Oxford Cambridge and RSA

Wednesday 5 June 2019 – Morning

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

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INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **8** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

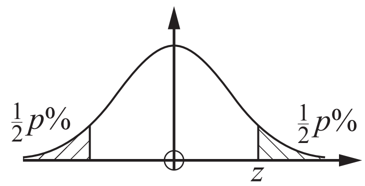
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \tfrac{1}{2}at^2$

$s = \tfrac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \tfrac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \tfrac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \tfrac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

Section A (25 marks)

1 **In this question you must show detailed reasoning.**

Show that $\int_4^9 (2x + \sqrt{x})\,\mathrm{d}x = \frac{233}{3}$. **[3]**

2 Show that the line which passes through the points $(2, -4)$ and $(-1, 5)$ does not intersect the line $3x + y = 10$. **[3]**

3 The function $\mathrm{f}(x)$ is given by $\mathrm{f}(x) = (1 - ax)^{-3}$, where a is a non-zero constant. In the binomial expansion of $\mathrm{f}(x)$, the coefficients of x and x^2 are equal.

(a) Find the value of a. **[3]**

(b) Using this value for a,

(i) state the set of values of x for which the binomial expansion is valid, **[1]**

(ii) write down the quadratic function which approximates $\mathrm{f}(x)$ when x is small. **[1]**

4 Fig. 4 shows a uniform beam of mass 4 kg and length 2.4 m resting on two supports P and Q. P is at one end of the beam and Q is 0.3 m from the other end.
Determine whether a person of mass 50 kg can tip the beam by standing on it. **[3]**

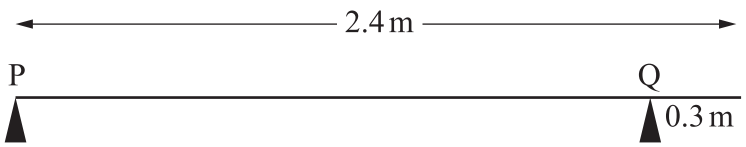


Fig. 4

5 A car of mass 1200 kg travels from rest along a straight horizontal road. The driving force is 4000 N and the total of all resistances to motion is 800 N.
Calculate the velocity of the car after 9 seconds. **[4]**

6 **(a)** Prove that $\frac{\sin\theta}{1 - \cos\theta} - \frac{1}{\sin\theta} = \cot\theta$. **[4]**

(b) Hence find the exact roots of the equation $\frac{\sin\theta}{1 - \cos\theta} - \frac{1}{\sin\theta} = 3\tan\theta$ in the interval $0 \leqslant \theta \leqslant \pi$. **[3]**

Answer **all** the questions.

Section B (75 marks)

7 The velocity $v\,\mathrm{m\,s^{-1}}$ of a particle at time t s is given by

$v = 0.5t(7-t)$.

Determine whether the **speed** of the particle is increasing or decreasing when $t = 8$. **[4]**

8 An arithmetic series has first term 9300 and 10th term 3900.

(a) Show that the 20th term of the series is negative. **[3]**

(b) The sum of the first n terms is denoted by S. Find the greatest value of S as n varies. **[4]**

9 A cannonball is fired from a point on horizontal ground at $100\,\mathrm{m\,s^{-1}}$ at an angle of 25° above the horizontal. Ignoring air resistance, calculate

(a) the greatest height the cannonball reaches, **[3]**

(b) the range of the cannonball. **[4]**

10 (a) Express $7\cos x - 2\sin x$ in the form $R\cos(x+\alpha)$ where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact value of R and the value of α correct to 3 significant figures. **[4]**

(b) Give details of a sequence of two transformations which maps the curve $y = \sec x$ onto the curve $y = \dfrac{1}{7\cos x - 2\sin x}$. **[3]**

11 In this question, the unit vector **i** is horizontal and the unit vector **j** is vertically upwards.

A particle of mass 0.8 kg moves under the action of its weight and two forces given by $(k\mathbf{i} + 5\mathbf{j})\,\mathrm{N}$ and $(4\mathbf{i} + 3\mathbf{j})\,\mathrm{N}$. The acceleration of the particle is vertically upwards.

(a) Write down the value of k. **[1]**

Initially the velocity of the particle is $(4\mathbf{i} + 7\mathbf{j})\,\mathrm{m\,s^{-1}}$.

(b) Find the velocity of the particle 10 seconds later. **[4]**

12 Fig. 12 shows a curve C with parametric equations $x = 4t^2$, $y = 4t$. The point P, with parameter t, is a general point on the curve. Q is the point on the line $x + 4 = 0$ such that PQ is parallel to the x-axis. R is the point (4, 0).

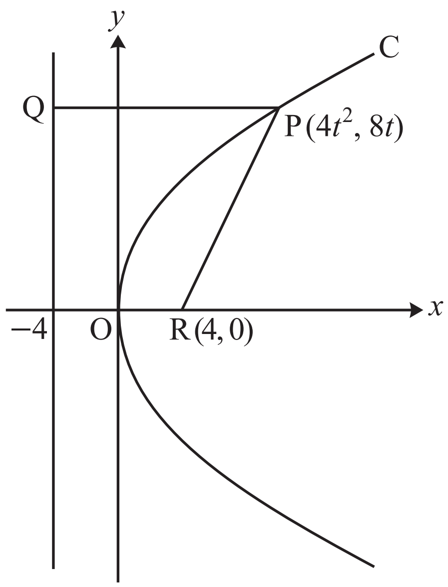


Fig. 12

(a) Show algebraically that P is equidistant from Q and R. **[4]**

(b) Find a cartesian equation of C. **[2]**

13 A 15 kg box is suspended in the air by a rope which makes an angle of 30° with the vertical. The box is held in place by a string which is horizontal.

(a) Draw a diagram showing the forces acting on the box. **[1]**

(b) Calculate the tension in the rope. **[2]**

(c) Calculate the tension in the string. **[2]**

14 Fig. 14 shows a circle with centre O and radius r cm. The chord AB is such that angle AOB $= x$ radians. The area of the shaded segment formed by AB is 5% of the area of the circle.

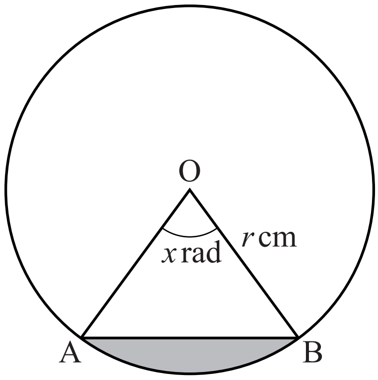


Fig. 14

(a) Show that $x - \sin x - \frac{1}{10}\pi = 0$. **[4]**

The Newton-Raphson method is to be used to find x.

(b) Write down the iterative formula to be used for the equation in part **(a)**. **[1]**

(c) Use three iterations of the Newton-Raphson method with $x_0 = 1.2$ to find the value of x to a suitable degree of accuracy. **[3]**

15 A model for the motion of a small object falling through a thick fluid can be expressed using the differential equation

$$\frac{dv}{dt} = 9.8 - kv,$$

where $v\,\text{m}\,\text{s}^{-1}$ is the velocity after t s and k is a positive constant.

(a) Given that $v = 0$ when $t = 0$, solve the differential equation to find v in terms of t and k. **[7]**

(b) Sketch the graph of v against t. **[2]**

Experiments show that for large values of t, the velocity tends to $7\,\text{m}\,\text{s}^{-1}$.

(c) Find the value of k. **[2]**

(d) Find the value of t for which $v = 3.5$. **[1]**

16 A particle of mass 2 kg slides down a plane inclined at 20° to the horizontal. The particle has an initial velocity of $1.4\,\mathrm{m\,s^{-1}}$ down the plane. Two models for the particle's motion are proposed.

In model A the plane is taken to be smooth.

(a) Calculate the time that model A predicts for the particle to slide the first 0.7 m. **[5]**

(b) Explain why model A is likely to underestimate the time taken. **[1]**

In model B the plane is taken to be rough, with a constant coefficient of friction between the particle and the plane.

(c) Calculate the acceleration of the particle predicted by model B given that it takes 0.8 s to slide the first 0.7 m. **[2]**

(d) Find the coefficient of friction predicted by model B, giving your answer correct to 3 significant figures. **[6]**

END OF QUESTION PAPER