

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**  
**A2 GCE**

**4723**

**MATHEMATICS**

**Core Mathematics 3**

**QUESTION PAPER**

**THURSDAY 14 JUNE 2012: Morning**

**DURATION: 1 hour 30 minutes**  
**plus your additional time allowance**

**MODIFIED ENLARGED**

**Candidates answer on the Printed Answer Book or any suitable paper provided by the Centre. The Printed Answer Book may be enlarged by the Centre.**

**OCR SUPPLIED MATERIALS:**

**Printed Answer Book 4723**  
**List of Formulae (MF1)**

**OTHER MATERIALS REQUIRED:**

**Scientific or graphical calculator**

**READ INSTRUCTIONS OVERLEAF**

## **INSTRUCTIONS TO CANDIDATES**

**These instructions are the same on the Printed Answer Book and the Question Paper.**

- **The Question Paper will be found in the centre of the Printed Answer Book.**
- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.**
- **WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Answer ALL the questions.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.**

## **INFORMATION FOR CANDIDATES**

**This information is the same on the Printed Answer Book and the Question Paper.**

- **The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**
- **YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.**
- **The total number of marks for this paper is 72.**

## **INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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- 1 Solve the inequality  $|2x - 5| > |x + 1|$ . [5]
- 2 It is given that  $p = e^{280}$  and  $q = e^{300}$ .
- (i) Use logarithm properties to show that  $\ln\left(\frac{ep^2}{q}\right) = 261$ . [3]
- (ii) Find the smallest integer  $n$  which satisfies the inequality  $5^n > pq$ . [3]
- 3 It is given that  $\theta$  is the acute angle such that  $\sec \theta \sin \theta = 36 \cot \theta$ .
- (i) Show that  $\tan \theta = 6$ . [3]
- (ii) Hence, using an appropriate formula in each case, find the exact value of
- (a)  $\tan(\theta - 45^\circ)$ , [2]
- (b)  $\tan 2\theta$ . [2]
- 4 (a) Show that  $\int_0^4 \frac{18}{\sqrt{6x+1}} dx = 24$ . [4]
- (b) Find  $\int_0^1 (e^x + 2)^2 dx$ , giving your answer in terms of  $e$ . [4]

- 5 (i) It is given that  $k$  is a positive constant. By sketching the graphs of

$$y = 14 - x^2 \text{ and } y = k \ln x$$

on a single diagram, show that the equation

$$14 - x^2 = k \ln x$$

has exactly one real root. [3]

- (ii) The real root of the equation  $14 - x^2 = 3 \ln x$  is denoted by  $\alpha$ .

(a) Find by calculation the pair of consecutive integers between which  $\alpha$  lies. [3]

(b) Use the iterative formula  $x_{n+1} = \sqrt{14 - 3 \ln x_n}$ , with a suitable starting value, to find  $\alpha$ . Show the result of each iteration, and give  $\alpha$  correct to 2 decimal places. [4]

- 6 The volume,  $V \text{ m}^3$ , of liquid in a container is given by

$$V = (3h^2 + 4)^{\frac{3}{2}} - 8,$$

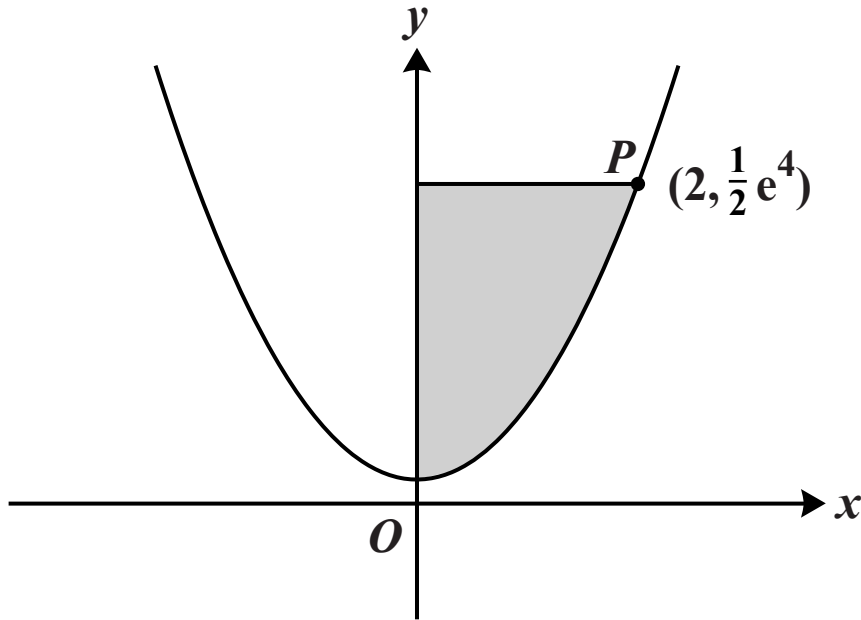
where  $h \text{ m}$  is the depth of the liquid.

- (i) Find the value of  $\frac{dV}{dh}$  when  $h = 0.6$ , giving your answer correct to 2 decimal places. [4]
- (ii) Liquid is leaking from the container. It is observed that, when the depth of the liquid is  $0.6 \text{ m}$ , the depth is decreasing at a rate of  $0.015 \text{ m}$  per hour. Find the rate at which the volume of liquid in the container is decreasing at the instant when the depth is  $0.6 \text{ m}$ . [3]

- 7** The function  $f$  is defined for all real values of  $x$  by  $f(x) = 2x + 5$ . The function  $g$  is defined for all real values of  $x$  and is such that  $g^{-1}(x) = \sqrt[3]{x - a}$ , where  $a$  is a constant. It is given that  $fg^{-1}(12) = 9$ . Find the value of  $a$  and hence solve the equation  $gf(x) = 68$ . [7]
- 8** (i) Express  $3 \sin \theta + 4 \cos \theta$  in the form  $R \sin(\theta + \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . [3]
- (ii) Hence
- (a) solve the equation  $3 \sin \theta + 4 \cos \theta + 1 = 0$ , giving all solutions for which  $-180^\circ < \theta < 180^\circ$ , [4]
- (b) find the values of the positive constants  $k$  and  $c$  such that
- $$-37 \leq k(3 \sin \theta + 4 \cos \theta) + c \leq 43$$
- for all values of  $\theta$ . [4]

- 9 (i) Show that the derivative with respect to  $y$  of  $y \ln(2y) - y$  is  $\ln(2y)$ . [3]

- (ii) Look at the following diagram.



The diagram shows the curve with equation  $y = \frac{1}{2} e^{x^2}$ . The point  $P(2, \frac{1}{2} e^4)$  lies on the curve. The shaded region is bounded by the curve and the lines  $x = 0$  and  $y = \frac{1}{2} e^4$ . Find the exact volume of the solid produced when the shaded region is rotated completely about the  $y$ -axis. [6]

- (iii) Hence find the volume of the solid produced when the region bounded by the curve and the lines  $x = 0$ ,  $x = 2$  and  $y = 0$  is rotated completely about the  $y$ -axis. [2]

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