Oxford Cambridge and RSA

Accredited

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

Sample Insert

Version 2

Date – Morning/Afternoon

Time allowed: 2 hours

INFORMATION FOR CANDIDATES

- This insert contains the article for Section B
- This document consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Archimedes's approximation of π

The constant π is defined to be the circumference of a circle divided by its diameter.

The value of π has been determined to an accuracy of more than twelve trillion decimal places. To the non-mathematician this may appear strange since it is not possible to measure the circumference and diameter of a circle to that degree of accuracy; this article explains how one of the greatest mathematicians of all time found the value of π to a high degree of accuracy without requiring any physical measurement.

Archimedes (287-212 BC) lived in Syracuse, Sicily. He developed many branches of mathematics, including calculus, in which he devised methods for finding areas under parabolas nearly 2000 years before Newton and Leibniz, and mechanics, in which he found the centres of gravity of various plane figures and solids and devised a method for calculating the weight of a body immersed in a liquid.

Whilst absorbed in a mathematical problem, Archimedes was killed by a soldier during the capture of Syracuse by the Romans.

Archimedes's method for determining the value of π is described below.

Fig. C1 shows a circle with unit radius and two regular hexagons.

The smaller regular hexagon has its vertices on the circle; it is called an *inscribed* polygon. Its perimeter is 6.

The larger regular hexagon has the midpoints of its edges on the circle; it is called an *escribed* polygon. Its perimeter is $4\sqrt{3}$.

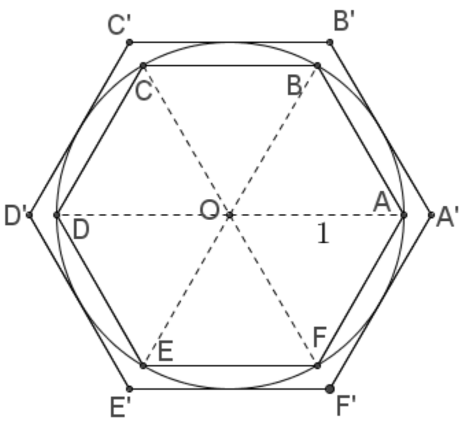

Fig. C1

The circumference of the circle is greater than the perimeter, ABCDEF, of the smaller hexagon but less than the perimeter, $A'B'C'D'E'F'$, of the larger hexagon.

Dividing the perimeters by the diameter of the circle gives lower and upper bounds for π of 3 and $2\sqrt{3}$, so that $3 < \pi < 2\sqrt{3}$.

To find tighter bounds, Archimedes repeatedly doubled the number of edges in the two regular polygons, from 6 to 12, 24, 48 and finally 96. The process of doubling the number of edges is described below.

Fig. C2 shows two adjacent vertices, P and Q, of a regular polygon inscribed in a circle with unit radius and centre O. PQ has length *a*. M is the midpoint of PQ. OM is extended to meet the circle at R. MR has length *h*. PR and RQ are adjacent edges of a regular polygon which has twice as many edges as the polygon which has PQ as an edge. PR has length *b*.

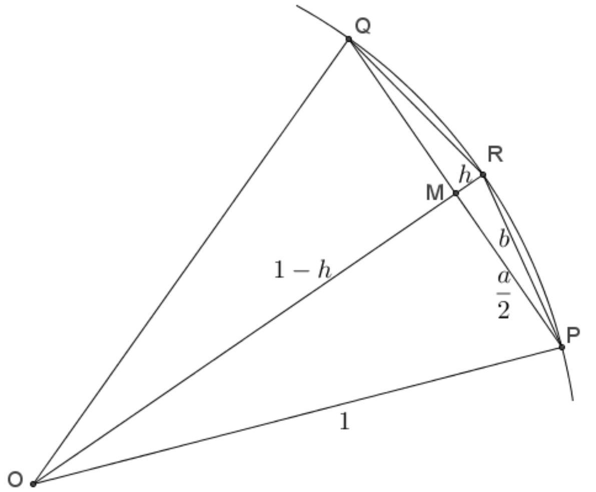

Fig. C2

Applying Pythagoras' Theorem

- to triangle OMP gives $1 = \frac{a^2}{4} + (1-h)^2$,
- to triangle PMR gives $b^2 = \frac{a^2}{4} + h^2$.

For the inscribed regular hexagon, $a = 1$. Substituting $a = 1$ in the equations above gives $h = \frac{2-\sqrt{3}}{2}$ and $b = \sqrt{2-\sqrt{3}}$. This can be written in the equivalent form $b = \frac{\sqrt{6}-\sqrt{2}}{2}$.

Therefore a regular polygon with 12 edges inscribed in a unit circle has edge length $\frac{\sqrt{6}-\sqrt{2}}{2}$.

Archimedes repeated this process to find the edge lengths of inscribed regular polygons with 24, 48 and 96 edges. He then used a similar technique for escribed regular polygons.

The inscribed and escribed regular polygons with 96 edges provide bounds for π which we now write, using decimal notation, as

$3.14103... < \pi < 3.14271...$.

Summary of Updates

Date	Version	Change
October 2018	2	We've reviewed the look and feel of our papers through text, tone, language, images and formatting. For more information please see our assessment principles in our "Exploring our question papers" brochures on our website.

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A Level Mathematics B (MEI)
H640/03 Pure Mathematics and Comprehension
Sample Question Paper Version 2.1

Date – Morning/Afternoon
Time allowed: 2 hours

You must have:
- Printed Answer Booklet
- the Insert

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r|<1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1\, a^{n-1}b + {}^nC_2\, a^{n-2}b^2 + \ \dots\ + {}^nC_r\, a^{n-r}b^r + \ \dots\ + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ \dots\ + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \ \dots \qquad (|x|<1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2\, x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \frac{u}{v}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h\to 0} \frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\left(\mathrm{f}(x)\right)^n \mathrm{d}x = \frac{1}{n+1}\left(\mathrm{f}(x)\right)^{n+1} + c$$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small Angle Approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Sample Variance

$s^2 = \dfrac{1}{n-1} S_{xx}$ where $S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \dfrac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The Binomial Distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

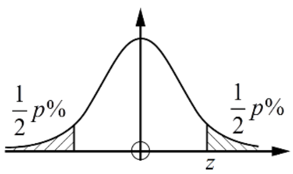
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \tfrac{1}{2}at^2$

$s = \tfrac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \tfrac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \tfrac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \tfrac{1}{2}\mathbf{a}t^2$

Answer **all** the questions

Section A (60 marks)

1 Express $\frac{2}{x-1}+\frac{5}{2x+1}$ as a single fraction. **[2]**

2 Find the first four terms of the binomial expansion of $(1-2x)^{\frac{1}{2}}$.

State the set of values of x for which the expansion is valid. **[4]**

3 Show that points A (1, 4, 9), B (0, 11, 17) and C (3, –10, –7) are collinear. **[4]**

4 Show that $\sum_{r=1}^{4} \ln \frac{r}{r+1} = -\ln 5$. **[3]**

5 **In this question you must show detailed reasoning.**

Fig. 5 shows the circle with equation $(x-4)^2+(y-1)^2=10$.
The points $(1, 0)$ and $(7, 0)$ lie on the circle. The point C is the centre of the circle.

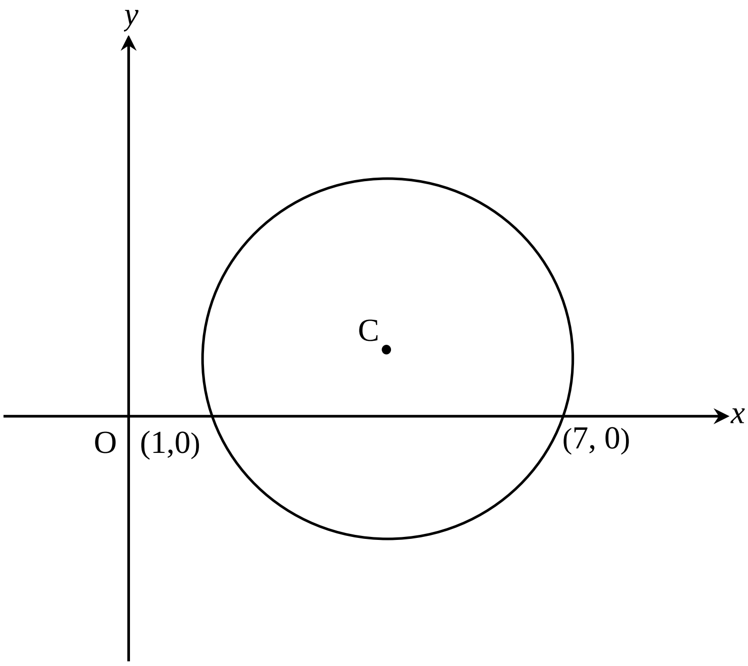


Fig. 5

Find the area of the part of the circle below the x-axis. **[5]**

6 **Fig. 6** shows the curve with equation $y = x^4 - 6x^2 + 4x + 5$.

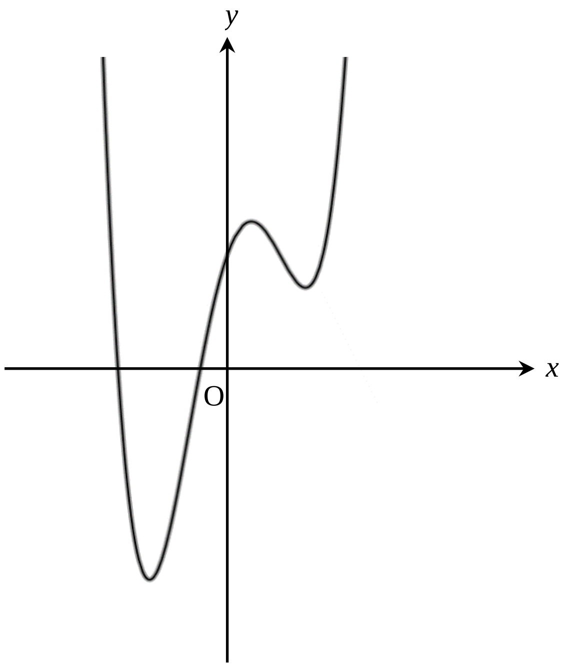


Fig. 6

Find the coordinates of the points of inflection. **[5]**

7 By finding a counter example, disprove the following statement.

If p and q are non-zero real numbers with $p < q$, then $\frac{1}{p} > \frac{1}{q}$. **[2]**

8 In **Fig. 8**, OAB is a thin bent rod, with OA = 1 m, AB = 2 m and angle OAB = 120°. Angles θ, ϕ and h are as shown in **Fig. 8**.

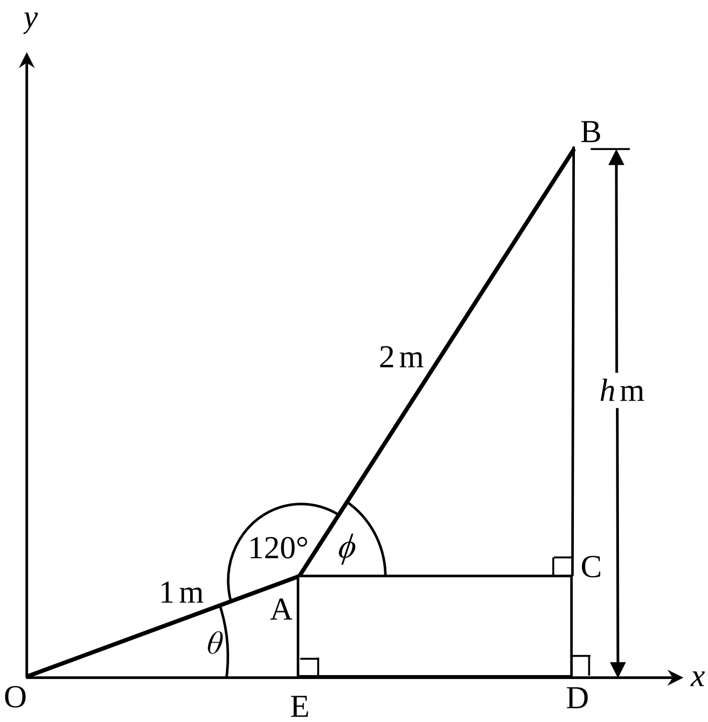


Fig. 8

(a) Show that $h = \sin\theta + 2\sin(\theta + 60°)$. **[3]**

The rod is free to rotate about the origin so that θ and ϕ vary. You may assume that the result for h in part **(a)** holds for all values of θ.

(b) Find an angle θ for which $h = 0$. **[5]**

9 **(a)** Express $\cos\theta + 2\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $0 < \alpha < \frac{1}{2}\pi$ and R is positive and given in exact form. **[4]**

The function $f(\theta)$ is defined by $f(\theta) = \dfrac{1}{(k + \cos\theta + 2\sin\theta)}$, $0 \le \theta \le 2\pi$, k is a constant.

(b) The maximum value of $f(\theta)$ is $\dfrac{(3+\sqrt{5})}{4}$.

Find the value of k. **[3]**

10 The function $f(x)$ is defined by $f(x) = x^4 + x^3 - 2x^2 - 4x - 2$.

(a) Show that $x = -1$ is a root of $f(x) = 0$. **[1]**

(b) Show that another root of $f(x) = 0$ lies between $x = 1$ and $x = 2$. **[2]**

(c) Show that $f(x) = (x+1)g(x)$, where $g(x) = x^3 + ax + b$ and a and b are integers to be determined. **[3]**

(d) Without further calculation, explain why $g(x) = 0$ has a root between $x = 1$ and $x = 2$. **[1]**

(e) Use the Newton-Raphson formula to show that an iteration formula for finding roots of $g(x) = 0$ may be written

$$x_{n+1} = \frac{2x_n^{\ 3} + 2}{3x_n^{\ 2} - 2}.$$

Determine the root of $g(x) = 0$ which lies between $x = 1$ and $x = 2$ correct to 4 significant figures. **[3]**

11 The curve $y = \mathrm{f}(x)$ is defined by the function $\mathrm{f}(x) = \mathrm{e}^{-x}\sin x$ with domain $0 \le x \le 4\pi$.

(a) **(i)** Show that the x-coordinates of the stationary points of the curve $y = \mathrm{f}(x)$, when arranged in increasing order, form an arithmetic sequence.

(ii) Show that the corresponding y-coordinates form a geometric sequence. **[9]**

(b) Would the result still hold with a larger domain? Give reasons for your answer. **[1]**

Answer **all** the questions

Section B (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

12 Explain why the smaller regular hexagon in **Fig. C1** has perimeter 6. **[1]**

13 Show that the larger regular hexagon in **Fig. C1** has perimeter $4\sqrt{3}$. **[3]**

14 Show that the two values of b given on line 36 are equivalent. **[3]**

15 **Fig. 15** shows a unit circle and the escribed regular polygon with 12 edges.

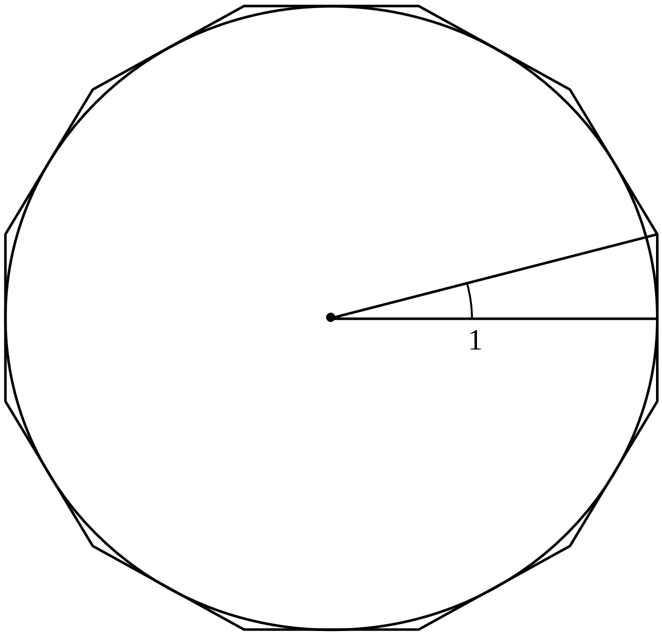


Fig. 15

(a) Show that the perimeter of the polygon is $24\tan 15°$. **[2]**

(b) Using the formula for $\tan(\theta - \phi)$ show that the perimeter of the polygon is $48 - 24\sqrt{3}$. **[3]**

16 On a unit circle, the inscribed regular polygon with 12 edges gives a lower bound for π, and the escribed regular polygon with 12 edges gives an upper bound for π.

Calculate the values of these bounds for π, giving your answers:

(i) in surd form

(ii) correct to 2 decimal places. **[3]**

END OF QUESTION PAPER

…day June 20XX – Morning/Afternoon

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

SAMPLE MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 75

This document consists of 16 pages

Text Instructions

1. **Annotations and abbreviations**

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

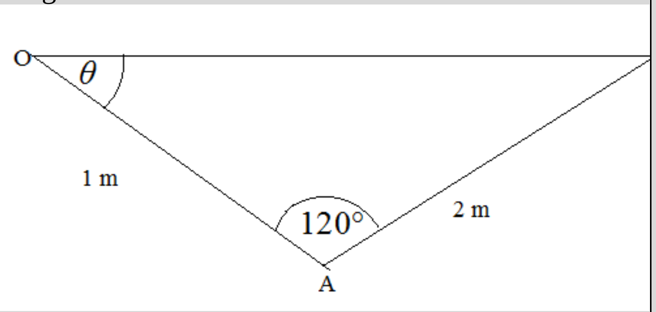
j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1			$\dfrac{2(2x+1)+5(x-1)}{(x-1)(2x+1)}$	**M1**	**1.1**		
			$=\dfrac{9x-3}{(x-1)(2x+1)}$	**A1**	**1.1**	Numerator should be simplified but need not be factorised, and denominator may be expanded, but mark final answer	
				[2]			
2			$(1-2x)^{\frac{1}{2}}$ $\approx 1+\frac{1}{2}(-2x)+\dfrac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}(-2x)^2+\dfrac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-2x)^3$	**M1**	**1.1**	binomial coefficients seen, allow one error	
			$=1-x-\frac{1}{2}x^2-\frac{1}{2}x^3$	**A2**	**1.1** **1.1**	$1-x$, $-\frac{1}{2}x^2$, $-\frac{1}{2}x^3$ or A1 for 2 correct terms	
			valid for $-\frac{1}{2}<x<\frac{1}{2}$	**B1**	**2.3**	or $\lvert x\rvert<\frac{1}{2}$	In this case, the series converges for $x=\pm\frac{1}{2}$ candidates are not expected to know this but allow $\leq$ for either or both inequalities.
				[4]			

Question			Answer	Marks	AOs	Guidance	
3			$\overrightarrow{AB} = \begin{pmatrix} -1 \\ 7 \\ 8 \end{pmatrix}$	**M1**	**3.1a**	Attempt to find vector between any two of the points	
			$\overrightarrow{AC} = \begin{pmatrix} 2 \\ -14 \\ -16 \end{pmatrix}$	**A1**	**1.1**	Correct pair of vectors with common point $\overrightarrow{BC} = \begin{pmatrix} 3 \\ -21 \\ -24 \end{pmatrix}$	
			AB is parallel to AC	**B1**	**1.1**		
			Common point A so collinear	**E1** **[4]**	**2.1**		
4			$\sum_{r=1}^{4} \ln \frac{r}{r+1} = \ln\frac{1}{2} + \ln\frac{2}{3} + \ln\frac{3}{4} + \ln\frac{4}{5}$	**B1**	**1.1**	soi	
			= ln 1– ln 2 + ln 2 – ln 3 + ln 3 – ln 4 + ln 4 – ln 5	**M1**	**3.1a**	use of $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ **or** ln *a* + ln *b* = ln *ab* $\ln\left(\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\right) = \ln\frac{1}{5} = \ln 1 - \ln 5 = -\ln 5$	
			= 0 – ~~ln 2~~ + ~~ln 2~~ – ~~ln 3~~ + ~~ln 3~~ – ~~ln 4~~ + ~~ln 4~~ – ln 5 = −ln 5	**E1** **[3]**	**2.2a**	AG	

Question			Answer	Marks	AOs	Guidance	
5			**DR** Radius $=\sqrt{10}$ $\cos C=\dfrac{10+10-(7-1)^2}{2\times\sqrt{10}\times\sqrt{10}}$ $C=2.50$ (3sf) Area $=\dfrac{1}{2}\times(\sqrt{10})^2\times 2.50-\dfrac{1}{2}\times(\sqrt{10})^2\times\sin 2.50$ Area = 9.49	**B1** **M1** **A1** **M1** **A1** **[5]**	**1.1** **3.1a** **1.1** **3.1a** **1.1**	Or use right angled triangle: M1 for $\cos x=\dfrac{3}{\sqrt{10}}$ and $\dfrac{1}{2}C=\dfrac{\pi}{2}-\cos^{-1}\left(\dfrac{3}{\sqrt{10}}\right)$	
6			$\dfrac{dy}{dx}=4x^3-12x+4$ $\dfrac{d^2y}{dx^2}=12x^2-12=0$ $x=\pm 1$ $(-1,-4)$ and $(1, 4)$	**M1** **A1** **M1** **A1** **A1** **[5]**	**1.1** **1.1** **1.2** **1.1** **2.1**	Differentiating once First derivative Differentiating a second time and equating to zero	
7			E.g. $p=-1$, $q=2$ $\dfrac{1}{p}=-1$, $\dfrac{1}{q}=\dfrac{1}{2}$ So $\dfrac{1}{p}<\dfrac{1}{q}$ for these values.	**B1** **E1** **[2]**	**3.1a** **2.1**	correct counter example stated shown	

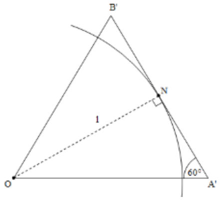
Question			Answer	Marks	AOs	Guidance	
8	(a)		BAC = 360 − 120 – 90 – (90 – θ) = θ + 60 $\Rightarrow$ BC = 2 sin(θ + 60) CD = AE = sin θ $\Rightarrow$ h = CD + BC = sin θ +2 sin (θ + 60°)	**B1** **M1** **E1** **[3]**	**3.1a** **1.1** **2.1**	AG	

Question			Answer	Marks	AOs	Guidance	
8	(b)		$h = \sin\theta + 2\sin(\theta + 60°)$				
			$= \sin\theta + 2(\sin\theta\cos 60 + \cos\theta\sin 60)$	**M1**	**3.1a**	use of compound angle formula	
			$= \sin\theta + \sin\theta + \sqrt{3}\cos\theta$	**A1**	**2.1**		
			$= 2\sin\theta + \sqrt{3}\cos\theta$				
			$h = 0 \Rightarrow 2\sin\theta + \sqrt{3}\cos\theta = 0$	**M1**	**1.1**	$h = 0$ soi	
			$\Rightarrow \tan\theta = -\frac{\sqrt{3}}{2}$	**M1**	**1.1**	Use of $\frac{\sin}{\cos} = \tan$	
			$\Rightarrow \theta = -40.9°$[so 40.9° below the horizontal]	**A1**	**1.1**	or 319.1° or 139.1°	
			Alternative method				
			Diagram with $h = 0$	**M1**	3.1a		
							
			$a^2 = 1^2 + 2^2 - 4\cos 120°$	**M1**	2.1		
			$a = \sqrt{7}$	**A1**	1.1		
			$\sin\theta = \frac{2\sin 120°}{\sqrt{7}} = \sqrt{\frac{3}{7}}$	**M1**	1.1		
			$\theta = -40.9°$[so 40.9° below the horizontal]	**A1**	1.1	For final mark, θ shown below horizontal in diagram together with 40.9° is acceptable	
				[5]			

Question			Answer	Marks	AOs	Guidance	
9	(a)		$\cos\theta + 2\sin\theta \equiv R\cos(\theta - \alpha)$ $\Rightarrow R\cos\alpha = 1,\ R\sin\alpha = 2$ $\Rightarrow R^2 = 5,\ R = \sqrt{5}$ $\tan\alpha = 2,\ \alpha = 1.107$	**M1** **B1** **M1** **A1** **[4]**	**1.1a** **1.1** **1.1** **1.1**		
9	(b)		max value is $\dfrac{1}{(k - \sqrt{5})}$ $\dfrac{1}{(k - \sqrt{5})} = \dfrac{(3+\sqrt{5})}{4}$ $4 = 3k - 5 + k\sqrt{5} - 3\sqrt{5}$ [This is indep of $\sqrt{5}$ so] $k = 3$	**M1** **M1** **A1** **[3]**	**3.1a** **1.1** **1.1**		
10	(a)		$f(-1) = (-1)^4 + (-1)^3 - 2(-1)^2 - 4(-1) - 2$ $= 1 - 1 - 2 + 4 - 2 = 0$	**E1** **[1]**	**1.1**		
10	(b)		f(1) = 1 + 1 – 2 – 4 – 2 = −6 or 'negative' f(2) = 16 + 8 – 8 – 8 – 2 = 6 or 'positive' change of sign $\Rightarrow$ root between 1 and 2	**B1** **E1** **[2]**	**1.1** **2.4**	both correct allow no mention of continuity of f AG	
10	(c)		long division or equating coeffts $\Rightarrow g(x) = x^3 - 2x - 2$ so $a = -2,\ b = -2$	**M1** **A1** **A1** **[3]**	**1.1** **2.2a** **1.1**		

Question			Answer	Marks	AOs	Guidance	
10	(d)		Clear explanation E.g. f(x) = (x + 1)g(x) For the root of f(x) = 0 between 1 and 2, RHS is also zero hence g(x) = 0	**E1** **[1]**	**2.4**		
10	(e)		$x_{n+1} = x_n - \frac{g(x_n)}{g'(x_n)}$ $= x_n - \frac{x_n^3 - 2x_n - 2}{3x_n^2 - 2}$ $= \frac{3x_n^3 - 2x_n - x_n^3 + 2x_n + 2}{3x_n^2 - 2}$ $= \frac{2x_n^3 + 2}{3x_n^2 - 2}$ Root 1.769 (4sf)	 **M1** **E1** **A1** **[3]**	 **1.1** **2.4** **2.2a**	 AG **BC**	

Question			Answer	Marks	AOs	Guidance	
11	(a)	(i)	$f'(x) = e^{-x}\cos x - e^{-x}\sin x$	M1 A1	3.1a 1.1	product rule correct	
			$f'(x) = 0$ and $e^{-x} \neq 0 \Rightarrow \cos x = \sin x$	E1	2.2a		
			$\Rightarrow \tan x = 1$	M1	1.1	Use of $\frac{\sin}{\cos} = \tan$	
			$\Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$	A1	1.1	$x = \frac{\pi}{4}$ (condone 45°) $\frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \ldots$	
			So an AP with $d = \pi$	E1FT	2.1	must state the common difference	FT their values of x
		(ii)	$y = \frac{\sqrt{2}}{2}e^{-\frac{\pi}{4}}, -\frac{\sqrt{2}}{2}e^{-\frac{5\pi}{4}}, \frac{\sqrt{2}}{2}e^{-\frac{9\pi}{4}}, -\frac{\sqrt{2}}{2}e^{-\frac{13\pi}{4}}$	M1 A1	3.1a 1.1	substituting one value of x into $f(x)$	
			This is a GP with $r = -e^{-\pi}$	E1FT [9]	2.1	must state common ratio, www	FT their values of y
11	(b)		Yes with explanation that values of x would continue to be separated by pi and so values of y would continue to have same common ratio.	E1 [1]	2.2a		
12			Each triangle (like OAB) is equilateral	E1 [1]	2.1	oe	

Question			Answer	Marks	AOs	Guidance	
13			 Show diagram which was previously fig 13 Angle $A'N = \tan 30^\circ$ OR $\tan 30^\circ = \frac{1}{A'N}$ $A'N = \tan 30^\circ = \frac{1}{\sqrt{3}}$	M1 A1	3.1a 1.1	soi	
			Alternative method using the equilateral triangle $OA'B'$ of side length $2a$: $(2a)^2 = a^2 + 1 \Rightarrow a^2 = \frac{1}{3}$ $a = A'N = \frac{1}{\sqrt{3}}$	M1 M1	3.1a 1.1		
			Evidence of $6 \times \text{A'B'}$ or $12 \times \text{A'N}$ $= 4\sqrt{3}$	E1 **[3]**	2.4	AG	
14			$\left(\frac{\sqrt{6}-\sqrt{2}}{2}\right)^2 = \frac{8-2\sqrt{12}}{4}$ $= \frac{8-4\sqrt{3}}{4} = 2-\sqrt{3}$ $\frac{\sqrt{6}-\sqrt{2}}{2}$ is positive so it is equal to $\sqrt{2-\sqrt{3}}$	M1 A1 E1 **[3]**	3.1a 1.1 2.1	Attempt to square Answer in exact form Completion of argument to show the two values are equal	

Question			Answer	Marks	AOs	Guidance	
15	(a)		Angle = 360 ÷ 24 = 15	M1	1.1		
			Edge length $= 2\tan 15^\circ$				
			Perimeter = $12 \times 2\tan 15^\circ$	E1	2.1		
			$= 24\tan 15^\circ$	[2]		AG	
15	(b)		$\tan 15^\circ = \tan\left(45^\circ - 30^\circ\right)$	B1	3.1a		
			$= \dfrac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$ $\left[= \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = \dfrac{\left(\sqrt{3}-1\right)^2}{2}\right]$	M1	1.1	Exact values of $\tan 45^\circ$ and $\tan 30^\circ$ used	
			Alternative method				
			$\tan 15^\circ = \tan\left(60^\circ - 45^\circ\right)$	B1	3.1a		
			$= \dfrac{\sqrt{3}-1}{1+\sqrt{3}}$ $\left[= \dfrac{2\sqrt{3}-4}{-2}\right]$	M1	1.1	Exact values of $\tan 60^\circ$ and $\tan 15^\circ$ used	
			Perimeter = $12 \times 2\tan 15^\circ$	E1	2.1	Correct completion	
			$= 48 - 24\sqrt{3}$	[3]		AG	
16		(i)	Lower bound: $3\left(\sqrt{6}-\sqrt{2}\right)$	B1	1.1	Half perimeter (from text)	
			Upper bound: $24 - 12\sqrt{3}$	B1	1.1		
		(ii)	$= 3.11$ and 3.22	B1 [3]	1.1	Both as decimals	

Question	AO1	AO2	AO3(PS)	AO3(M)	Total
1	2				**2**
2	3	1			**4**
3	2	1	1		**4**
4	1	1	1		**3**
5	3	0	2		**5**
6	4	1			**5**
7		1	1		**2**
8 a	1	1	1		**3**
8 b	3	1	1		**5**
9 a	4	0			**4**
9 b	2		1		**3**
10 a	1				**1**
10 b	1	1			**2**
10 c	2	1			**3**
10 d		1			**1**
10 e	1	2			**3**
11 a	4	3	2		**9**
11 b		1	0		**1**
12		1			**1**
13	1	1	1		**3**
14	1	1	1		**3**
15 a	1	1			**2**
15 b	1	1	1		**3**
16 i	2				**2**
16 ii	1				**1**
Totals	**41**	**21**	**13**	**0**	**75**

Summary of Updates

Date	Version	Change
October 2018	2	We’ve reviewed the look and feel of our papers through text, tone, language, images and formatting. For more information please see our assessment principles in our “Exploring our question papers” brochures on our website.
June 2019	2.1	Correction of typographical error in the Integration fraction on page 2.