

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4724/01

MATHEMATICS

Core Mathematics 4

QUESTION PAPER

FRIDAY 18 JANUARY 2013: Afternoon

**DURATION: 1 hour 30 minutes
plus your additional time allowance**

MODIFIED ENLARGED 18pt

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

**Printed Answer Book 4724/01
List of Formulae (MF1)**

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- **The Question Paper will be found in the centre of the Printed Answer Book.**
- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.**
- **WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Answer ALL the questions.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.**

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- **The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.**
- **YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.**
- **The total number of marks for this paper is 72.**
- **The Printed Answer Book consists of 12 pages. The Question Paper consists of 8 pages. Any blank pages are indicated.**

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- **Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.**

- 1 Find $\int x \cos 3x \, dx$. [4]
- 2 Find the first three terms in the expansion of $(9 - 16x)^{\frac{3}{2}}$ in ascending powers of x , and state the set of values for which this expansion is valid. [5]
- 3 The equation of a curve is $xy^2 = x^2 + 1$. Find $\frac{dy}{dx}$ in terms of x and y , and hence find the coordinates of the stationary points on the curve. [7]
- 4 The equations of two lines are
- $$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \quad \text{and}$$
- $$\mathbf{r} = 6\mathbf{i} + 8\mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}).$$
- (i) Show that these lines meet, and find the coordinates of the point of intersection. [5]
- (ii) Find the acute angle between these lines. [3]

5 The parametric equations of a curve are

$$x = 2 + 3 \sin \theta \text{ and } y = 1 - 2 \cos \theta \text{ for } 0 \leq \theta \leq \frac{1}{2}\pi.$$

(i) Find the coordinates of the point on the curve where the gradient is $\frac{1}{2}$. [5]

(ii) Find the cartesian equation of the curve. [2]

6 Use the substitution $u = 2x + 1$ to evaluate $\int_0^{\frac{1}{2}} \frac{4x - 1}{(2x + 1)^5} dx$. [7]

7 (i) Given that $y = \ln(1 + \sin x) - \ln(\cos x)$,

show that $\frac{dy}{dx} = \frac{1}{\cos x}$. [4]

(ii) Using this result, evaluate $\int_0^{\frac{1}{3}\pi} \sec x dx$, giving

your answer as a single logarithm. [3]

8 The points $A(3, 2, 1)$, $B(5, 4, -3)$, $C(3, 17, -4)$ and $D(1, 6, 3)$ form a quadrilateral $ABCD$.

(i) Show that $AB = AD$. [2]

(ii) Find a vector equation of the line through A and the mid-point of BD . [3]

(iii) Show that C lies on the line found in part (ii). [1]

(iv) What type of quadrilateral is $ABCD$? [1]

- 9 The temperature of a freezer is -20°C . A container of a liquid is placed in the freezer. The rate at which the temperature, $\theta^{\circ}\text{C}$, of a liquid decreases is proportional to the difference in temperature between the liquid and its surroundings. The situation is modelled by the differential equation**

$$\frac{d\theta}{dt} = -k(\theta + 20),$$

where time t is in minutes and k is a positive constant.

- (i) Express θ in terms of t , k and an arbitrary constant. [3]**

Initially the temperature of the liquid in the container is 40°C and, at this instant, the liquid is cooling at a rate of 3°C per minute. The liquid freezes at 0°C .

- (ii) Find the value of k and find also the time it takes (to the nearest minute) for the liquid to freeze. [5]**

The procedure is repeated on another occasion with a different liquid. The initial temperature of this liquid is 90°C . After 19 minutes its temperature is 0°C .

- (iii) Without any further calculation, explain what you can deduce about the value of k in this case. [1]**

10 (i) Use algebraic division to express $\frac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6}$
in the form $Ax + B + \frac{Cx + D}{x^2 - x - 6}$,

where A, B, C and D are constants. [4]

(ii) Hence find $\int_4^6 \frac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6} dx$, giving your
answer in the form $a + \ln b$. [7]

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