



Oxford Cambridge and RSA

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Friday 15 June 2018 – Afternoon

Time allowed: 2 hours

**You must have:**

- Printed Answer Booklet

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$\sin \theta \approx \theta$, $\cos \theta \approx 1 - \frac{1}{2}\theta^2$, $\tan \theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, Mean of X is np , Variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Section A: Pure Mathematics

Answer all the questions.

- 1 A circle with centre C has equation $x^2 + y^2 + 8x - 2y - 7 = 0$.

Find

- (i) the coordinates of C , [2]
(ii) the radius of the circle. [1]

- 2 Solve the equation $|2x - 1| = |x + 3|$. [3]

- 3 In this question you must show detailed reasoning.

A gardener is planning the design for a rectangular flower bed. The requirements are:

- the length of the flower bed is to be 3 m longer than the width,
- the length of the flower bed must be at least 14.5 m,
- the area of the flower bed must be less than 180 m^2 .

The width of the flower bed is x m.

By writing down and solving appropriate inequalities in x , determine the set of possible values for the width of the flower bed. [6]

- 4 In this question you must show detailed reasoning.

The functions f and g are defined for all real values of x by

$$f(x) = x^3 \quad \text{and} \quad g(x) = x^2 + 2.$$

- (i) Write down expressions for

(a) $fg(x)$, [1]

(b) $gf(x)$. [1]

- (ii) Hence find the values of x for which $fg(x) - gf(x) = 24$. [6]

- 5 (i) Use the trapezium rule, with two strips of equal width, to show that

$$\int_0^4 \frac{1}{2 + \sqrt{x}} dx \approx \frac{11}{4} - \sqrt{2}. \quad [5]$$

- (ii) Use the substitution $x = u^2$ to find the exact value of

$$\int_0^4 \frac{1}{2 + \sqrt{x}} dx. \quad [6]$$

- (iii) Using your answers to parts (i) and (ii), show that

$$\ln 2 \approx k + \frac{\sqrt{2}}{4},$$

where k is a rational number to be determined. [2]

- 6 It is given that the angle θ satisfies the equation $\sin\left(2\theta + \frac{1}{4}\pi\right) = 3 \cos\left(2\theta + \frac{1}{4}\pi\right)$.

(i) Show that $\tan 2\theta = \frac{1}{2}$. [3]

- (ii) Hence find, in surd form, the exact value of $\tan \theta$, given that θ is an obtuse angle. [5]

- 7 The gradient of the curve $y = f(x)$ is given by the differential equation

$$(2x - 1)^3 \frac{dy}{dx} + 4y^2 = 0$$

and the curve passes through the point $(1, 1)$. By solving this differential equation show that

$$f(x) = \frac{ax^2 - ax + 1}{bx^2 - bx + 1},$$

where a and b are integers to be determined. [9]

Section B: Mechanics

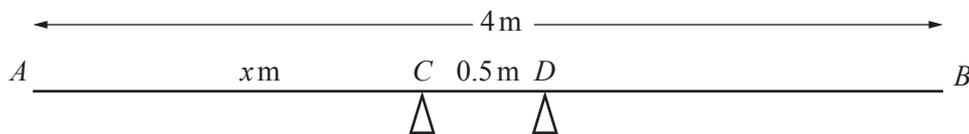
Answer **all** the questions.

- 8 In this question $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ denote unit vectors which are horizontal and vertically upwards respectively.

A particle of mass 5 kg, initially at rest at the point with position vector $\begin{pmatrix} 2 \\ 45 \end{pmatrix}$ m, is acted on by gravity and also by two forces $\begin{pmatrix} 15 \\ -8 \end{pmatrix}$ N and $\begin{pmatrix} -7 \\ -2 \end{pmatrix}$ N.

- (i) Find the acceleration vector of the particle. [3]
- (ii) Find the position vector of the particle after 10 seconds. [3]

- 9 A uniform plank AB has weight 100 N and length 4 m. The plank rests horizontally in equilibrium on two smooth supports C and D , where $AC = x$ m and $CD = 0.5$ m (see diagram).



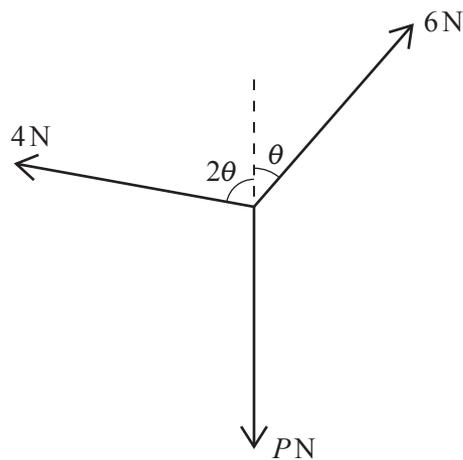
The magnitude of the reaction of the support on the plank at C is 75 N. Modelling the plank as a rigid rod, find

- (i) the magnitude of the reaction of the support on the plank at D , [1]
- (ii) the value of x . [3]

A stone block, which is modelled as a particle, is now placed at the end of the plank at B and the plank is on the point of tilting about D .

- (iii) Find the weight of the stone block. [3]
- (iv) Explain the limitation of modelling
- (a) the stone block as a particle, [1]
- (b) the plank as a rigid rod. [1]

- 10 Three forces, of magnitudes 4 N, 6 N and P N, act at a point in the directions shown in the diagram.



The forces are in equilibrium.

- (i) Show that $\theta = 41.4^\circ$, correct to 3 significant figures. [4]
- (ii) Hence find the value of P . [2]

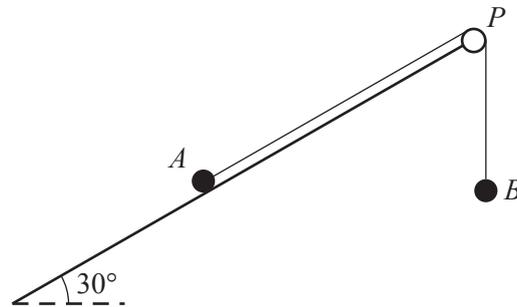
The force of magnitude 4 N is now removed and the force of magnitude 6 N is replaced by a force of magnitude 3 N acting in the same direction.

- (iii) Find
- (a) the magnitude of the resultant of the two remaining forces, [3]
- (b) the direction of the resultant of the two remaining forces. [2]

- 11 The velocity $v \text{ m s}^{-1}$ of a car at time t s, during the first 20 s of its journey, is given by $v = kt + 0.03t^2$, where k is a constant. When $t = 20$ the acceleration of the car is 1.3 m s^{-2} . For $t > 20$ the car continues its journey with constant acceleration 1.3 m s^{-2} until its speed reaches 25 m s^{-1} .

- (i) Find the value of k . [3]
- (ii) Find the total distance the car has travelled when its speed reaches 25 m s^{-1} . [7]

- 12 One end of a light inextensible string is attached to a particle A of mass m kg. The other end of the string is attached to a second particle B of mass λm kg, where λ is a constant. Particle A is in contact with a rough plane inclined at 30° to the horizontal. The string is taut and passes over a small smooth pulley P at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane. The particle B hangs freely below P (see diagram).



The coefficient of friction between A and the plane is μ .

- (i) It is given that A is on the point of moving down the plane.
- (a) Find the exact value of μ when $\lambda = \frac{1}{4}$. [7]
- (b) Show that the value of λ must be less than $\frac{1}{2}$. [2]
- (ii) Given instead that $\lambda = 2$ and that the acceleration of A is $\frac{1}{4}g \text{ m s}^{-2}$, find the exact value of μ . [5]

END OF QUESTION PAPER

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