** **M1** **A1**	**1.1a** **1.1** **3.4**	Correct first iterate Use correct iterative process to find at least two further values Obtain $X = 30$	Allow 19 or 18.8… Allow integer values Could be truncated not rounded Must be an integer

Question			Answer	Marks	AO	Guidance	
				[3]			
	(d)	**(i)**	Eventually there will not be enough hours in the day for revision	**B1** **[1]**	**3.5b**	Comment on long-term behaviour	Allow other sensible reason
		(ii)	Increasing by 10% will involve decimals of minutes so will no longer be accurate	**B1**	**3.5b**	Comment on not being able to measure time that accurately	Allow long-term behaviour if not already given in **(i)** Allow other sensible reason B0 if referring to reasons that may prevent revision from happening eg illness If a correct reason is given then ISW an incorrect numerical value

Question			Answer	Marks	AO	Guidance	
				[1]			
8	**(a)**		$\frac{d}{dx}\left(e^{2x}\right) = 2e^{2x}$	**B1**	**1.1**	Seen anywhere in solution	
			$6xe^{2x} + (2 + 3x^2)(2e^{2x})$	**M1**	**1.1a**	Attempt product rule	Could expand first
			$e^{2x}(6x^2 + 6x + 4)$	**A1**	**1.1**	Obtain any fully correct expression	
				[3]			
	(b)		$e^{2x} > 0$ for all x	**B1**	**2.1**		B0 if clearly considering f(x) or $f''(x)$ and not $f'(x)$
			$6x^2 + 6x + 4 = 6\left(x + \frac{1}{2}\right)^2 + \frac{5}{2}$ minimum value is $\frac{5}{2}$ so > 0 for all x	**M1**	**2.1**	Attempt to show that their 3 term quadratic factor is > 0 for all x	Complete the square or consider discriminant Could be multiple or fraction of their quadratic

Question			Answer	Marks	AO	Guidance	
				A1	**2.4**	Full justification that quadratic factor is always positive	Show minimum point > 0, or show that quadratic is always positive
			Gradient $e^{2x}(6x^2 + 6x + 4) > 0$ for all x so it is increasing for all x	**A1**	**2.4**	Justify increasing function as $f'(x) > 0$ for all x	
				[4]			**OR** B1 $e^{2x} \neq 0$ M1 Show that quadratic $\neq 0$ (detail required) M1 Show gradient is positive at one point, as part of attempt to show $f'(x) \neq 0$ A1 Conclude that gradient must hence be positive for all x, so increasing function
9	**(a)**		(1.5, 0)	**B1**	**1.1**	Allow $x = 1.5$	Unless contradicted with non-zero y-coord
			(0, 3)	**B1**	**1.1**	Allow $y = 3$	Unless contradicted with non-zero x-coord
				[2]			
	(b)	**(i)**	$a < 2$	**B1**	**3.1a**	Allow for answer of form $k < a < 2$	

Question			Answer	Marks	AO	Guidance	
			$0 = 1.5a + 2$	**M1**	**3.1a**	Attempt to find value of a at their x intersection	eg Use equation of line to find a Use gradient of line to find a Use a point of intersection of the two lines = their 1.5 Equate two points of intersection and solve for a Square both sides and link discriminant to 0
			$a = -\frac{4}{3}$	**A1**	**1.1**	Obtain $-\frac{4}{3}$ (condone any inequality sign, an equals sign or no sign)	Question is 'determine' so method required for this value of a
			$-\frac{4}{3} < a < 2$	**A1** **[4]**	**1.1**	Correct final inequality	Formal set notation not required
		(ii)	$2x - 3 = ax + 2$ $x = \frac{5}{2-a}$	**B1**	**1.1**	Correct point of intersection – allow any exact equiv	**OR** M1 – square both sides and attempt to solve – as far as substituting into quadratic formula A1 A1 for each root
			$3 - 2x = ax + 2$ $(2 + a)x = 1$	**M1**	**1.1a**	Attempt to solve linear equation with $2x$ and ax of different signs	Method may be seen in (i), only credit if answers seen in (ii)
			$x = \frac{1}{2+a}$	**A1** **[3]**	**1.1**	Correct point of intersection – allow any exact equiv	Max of 2 out of 3 if additional roots as well.
10	**(a)**		$0.25\{\sin 0 + \sin\left(\frac{1}{2}\sqrt{0.25}\right) + \sin\left(\frac{1}{2}\sqrt{0.5}\right)$ $+ \sin\left(\frac{1}{2}\sqrt{0.75}\right)\}$	**M1**	**1.2**	Attempt four rectangles of width 0.25, with height on left-hand side	No need to see sin0 Allow M1 if evaluated in degrees (0.00452)
			Lower bound = 0.253	**A1**	**1.1**	Obtain 0.253, or better	soi as lower bound

Question			Answer	Marks	AO	Guidance	
			$0.25\{\sin\left(\frac{1}{2}\sqrt{0.25}\right)+\sin\left(\frac{1}{2}\sqrt{0.5}\right)+\sin\left(\frac{1}{2}\sqrt{0.75}\right)+\sin\frac{1}{2}\}$	**M1**	**1.1a**	Attempt rectangles of width 0.25, with height on right-hand side	Or subtract sin0 from part (ii) and add sin0.5 Allow M1 if evaluated in degrees (0.00670) M0 if sin0 explicitly included
			Upper bound= 0.373 or 0.374	**A1** **[4]**	**1.1**	Obtain 0.373 or 0.374 (from rounding upper bound up), or better	soi as upper bound
	(b)	**(i)**	$t^2 = x-1$	**M1**	**2.1**	Attempt to link dt and dx	
			$2t\mathrm{d}t = \mathrm{d}x$	**A1**	**2.1**	Obtain correct equation linking dt and dx	Allow for $\mathrm{d}t = \frac{1}{2}(x-1)^{-\frac{1}{2}}\mathrm{d}x$ oe
			$\int \sin\left(\frac{1}{2}\sqrt{x-1}\right)\mathrm{d}x = \int \sin\left(\frac{1}{2}t\right)2t\mathrm{d}t$ $= \int 2t\sin(\frac{1}{2}t)\mathrm{d}t$ **A.G.**	**A1** **[3]**	**2.5**	Attempt integrand in terms of t to obtain given answer	Award A1 once all elements are correct, even if not in same order as given answer BOD if no brackets
	(b)	**(ii)**	$-4t\cos\left(\frac{1}{2}t\right)+\int 4\cos\left(\frac{1}{2}t\right)\mathrm{d}t$	**M1***	**3.1a**	Attempt integration by parts	Correct parts As far as first stage
			$-4t\cos\left(\frac{1}{2}t\right)+8\sin\left(\frac{1}{2}t\right)$	**A1**	**2.1**	Correct integral	
			$(-4\cos\frac{1}{2}+8\sin\frac{1}{2})-(-0+0)$	**M1d***	**2.1**	Attempt use of limits	Using either t or x, but must be consistent Condone no clear use of the lower limit for M1
			$8\sin\frac{1}{2}-4\cos\frac{1}{2}$ **AG**	**A1** **[4]**	**2.4**	Obtain given answer	Must see some indication that lower limit considered
11			**DR**				

Question			Answer	Marks	AO	Guidance	
	(a)	**(i)**	$x^2 + (mx + 2)^2 - 10x - 14(mx + 2) + 64 = 0$ $x^2 + m^2x^2 + 4mx + 4 - 10x - 14mx - 28 + 64 = 0$ $(m^2 + 1)x^2 - 10(m + 1)x + 40 = 0$ **A.G.**	**M1** **A1** **[2]**	**1.1a** **1.1**	Substitute eqn of tangent into eqn of circle Expand and tidy to given answer, including '= 0' in final answer	Could work backwards, eliminating m to obtain equation of circle **AG** so unsimplified expansion needs to be seen
		(ii)	$100(m + 1)^2 - 160(m^2 + 1) = 0$ $60m^2 - 200m + 60 = 0$ $(3m - 1)(m - 3) = 0$ $m = 3,\ m = \frac{1}{3}$ $y = 3x + 2$	**M1*** **A1** **M1d*** **A1** **[4]**	**3.1a** **1.1** **1.1a** **1.1**	Use $b^2 - 4ac = 0$ Obtain correct equation Attempt to solve quadratic Obtain correct equation	M1 only awarded when '= 0' soi Any correct 3 term equation DR so method for solving the quadratic must be shown SC B1for correct equation if roots not justified A0 if second equation also given **OR** (for first 2 marks) M1 – Attempt two equations in m and x (eg use lengths and gradients) and eliminate one variable A1 – correct quadratic in m or x
	(b)		radius $= \sqrt{10}$, $PC = 5\sqrt{2}$, $PA = PB = 2\sqrt{10}$, $AB = 4\sqrt{2}$ $\tan\left(\frac{1}{2}APB\right) = \frac{1}{2}$ $\tan APB = \frac{1}{1 - \frac{1}{4}}$	**M1** **A1** **M1**	**3.1a** **1.1** **3.1a**	Attempt (at least 2) useful lengths Obtain a correct related trig ratio Attempt tan APB	NB points of intersection are (2, 8) and (6, 4) $\cos APB = \frac{3}{5}$, from cosine rule **DR** so need to see use of identity or relevant triangle to find tan APB

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			$\tan APB = \frac{4}{3}$	**A1** **[4]**	**1.1**	Obtain $\frac{4}{3}$	From explicit, exact, working
							OR M2 – attempt $\frac{\pm m \pm n}{1 \pm mn}$ with their values for m and n A1FT – correct $\frac{m-n}{1+mn}$ for their values of m and n A1 – obtain $\tan APB = \frac{4}{3}$
12			$\int \frac{1}{y}\,dy = \int \frac{20x-35}{2x^3-3x^2-11x+6}\,dx$	**M1**	**1.1**	Separate variables	Correct process to deal with algebraic fractions, with BOD on integral notation
			$f(x) = 2x^3-3x^2-11x+6$ $= (x-3)(2x^2+3x-2)$	**M1**	**3.1a**	Attempt to factorise cubic	Possibly **BC**, so correct factorised cubic implies M1A1 If incorrect factorised cubic then method must be seen for M1 Allow M1A0 for $(x-3)(x+2)(x-0.5)$
			$= (x-3)(x+2)(2x-1)$	**A1**	**1.1**	Correct factorised cubic	
			$\frac{20x-35}{2x^3-3x^2-11x+6} = \frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{2x-1}$	**M1**	**1.1a**	Attempt partial fractions, using their 3 linear factors	Must be correct structure, attempting at least one numerator
			$= -\frac{3}{x+2} + \frac{1}{x-3} + \frac{4}{2x-1}$	**A1**	**1.1**	Obtain any one correct fraction www	Possibly implied by eg $A = -3$
				A1	**1.1**	Obtain fully correct partial fractions	Could be implied by $A = -3$ etc, if subsequent slip when writing out partial fractions
			$\int \frac{1}{y}\,dy = \ln\lvert y\rvert$	**B1**	**1.1**	Correct integration of $\frac{1}{y}$	Condone no modulus sign

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			$-3\ln\lvert x+2\rvert+\ln\lvert x-3\rvert+2\ln\lvert 2x-1\rvert+\ln A$	**A1FT**	**1.1**	Obtain correct integral following their 3 linear partial fractions	Condone no constant of integration Condone brackets and not modulus FT from point that partial fractions were credited, and not on subsequent errors
			$y=\dfrac{A(x-3)(2x-1)^2}{(x+2)^3}$	**A1**	**1.1**	Obtain correct equation	Any correct form not involving ln May be e^c not A, but A0 if fraction $+c$ Could have $(x+2)^{-3}$ in a product
				[9]			

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