

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**A2 GCE**

**4723/01**

**MATHEMATICS**

**Core Mathematics 3**

**QUESTION PAPER**

**THURSDAY 13 JUNE 2013: Morning**

**DURATION: 1 hour 30 minutes  
plus your additional time allowance**

**MODIFIED ENLARGED**

**Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.**

**OCR SUPPLIED MATERIALS:**

**Printed Answer Book 4723/01**

**List of Formulae (MF1)**

**OTHER MATERIALS REQUIRED:**

**Scientific or graphical calculator**

**READ INSTRUCTIONS OVERLEAF**

## **INSTRUCTIONS TO CANDIDATES**

- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.**
- **If you use the Printed Answer Book, write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **Answer ALL the questions.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.**

## **INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.**
- The total number of marks for this paper is 72.
- Any blank pages are indicated.

## **INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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**1 Find**

(i)  $\int (4 - 3x)^7 dx,$

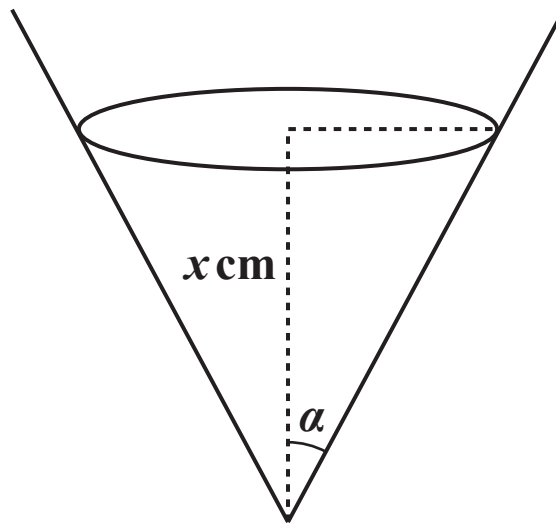
(ii)  $\int (4 - 3x)^{-1} dx.$  [5]

**2 Using an appropriate identity in each case, find the possible values of**

(i)  $\sin \alpha$  given that  $4 \cos 2\alpha = \sin^2 \alpha,$  [3]

(ii)  $\sec \beta$  given that  $2 \tan^2 \beta = 3 + 9 \sec \beta.$  [4]

**3 Look at the following diagram.**



The diagram shows a container in the form of a right circular cone. The angle between the axis and the slant height is  $\alpha$ , where  $\alpha = \tan^{-1}(\frac{1}{2})$ . Initially the container is empty, and then liquid is added at the rate of  $14 \text{ cm}^3$  per minute. The depth of liquid in the container at time  $t$  minutes is  $x \text{ cm}$ .

- (i) In this question you may need to use the following formula.

The volume of a cone is  $\frac{1}{3}\pi r^2 h$ .

Show that the volume,  $V\text{ cm}^3$ , of liquid in the container when the depth is  $x\text{ cm}$  is given by

$$V = \frac{1}{12}\pi x^3. \quad [2]$$

- (ii) Find the rate at which the depth of the liquid is increasing at the instant when the depth is 8 cm. Give your answer in cm per minute correct to 2 decimal places. [3]

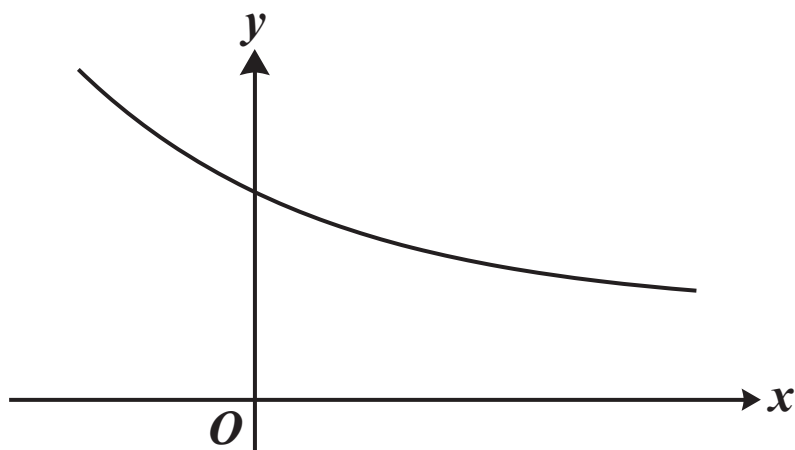
- 4 Find the exact value of the gradient of the curve

$$y = \sqrt{4x - 7} + \frac{4x}{2x + 1}$$

at the point for which  $x = 4$ . [6]

- 5 (i) Give full details of a sequence of two transformations needed to transform the graph of  $y = |x|$  to the graph of  $y = |2(x + 3)|$ . [3]
- (ii) Solve the inequality  $|x| > |2(x + 3)|$ , showing all your working. [5]

- 6 The value of  $\int_0^8 \ln(3 + x^2) dx$  obtained by using Simpson's rule with four strips is denoted by  $A$ .
- (i) Find the value of  $A$  correct to 3 significant figures. [4]
- (ii) Explain why an approximate value of  $\int_0^8 \ln(9 + 6x^2 + x^4) dx$  is  $2A$ . [2]
- (iii) Explain why an approximate value of  $\int_0^8 \ln(3e + ex^2) dx$  is  $A + 8$ . [2]
- 7 Look at the following diagram.



The diagram above shows the curve  $y = f(x)$ , where  $f$  is the function defined for all real values of  $x$  by

$$f(x) = 3 + 4e^{-x}.$$

- (i) State the range of  $f$ . [1]
- (ii) Find an expression for  $f^{-1}(x)$ , and state the domain and range of  $f^{-1}$ . [4]

- (iii) The straight line  $y = x$  meets the curve  $y = f(x)$  at the point  $P$ . By using an iterative process based on the equation  $x = f(x)$ , with a starting value of 3, find the coordinates of the point  $P$ . Show all your working and give each coordinate correct to 3 decimal places. [4]
- (iv) How is the point  $P$  related to the curves  $y = f(x)$  and  $y = f^{-1}(x)$ ? [1]

8 (i) Express  $4 \cos \theta - 2 \sin \theta$  in the form  $R \cos(\theta + \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . [3]

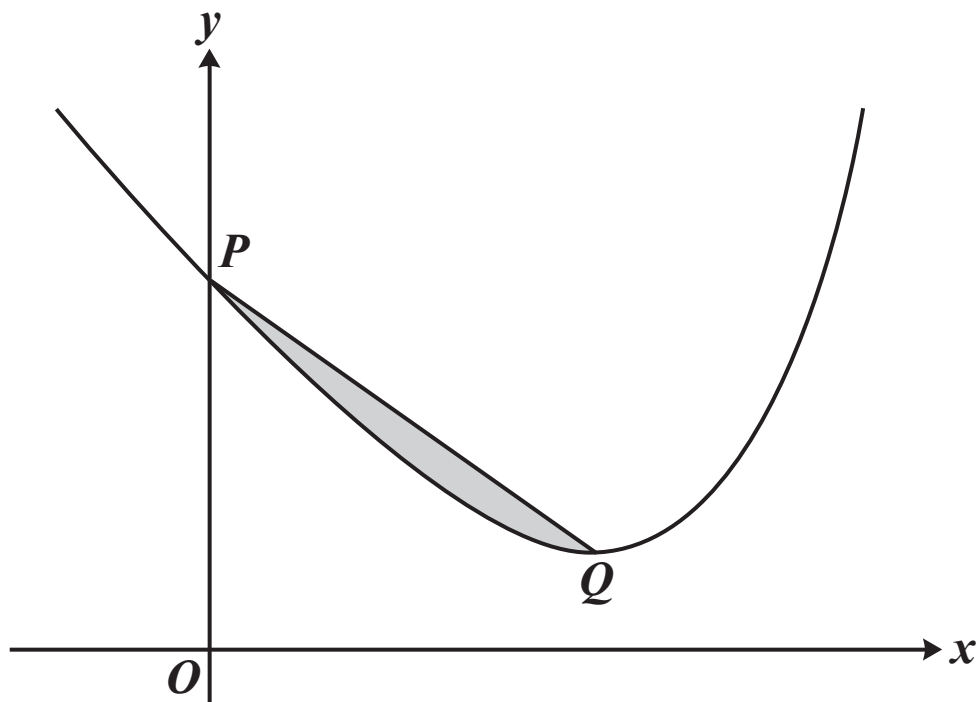
(ii) Hence

(a) solve the equation  $4 \cos \theta - 2 \sin \theta = 3$  for  $0^\circ < \theta < 360^\circ$ , [4]

(b) determine the greatest and least values of  $25 - (4 \cos \theta - 2 \sin \theta)^2$

as  $\theta$  varies, and, in each case, find the smallest positive value of  $\theta$  for which that value occurs. [5]

**9 Look at the following diagram.**



**The diagram above shows the curve**

$$y = e^{2x} - 18x + 15.$$

**The curve crosses the  $y$ -axis at  $P$  and the minimum point is  $Q$ . The shaded region is bounded by the curve and the line  $PQ$ .**

- (i) Show that the  $x$ -coordinate of  $Q$  is  $\ln 3$ . [3]**
- (ii) Find the exact area of the shaded region. [8]**



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