



GCE

# Mathematics

Advanced GCE 4726

Further Pure Mathematics 2

## Mark Scheme for June 2010

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- 1** Derive/quote  $g'(x) = p/(1+x^2)$   
 Attempt  $f'(x)$  as  $a/(1+bx^2)$   
 Use  $x = 1/2$  to set up a solvable equation in  $p$ , leading to at least one solution  
 Get  $p = 5/4$  only
- 2** Reasonable attempt at  $e^{2x}(1+2x+2x^2)$   
 Multiply out their expressions to get all terms up to  $x^2$   
 Get  $1+3x+4x^2$   
 Use binomial, equate coefficients to get 2 solvable equations in  $a$  and  $n$   
 Reasonable attempt to eliminate  $a$  or  $n$   
 Get  $n=9$ ,  $a=1/3$  cwo
- 3** Quote/derive correct  $dx=2dt/(1+t^2)$   
 Replace all  $x$  (not  $dx=dt$ )  
 Get  $2/(t-1)^2$  or equivalent  
 Reasonable attempt to integrate their expression  
 Use correct limits in their correct integral  
 Clearly tidy to  $\sqrt{3}+1$  from cwo
- 4 (i)** Get  $a = -2$   
 Get  $b = 6$   
 Get  $c = 1$

B1  
 M1 Allow any  $a$ ,  $b=2$  or  $4$   
 M1  
 A1 AEEF

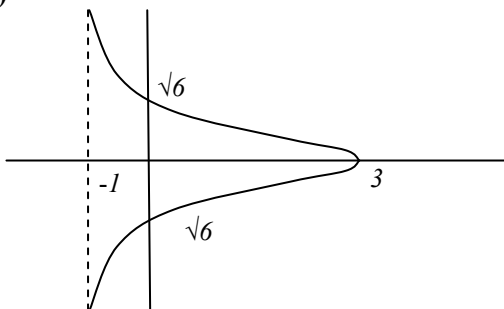
M1 3 terms of the form  $1+2x+ax^2$ ,  $a \neq 0$   
 M1 (3 terms) x (minimum of 2 terms)  
 A1 cao  
 Reasonable attempt at binomial, each term  
 M1 involving  $a$  and  $n$  ( $an=3$ ,  $a^2n(n-1)/2=4$ )  
 M1  
 A1 cao  
 SC Reasonable  $f'(x)$  and  $f''(x)$  using product rule (2 terms) M1  
 Use their expressions to find  $f'(0)$  and  $f''(0)$  M1  
 Get  $1+3x+4x^2$  cao A1

B1  
 M1 From their expressions  
 A1  
 M1  
 A1  $\sqrt{\quad}$  Must involve  $\sqrt{3}$   
 A1 A.G.

B1 May be quoted  
 B1 May be quoted  
 B1 May be quoted

(from correct working)

(ii)



B1 Correct shape in  $-1 < x \leq 3$  only  
 (allow just top or bottom half)

B1  $90^\circ$  (at  $x=3$ ) (must cross  $x$ -axis i.e. symmetry)

B1 Asymptote at  $x = -1$  only (allow  $-1$  seen)

B1  $\sqrt{\quad}$  Correct crossing points;  $\pm\sqrt{(b/c)}$  from their  $b, c$

- 5 (i)** Reasonable attempt at parts  
Get  $e^x(1-2x)^n - \int e^x \cdot n(1-2x)^{n-1} \cdot -2 dx$   
Evidence of limits used in integrated part  
Tidy to A.G.
- M1 Leading to second integral  
A1 Or  $(1-2x)^{n+1}/(-2(n+1))e^x - \int (1-2x)^{n+1}/(-2(n+1))e^x dx$   
M1 Should show  $\pm 1$   
A1 Allow  $I_{n+1} = 2(n+1)I_n - 1$
- (ii)** Show any one of  $I_3=6I_2-1$ ,  $I_2=4I_1-1$ ,  
 $I_1=2I_0-1$   
Get  $I_0(=e^{1/2}-1)$  or  $I_1(=2e^{1/2}-3)$   
Substitute their values back for their  $I_3$   
Get  $48e^{1/2} - 79$
- B1 May be implied  
B1  
M1 Not involving  $n$   
A1
- 6 (i)** Reasonable attempt to differentiate  
 $\sinh y = x$  to get  $dy/dx$  in terms of  $y$   
Replace  $\sinh y$  to A.G.
- M1 Allow  $\pm \cosh y \, dy/dx = 1$   
A1 Clearly use  $\cosh^2 - \sinh^2 = 1$   
SC Attempt to diff.  $y = \ln(x + \sqrt{x^2+1})$   
using chain rule M1  
Clearly tidy to A.G. A1
- (ii)** Reasonable attempt at chain rule  
Get  $dy/dx = a \sinh(asinh^{-1}x)/\sqrt{x^2+1}$   
Reasonable attempt at product/quotient  
Get  $d^2y/dx^2$  correctly in some form  
Substitute in and clearly get A.G.
- M1 To give a product  
A1  
M1 Must involve  $\sinh$  and  $\cosh$   
A1  $\sqrt{\text{From } dy/dx = k \sinh(asinh^{-1}x)/\sqrt{x^2+1}}$   
A1  
SC Write  $\sqrt{x^2+1} dy/dx = k \sinh(asinh^{-1}x)$   
or similar  
Derive the A.G.
- 7 (i)** Get 5.242, 5.239, 5.237  
Get 5.24
- B1  $\sqrt{\text{Any 3(minimum) correct from previous value}}$   
B1 Allow one B1 for 5.24 seen if 2 d.p.used
- (ii)** Show reasonable staircase for any region  
Describe any one of the three cases  
Describe all three cases
- B1 Drawn curve to line  
B1  
B1
- (iii)** Reasonable attempt to use log/expo. rules  
Clearly get A.G.  
Attempt  $f'(x)$  and use at least once in correct N-R formula  
Get answers that lead to 1.31
- M1 Allow derivation either way  
A1  
M1  
A1 Minimum of 2 answers; allow truncation/rounding to at least 3 d.p.
- (iv)** Show  $f'(\ln 36) = 0$   
Explain why N-R would not work
- B1  
B1 Tangent parallel to  $Ox$  would not meet  $Ox$  again or divide by 0 gives an error

- 8 (i)** Use correct definition of  $\cosh x$  B1  
 Attempt to cube their definition involving  $e^x$  and  $e^{-x}$  (or  $e^{2x}$  and  $e^x$ ) M1 Must be 4 terms  
 Put their 4 terms into LHS and attempt to simplify M1  
 Clearly get A.G. A1  
 SC Allow one B1 for correct derivation from  $\cosh 3x = \cosh(2x+x)$
- (ii)** Rewrite as  $k\cosh 3x = 13$  M1  
 Use  $\ln$  equivalent on  $13/k$  M1 Allow  $\pm \ln$  or  $\ln(13/k \pm \sqrt{(13/k)^2 - 1})$  for their  $k$  or attempt to set up and solve quadratic via exponentials  
 Get  $x = (\pm) \frac{1}{3} \ln 5$  A1  
 Replace in  $\cosh x$  for  $u$  M1  
 Use  $e^{aln b} = b^a$  at least once M1  
 Get  $\frac{1}{2}(5^{1/3} + 5^{-1/3})$  A1
- 9 (i)** Attempt integral as  $k(2x+1)^{1.5}$  M1  
 Get 9 A1 cao  
 Attempt subtraction of areas M1 Their answer – triangle  
 Get 3 A1√ Their answer – 6 (>0)
- (ii)** Use  $r^2 = x^2 + y^2$  and  $x = r\cos\theta, y = r\sin\theta$  B1  
 Eliminate  $x$  and  $y$  to produce quadratic equation ( $=0$ ) in  $r$  (or  $\cos\theta$ ) M1  
 Solve their quadratic to get  $r$  in terms of  $\theta$  (or vice versa) A1√  
 Clearly get A.G. A1  $r > 0$  may be assumed  
 Clearly show  $\theta_1$  (at  $B$ ) =  $\tan^{-1} \frac{3}{4}$  and  $\theta_2$  (at  $A$ ) =  $\pi$  B1  
 SC Eliminate  $y$  to get  $r$  in terms of  $x$  only M1  
 Get  $r = x + 1$  A1  
 SC Start with  $r = 1/(1 - \cos\theta)$  and derive cartesian
- (iii)** Use area =  $\frac{1}{2} \int r^2 d\theta$  with correct  $r$  B1 cwo; ignore limits  
 Rewrite as  $k\operatorname{cosec}^4(\frac{1}{2}\theta)$  M1 Not just quoted  
 Equate to their part (i) and tidy M1 To get  $\int =$  some constant  
 Get 24 A1 A.G.

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