A LEVEL
Further Mathematics B (MEI)

Qualification Accredited

Oxford Cambridge and RSA

A LEVEL
Specification

FURTHER MATHEMATICS B (MEI)

H645
For first assessment in 2019

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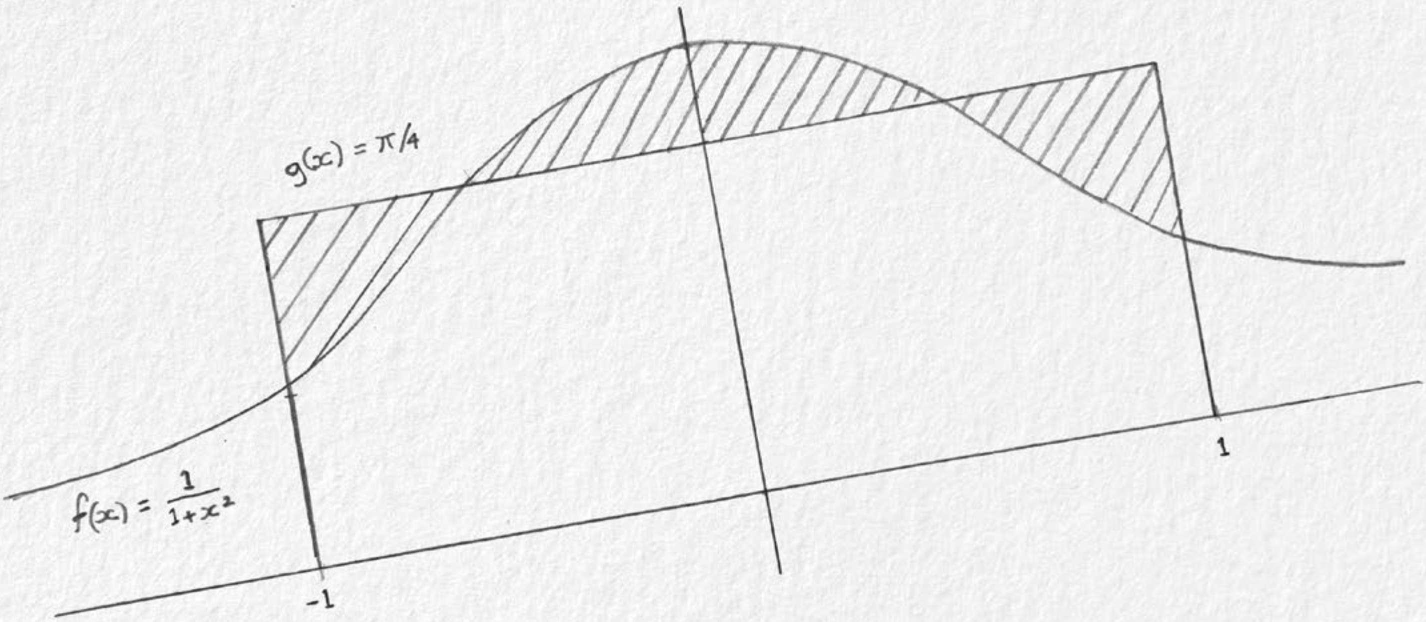

Innovators in Mathematics Education

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We will inform centres about changes to specifications. We will also publish changes on our website. The latest version of our specifications will always be those on our website (**ocr.org.uk**) and these may differ from printed versions.

Contents

1 Why choose an OCR A Level in Further Mathematics B (MEI)?

1a. Why choose an OCR qualification?

Choose OCR and you've got the reassurance that you're working with one of the UK's leading exam boards. Our new A Level in Further Mathematics B (MEI) course has been developed in consultation with teachers, employers and Higher Education to provide learners with a qualification that's relevant to them and meets their needs.

We're part of the Cambridge Assessment Group, Europe's largest assessment agency and a department of the University of Cambridge. Cambridge Assessment plays a leading role in developing and delivering assessments throughout the world, operating in over 150 countries.

We work with a range of education providers, including schools, colleges, workplaces and other institutions in both the public and private sectors. Over 13,000 centres choose our A Levels, GCSEs and vocational qualifications including Cambridge Nationals and Cambridge Technicals.

Our Specifications

We believe in developing specifications that help you bring the subject to life and inspire your students to achieve more.

We've created teacher-friendly specifications based on extensive research and engagement with the teaching community. They're designed to be straightforward and accessible so that you can tailor the delivery of the course to suit your needs. We aim to encourage learners to become responsible for their own learning, confident in discussing ideas, innovative and engaged.

We provide a range of support services designed to help you at every stage, from preparation through to the delivery of our specifications. This includes:

- A wide range of high-quality creative resources including:
  - Delivery Guides
  - Transition Guides
  - Topic Exploration Packs
  - Lesson Elements
  - ...and much more.
- Access to Subject Advisors to support you through the transition and throughout the lifetime of the specifications.
- CPD/Training for teachers including events to introduce the qualifications and prepare you for first teaching.
- Active Results – our free results analysis service to help you review the performance of individual learners or whole schools.
- ExamBuilder – our new free online past papers service that enables you to build your own test papers from past OCR exam questions can be found at www.ocr.org.uk/exambuilder

All A level qualifications offered by OCR are accredited by Ofqual, the Regulator for qualifications offered in England. The accreditation number for OCR A Level in Further Mathematics B (MEI) is QN603/1364/X.

1b. Why choose an OCR A Level in Further Mathematics B (MEI)

OCR A Level in Further Mathematics B (MEI) has been developed by Mathematics in Education and Industry (MEI) and is assessed by OCR. This is a well-established partnership which provides a firm foundation for curriculum and qualification development.

MEI is a long-established, independent curriculum development body; in developing this specification, MEI has consulted with teachers and representatives from Higher Education to decide how best to meet the long-term needs of learners.

MEI provides advice and CPD relating to all the curriculum and teaching aspects of the course. It also provides teaching resources, which for this specification can be found on the website (www.mei.org.uk).

Aims and learning outcomes

OCR A Level in Further Mathematics B (MEI) will encourage learners to:

- understand mathematics and mathematical processes in ways that promote confidence, foster enjoyment and provide a strong foundation for progress to further study
- extend their range of mathematical skills and techniques
- understand coherence and progression in mathematics and how different areas of mathematics are connected
- apply mathematics in other fields of study and be aware of the relevance of mathematics to the world of work and to situations in society in general
- use their mathematical knowledge to make logical and reasoned decisions in solving problems both within pure mathematics and in a variety of contexts, and communicate the mathematical rationale for these decisions clearly
- reason logically and recognise incorrect reasoning
- generalise mathematically
- construct mathematical proofs
- use their mathematical skills and techniques to solve challenging problems which require them to decide on the solution strategy
- recognise when mathematics can be used to analyse and solve a problem in context
- represent situations mathematically and understand the relationship between problems in context and mathematical models that may be applied to solve them
- draw diagrams and sketch graphs to help explore mathematical situations and interpret solutions
- make deductions and inferences and draw conclusions by using mathematical reasoning
- interpret solutions and communicate their interpretation effectively in the context of the problem
- read and comprehend mathematical arguments, including justifications of methods and formulae, and communicate their understanding
- read and comprehend articles concerning applications of mathematics and communicate their understanding
- use technology such as calculators and computers effectively, and recognise when such use may be inappropriate
- take increasing responsibility for their own learning and the evaluation of their own mathematical development.

OCR A Level in Further Mathematics B (MEI) is designed for students with an enthusiasm for mathematics, many of whom will go on to degrees in mathematics, engineering, the sciences and economics. or any subject where mathematics is developed further than in A level Mathematics.

OCR A Level in Further Mathematics B (MEI) is both deeper and broader than A level mathematics. AS and A Level Further Mathematics build from GCSE Level and AS and A Level Mathematics. As well as building on algebra and calculus introduced in A Level Mathematics, the A Level Further Mathematics core content introduces complex numbers and matrices, fundamental mathematical ideas with wide applications in mathematics, engineering, physical sciences and computing. The non-core content includes different options that can enable students to specialise in areas of mathematics that are particularly relevant to their interests and future aspirations. A Level Further Mathematics prepares students for further study and employment in highly mathematical disciplines that require knowledge and understanding of sophisticated mathematical ideas and techniques.

1c. What are the key features of this specification?

OCR A Level in Further Mathematics B (MEI) has been designed to help learners to fulfil their potential in mathematics and to support teachers in enabling them to do this. The qualification:

- encourages learners to develop a deep understanding of mathematics and an ability to use it in a variety of contexts
- allows a choice of options to enable teachers to create the most appropriate course for their students: choosing a major and a minor option focuses on depth of study; choosing three minor options focuses on breadth of study
- encourages learners to use appropriate technology to deepen their mathematical understanding and extend the range of problems which they are able to solve
- includes an option (Further Pure with Technology) which is assessed with learners having access to appropriate software in the examination
- is assessed in a way which is designed to enable all learners to show what they are able to do
- is clearly laid out with detailed guidance regarding what learners need to be able to do
- is resourced and supported by MEI in line with the aims and learning outcomes of the qualification.

This specification is designed to be co-teachable with A Level Mathematics B (MEI) and with AS Level Further Mathematics B (MEI). Clear labelling of the material allows teachers to know which parts of the A Level Further Mathematics specification could be taught in the first year of the course, alongside AS Mathematics and AS Further Mathematics.

1d. How do I find out more information?

If you are already using OCR specifications you can contact us at: www.ocr.org.uk

If you are not already a registered OCR centre then you can find out more information on the benefits of becoming one at: www.ocr.org.uk

If you are not yet an approved centre and would like to become one go to: www.ocr.org.uk

Want to find out more?

Get in touch with one of OCR's Subject Advisors:

Email: maths@ocr.org.uk

Customer Contact Centre: 01223 553998

Teacher support: www.ocr.org.uk

Advice is also available from MEI; contact details can be found on www.mei.org.uk

2 The specification overview

2a. OCR A Level in Further Mathematics B (MEI) (H645)

OCR's A Level in Further Mathematics B is a linear qualification in which all papers must be taken in the same examination series. To be awarded OCR's A Level in Further Mathematics B (MEI) learners must take one of three routes through the qualification, Route A, Route B or Route C.

Route A: Candidates must take the mandatory Core Pure and Mechanics Major papers and then one further optional minor paper. This paper must not be Mechanics Minor.
Route B: Candidates must take the mandatory Core Pure and Statistics Major papers and then one further optional minor paper. This paper must not be Statistics Minor.
Route C: Candidates must take the mandatory Core Pure paper and then three further minor optional papers.

Learners may **not** enter for Mechanics Major Y421 and Mechanics Minor Y431, Statistics Major Y422 and Statistics Minor Y432 or Mechanics Major Y421 and Statistics Major Y422.

Learners may take more than the required number of minor papers to increase the breadth of their course. For details of how their grade will be awarded, see Section 3g.

Content Overview

The qualification comprises one mandatory Core Pure paper and then a combination of optional papers:

- Core Pure content[1]
- Major options
  - Mechanics Major (Y421)[1]
  - Statistics Major (Y422)[1]
- Minor options
  - Mechanics Minor (Y431)[2]
  - Statistics Minor (Y432)[2]
  - Modelling with Algorithms (Y433)[2]
  - Numerical Methods (Y434)[2]
  - Extra Pure (Y435)
  - Further Pure with Technology (Y436)

The Overarching Themes must be applied along with associated mathematical thinking and understanding, across the whole of the subject content. See Section 2b.

[1]One third of the Core Pure content, and one half of the content of each major option can be co-taught with AS Further Mathematics. This material is labelled (a) throughout Sections 2c to 2e.

[2]These minor options can be co-taught with AS Further Mathematics.

Assessment Overview

Mandatory paper: Core Pure (Y420) 144 raw marks (180 scaled) 2 hour 40 mins Written paper	**50%** of total A level
Major Option 120 raw marks (120 scaled) 2 hour 15 mins Written paper	**$33\frac{1}{3}\%$** of total A level
Minor Option 60 raw marks (60 scaled) 1 hour 15 mins Written paper (1 hour 45 mins Written paper for Y436)	**$16\frac{2}{3}\%$** of total A level

2b. Content of A Level in Further Mathematics B (MEI) (H645)

This A level qualification builds on the skills, knowledge and understanding set out in the whole GCSE subject content for mathematics and the subject content for AS and A Level mathematics. Problem solving, proof and mathematical modelling will be assessed in further mathematics in the context of the wider knowledge which students taking A Level Further Mathematics will have studied.

A Level Further Mathematics B (MEI) is a linear qualification. Learners enter for the mandatory paper Core Pure (Y420) and then a combination of optional papers.

Route A: Candidates must take the mandatory Core Pure and Mechanics Major papers and then one further optional minor paper. This paper **must not** be Mechanics Minor.

Route B: Candidates must take the mandatory Core Pure and Statistics Major papers and then one further optional minor paper. This paper **must not** be Statistics Minor.

Route C: Candidates must take the mandatory Core Pure paper and then three further minor optional papers.

Learners may take more than the required number of minor papers to increase the breadth of their course. For details of how their grade will be awarded, see Section 3g.

Learners may **not** enter for Mechanics Major (Y421) and Mechanics Minor (Y431).

Learners may **not** enter for Statistics Major (Y422) and Statistics Minor (Y432).

Learners may **not** enter for Mechanics Major (Y421) and Statistics Major (Y422).

The content is listed below, under three headings:

1. Core Pure content

2. Major options
   - Mechanics Major (Y421)
   - Statistics Major (Y422)

3. Minor options
   - Mechanics Minor (Y431)
   - Statistics Minor (Y432)
   - Modelling with Algorithms (Y433)
   - Numerical Methods (Y434)
   - Extra Pure (Y435)
   - Further Pure with Technology (Y436)

The overarching themes should be applied, along with associated mathematical thinking and understanding, across every permissible combination of papers in this specification.

The applied optional papers (Mechanics Major, Mechanics Minor, Statistics Major, Statistics Minor and Modelling with Algorithms) should be regarded as applications of pure maths as well as ways of thinking about the world in their own right. The pure optional papers (Extra Pure and Further Pure with Technology) extend the content of the Core Pure paper. The Numerical Methods paper extends the range of non-analytic techniques for solving a wider class of problems from within pure mathematics. In all of these cases appropriate links should be made with the content of A Level Mathematics and the content of the Core pure paper in this A Level Further Mathematics.

Formulae and statistical tables

Some formulae will be available to learners in the examination, in a separate formulae booklet. A list of these formulae can be found in Section 5d. This list also contains the statistical tables which will be available to learners in the examination.

Use of calculators

Learners are permitted to use a scientific or graphical calculator for all papers. Generally, calculators are subject to the rules in the document *Instructions for Conducting Examinations*, published annually by JCQ (www.jcq.org.uk).

It is expected that calculators available in the assessment will include the following features:

- An iterative function such as an ANS key.
- The ability to perform calculations, including inversion, with matrices up to at least order 3×3.
- The ability to compute summary statistics and access probabilities from the binomial, Poisson and Normal distributions.

The Further Pure with Technology optional paper has different rules, requiring learners to have additional technology. In the examination for the Further Pure with Technology learners have access to a spreadsheet, graph-drawing software, a computer algebra system and a programming language on a computer or calculator.

Calculators with spreadsheets and graph-drawing functionality are permitted in all examination papers, but this functionality is only **required** in the Further Pure with Technology, where it may be available on either a computer or a calculator.

Generally, permitted calculators may be used for any function they can perform. When using calculators, learners should bear in mind the following:

1. Learners are advised to write down explicitly any expressions, including integrals, that they use the calculator to evaluate.
2. Learners are advised to write down the values of any parameters and variables that they input into the calculator. Learners are not expected to write down data transferred from question paper to calculator.
3. Correct mathematical notation (rather than "calculator notation") should be used; incorrect notation may result in loss of marks.

In the Numerical Methods optional paper, candidates are expected to show evidence of working through methods rather than just writing down solutions provided by equation solvers or numerical differential or integration functions on calculators.

Example for Numerical Methods:
Show that the equation $x^5 - 5x + 1 = 0$ has a root in the interval $[0, 1]$.

Using a calculator equation solver to find the three real roots 1.44, 0.2 and -1.54 and stating that one of them lies in the required interval would not be awarded marks. An acceptable method would be to evaluate $x^5 - 5x + 1$ at 0 and 1 and explain that the change of sign indicates that there is a root in the interval.

These are not restrictions on a learner's use of a calculator when tackling the question, e.g. for checking an answer or evaluating a function at a given point, but it is a restriction on what will be accepted as evidence of a complete method.

Use of technology

It is expected that learners will have used appropriate technology including mathematical graphing tools and spreadsheets when studying A Level Further Mathematics B (MEI). Several options have their own requirements for generic software which learners will have used; the content sections give more detail, including what is expected in the examination. In general, learners are not expected to be familiar with particular software, nor will they be expected to use the syntax associated with particular software but examination questions may include output from software which learners will need to complete or interpret. However, the Numerical Methods optional paper will also assess learners' ability to write some spreadsheet formulae; the Further Pure with Technology optional paper will assess learners' ability to use a computer algebra system, a graph plotter, a spreadsheet and a programming language on a computer or calculator in the examination.

Use of a computer in Further Pure with Technology

Learners require access to a computer and/or calculator with suitable software in the examination for Further Pure with Technology (Y436). Details of the software requirements and lists of approved software and approved programming languages may be found in Appendix 5e.

Any computer or calculator used in the examination must not be connected to any other computer or device, including a printer, whether wirelessly or by cable.

The learner must not have access to any stored files or documents at the beginning of the examination, but must be able to save and access files or documents produced during the examination.

Simplifying expressions

It is expected that learners will simplify algebraic and numerical expressions when giving their final answers, even if the examination question does not explicitly ask them to do so.

- $80\frac{\sqrt{3}}{2}$ should be written as $40\sqrt{3}$.
- $\frac{1}{3-\sqrt{2}}$ should be written as $\frac{3+\sqrt{2}}{7}$.
- $\frac{1}{2}(1+2x)^{-\frac{1}{2}} \times 2$ should be written as either $(1+2x)^{-\frac{1}{2}}$ or $\frac{1}{\sqrt{1+2x}}$.
- $\ln 2 + \ln 3 - \ln 1$ should be written as $\ln 6$.
- The equation of a straight line should be given in the form $y = mx + c$ or $ax + by = c$ unless otherwise stated.

The meanings of some instructions used in examination questions

In general, learners should show sufficient detail of their working and reasoning to indicate that a correct method is being used. The following command words are used to indicate when more, or less, specific detail is required.

Exact
An exact answer is one where numbers are not given in rounded form. The answer will often contain an irrational number such as $\sqrt{3}$, e or π and these numbers should be given in that form when an exact answer is required. The use of the word 'exact' also tells learners that rigorous (exact) working is expected in the answer to the question.
e.g. Find the exact solution of $\ln x = 2$.
The correct answer is e^2 and not $7.389\,056$.

e.g. Find the exact solution of $3x = 2$.
The correct answers are $x = \frac{2}{3}$ or $x = 0.\dot{6}$, not $x = 0.67$ or similar.

Prove
Learners are given a statement and must provide a formal mathematical argument which demonstrates its validity. A formal proof requires a high level of mathematical detail, with candidates clearly defining variables, correct algebraic manipulation and a concise conclusion.

Show that
Learners are given a result and have to get to the given result from the starting information. Because they are given the result, the explanation has to be sufficiently detailed to cover every step of their working.
e.g. Show that the curve $y = x\ln x$ has a stationary point $\left(\frac{1}{e}, -\frac{1}{e}\right)$.

Determine
This command word indicates that justification should be given for any results found, including working where appropriate.

Verify
A clear substitution of the given value to justify the statement is required.

Find, Solve, Calculate
These command words indicate, while working may be necessary to answer the question, no justification is required. A solution could be obtained from the efficient use of a calculator, either graphically or using a numerical method.

Give, State, Write down
These command words indicate that neither working nor justification is required.

In this question you must show detailed reasoning.
When a question includes this instruction learners must give a solution which leads to a conclusion showing a detailed and complete analytical method. Their solution should contain sufficient detail to allow the line of their argument to be followed. This is not a restriction on a learner's use of a calculator when tackling the question, e.g. for checking an answer or evaluating a function at a given point, but it is a restriction on what will be accepted as evidence of a complete method.

In these examples below variations in the structure of the answers are possible, for example; giving the integral as $\ln(x+\sqrt{x^2-16})$ in example 2, and different intermediate steps may be given.

Example 1:
Express $-4+2\mathrm{i}$ in modulus-argument form.

The answer is $\sqrt{20}(\cos 2.68+\mathrm{i}\sin 2.68)$, but the learner *must* include the steps $|-4+2\mathrm{i}|=\sqrt{16+4}=\sqrt{20}$, $\arg(-4+2\mathrm{i})=\pi-\tan^{-1}(0.5)=2.68$. Using a calculator in complex mode to convert to modulus-argument form would not result in a complete analytical method.

Example 2:
Evaluate $\int_4^5 \frac{1}{\sqrt{x^2-16}}\,\mathrm{d}x$.

The answer is $\ln(2)$, but the learner *must* include at least $\lim_{a\to 4}\left[\operatorname{arcosh}\left(\frac{X}{4}\right)\right]_a^5$ and the substitution $\ln\left(\frac{5}{4}+\sqrt{\frac{25}{16}-1}\right)-\ln(1+\sqrt{0})$. Just writing down the answer using the definite integral function on a calculator would therefore not be awarded any marks.

Example 3:
Solve the equation $2x^3-11x^2+22x-15=0$.

The answer is $1.5,\ 2\pm\mathrm{i}$, but the learner *must* include steps to find a real root or corresponding factor, find the factor $(2x-3)$ and factorise the cubic then solve the quadratic. Just writing down the three roots by using the cubic equation solver on a calculator would not be awarded any marks.

Hence
When a question uses the word 'hence', it is an indication that the next step should be based on what has gone before. The intention is that learners should start from the indicated statement.

e.g. You are given that $\mathrm{f}(x)=2x^3-x^2-7x+6$. Show that $(x-1)$ is a factor of $\mathrm{f}(x)$. Hence find the three factors of $\mathrm{f}(x)$.

Hence or otherwise is used when there are multiple ways of answering a given question. Learners starting from the indicated statement may well gain some information about the solution from doing so, and may already be some way towards the answer. The command phrase is used to direct learners towards using a particular piece of information to start from or to a particular method. It also indicates to learners that valid alternative methods exist which will be given full credit, but that they may be more time-consuming or complex.

e.g. Show that $(\cos x+\sin x)^2=1+\sin 2x$ for all x. Hence, or otherwise, find the derivative of $(\cos x+\sin x)^2$.

You may use the result
When this phrase is used it indicates a given result that learners would not always be expected to know, but which may be useful in answering the question. The phrase should be taken as permissive; use of the given result is not *necessarily* required.

Plot
Learners should mark points accurately on graph paper provided in the Printed Answer Booklet. They will either have been given the points or have had to calculate them. They may also need to join them with a curve or a straight line, or draw a line of best fit through them.
e.g. Plot this additional point on the scatter diagram.

Sketch (a graph)
Learners should draw a diagram, not necessarily to scale, showing the main features of a curve. These are likely to include at least some of the following.

- Turning points
- Asymptotes
- Intersection with the y-axis
- Intersection with the x-axis
- Behaviour for large x ($+$ or $-$)

Any other important features should also be shown.
E.g. Sketch the curve with equation $y = \frac{1}{(x-1)}$.

Draw
Learners should draw to an accuracy appropriate to the problem. They are being asked to make a sensible judgement about the level of accuracy which is appropriate.
e.g. Draw a diagram showing the forces acting on the particle.
e.g. Draw a line of best fit for the data.

Other command words
Other command words, for example “explain”, will have their ordinary English meaning.

Overarching Themes

These must be applied, along with associated mathematical thinking and understanding, across the whole of the detailed content set out below. These statements, similar to those in A Level Mathematics, are intended to direct the teaching and learning of A Level Further Mathematics, and they will be reflected in assessment tasks.

2

OT1 Mathematical argument, language and proof

	Knowledge/Skill
OT1.1	Construct and present mathematical arguments through appropriate use of diagrams; sketching graphs; logical deduction; precise statements involving correct use of symbols and connecting language, including: constant, coefficient, expression, equation, function, identity, index, term, variable.
OT1.2	Understand and use mathematical language and syntax as set out in the glossary.
OT1.3	Understand and use language and symbols associated with set theory, as set out in the glossary.
OT1.4	Understand and use the definition of a function; domain and range of functions.
OT1.5	Comprehend and critique mathematical arguments, proofs and justifications of methods and formulae, including those relating to applications of mathematics.

OT2 Mathematical problem solving

	Knowledge/Skill
OT2.1	Recognise the underlying mathematical structure in a situation and simplify and abstract appropriately to enable problems to be solved.
OT2.2	Construct extended arguments to solve problems presented in an unstructured form, including problems in context.
OT2.3	Interpret and communicate solutions in the context of the original problem.
OT2.4	Not Applicable to A Level Further Mathematics.
OT2.5	Not Applicable to A Level Further Mathematics.
OT2.6	Understand the concept of a mathematical problem solving cycle, including specifying the problem, collecting information, processing and representing information and interpreting results, which may identify the need to repeat the cycle.
OT2.7	Understand, interpret and extract information from diagrams and construct mathematical diagrams to solve problems.

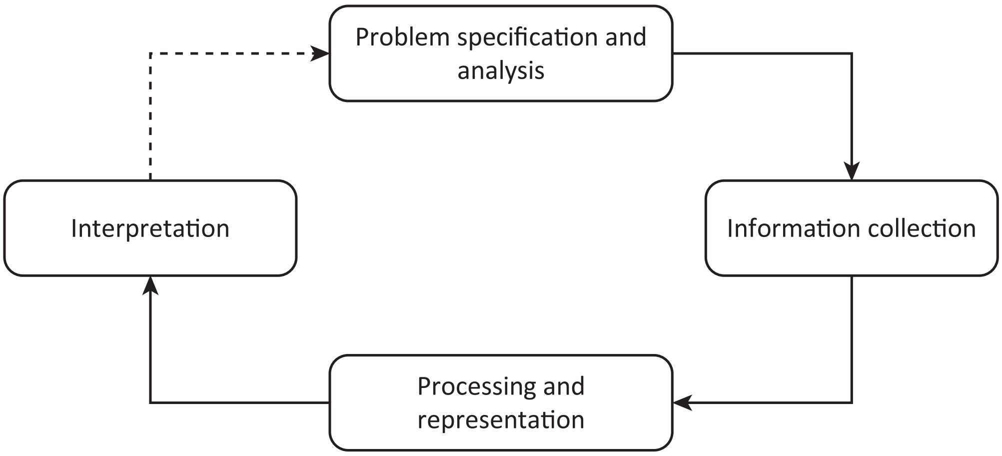
OT3 Mathematical modelling

	Knowledge/Skill
OT3.1	Translate a situation in context into a mathematical model, making simplifying assumptions.
OT3.2	Use a mathematical model with suitable inputs to engage with and explore situations (for a given model or a model constructed or selected by the student).
OT3.3	Interpret the outputs of a mathematical model in the context of the original situation (for a given model or a model constructed or selected by the student).
OT3.4	Understand that a mathematical model can be refined by considering its outputs and simplifying assumptions; evaluate whether the model is appropriate.
OT3.5	Understand and use modelling assumptions.

2

Mathematical Problem Solving Cycle

Mathematical problem solving is a core part of mathematics. The problem solving cycle gives a general strategy for dealing with problems which can be solved using mathematical methods; it can be used for problems within mathematical contexts and for problems in real-world contexts.

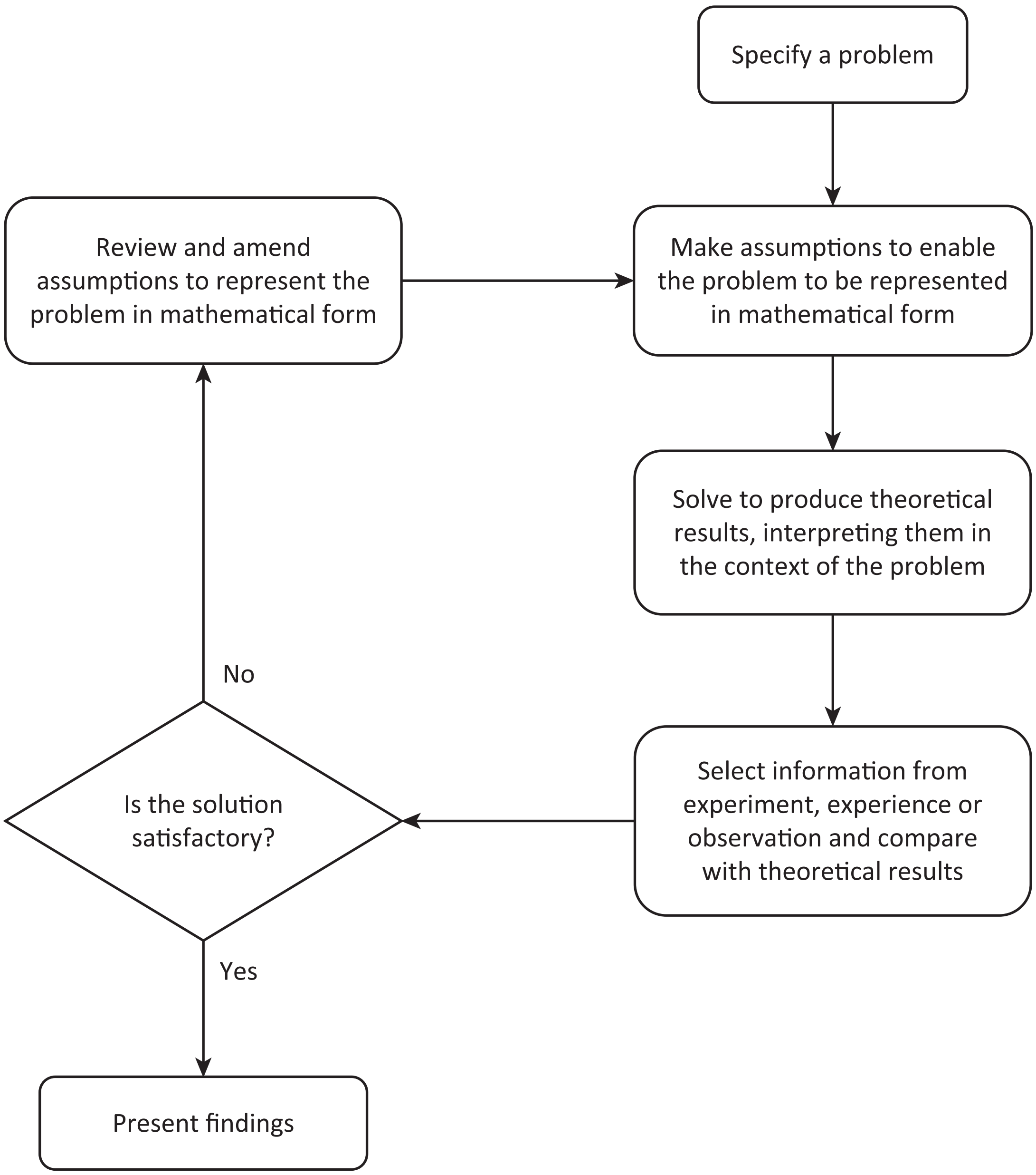


Process	Description
Problem specification and analysis	The problem to be addressed needs to be formulated in a way which allows mathematical methods to be used. It then needs to be analysed so that a plan can be made as to how to go about it. The plan will almost always involve the collection of information in some form. The information may already be available (e.g. online) or it may be necessary to carry out some form of experimental or investigational work to gather it. In some cases the plan will involve considering simple cases with a view to generalising from them. In others, physical experiments may be needed. In statistics, decisions need to be made at this early stage about what data will be relevant and how they will be collected. The analysis may involve considering whether there is an appropriate standard model to use (e.g. the Normal distribution or the particle model) or whether the problem is similar to one which has been solved before. At the completion of the problem solving cycle, there needs to be consideration of whether the original problem has been solved in a satisfactory way or whether it is necessary to repeat the problem solving cycle in order to gain a better solution. For example, the solution might not be accurate enough or only apply in some cases.
Information collection	This stage involves getting the necessary inputs for the mathematical processing that will take place at the next stage. This may involve deciding which are the important variables, finding key measurements or collecting data.
Processing and representation	This stage involves using suitable mathematical techniques, such as calculations, graphs or diagrams, in order to make sense of the information collected in the previous stage. This stage ends with a provisional solution to the problem.
Interpretation	This stage of the process involves reporting the solution to the problem in a way which relates to the original situation. Communication should be in clear plain English which can be understood by someone who has an interest in the original problem but is not an expert in mathematics. This should lead into reflection on the solution to consider whether it is satisfactory or further work is needed.

The Modelling Cycle

The examinations will assume that learners have used the full modelling cycle during the course.

Mathematics can be applied to a wide variety of **problems** arising from real situations but real life is complicated, and can be unpredictable, so some **assumptions** need to be made to simplify the situation and allow mathematics to be used. Once answers have been obtained, we need to **compare with experience** to make sure that the answers are useful. For example, the government might want to know the effects of different possible regulations on catching fish so that they can put the right rules in place to safeguard fish stocks for the future. To model the effects of catching fish, they might **assume** that the population growth rate depends on the number of fish of breeding age. They would **evaluate** these assumptions by checking whether they fit in with **past data** and then model the effects of removing fish from the population in accordance with the proposed rules. **New data** about fish populations after new rules are put in place would be **reviewed** to check whether the model is making reasonable predictions.



2

Learning Outcomes

Learning outcomes are designed to help users by clarifying the requirements, but the following points need to be noted:

- Content that is covered by a learning outcome with a reference code may be tested in an examination question without further guidance being given.
- Learning outcomes marked with an asterisk * are assumed knowledge and will not form the focus of any examination questions. These outcomes are included for clarity and completeness.
- Many examination questions will require learners to use two or more learning outcomes at the same time without further guidance being given. Learners are expected to be able to make links between different areas of mathematics.
- Learners are expected to be able to use their knowledge to reason mathematically and solve problems both within mathematics and in context. Content that is covered by any learning outcome may be required in problem solving, modelling and reasoning tasks even if that is not explicitly stated in the learning outcome.
- Learning outcomes have an implied prefix: 'A learner should …'
- Each reference code for a learning outcome is unique. For example, in the code MG1, M refers to Mechanics, G refers to 'centre of mass' (see below) and 1 means that it is the first such learning outcomes in the list.
- The letters used in assigning reference codes to learning outcomes are shown below.

a	algebra	A	Algorithms
b	bivariate data	B	
c	calculus	C	Curves, curve sketching
d	dynamics	D	Data presentation and interpretation
e	equations	E	Exponentials and logarithms
f	functions	F	Forces
g	geometry, graphs	G	Centre of mass
h	Hooke's law	H	Hypothesis testing
i	impulse and momentum	I	Inference
j	complex numbers	J	
k	kinematics	K	
l		L	Linear programming
m	matrices	M	
n	Newton's laws	N	Networks
o		O	
p	mathematical processes (modelling, proof, etc)	P	Polar coordinates
q	dimensions (quantities)	Q	Technology
r	rotation	R	Random variables
s	sequences and series	S	Sets and logic
t	trigonometry	T	Number theory
u	probability (uncertainty)	U	Errors (uncertainty)
v	vectors	V	
w	work, energy and power	W	
x	experimental design	X	
y	projectiles	Y	
z		Z	Simulation

Notes, notation and exclusions

The notes, notation and exclusions columns in the specification are intended to assist teachers and learners.

- The notes column provides examples and further detail for some learning outcomes . All exemplars contained in the specification are for illustration only and do not constitute an exhaustive list.
- The notation column shows the notation and terminology that learners are expected to know, understand and be able to use.
- The exclusions column lists content which will not be tested, for the avoidance of doubt when interpreting learning outcomes.

2c. Content of Core Pure (Y420) – mandatory paper

Description	In this mandatory paper some pure topics from A Level mathematics are studied in greater depth, while some new topics are introduced. Algebraic work with series is extended. The powerful technique of proof by induction is used in various contexts. Complex numbers are introduced and lead to solutions of problems in algebra, geometry and trigonometry. Matrices are used to solve systems of equations and to explore transformations. Vector methods are applied to problems involving lines and planes. Calculus techniques are extended, including the use of hyperbolic functions and polar coordinates, and culminate in the solution of differential equations.
Assumed knowledge	Learners are expected to know the content of A Level Mathematics. The unshaded sections, labelled (a), can be co-taught with AS Further Mathematics.
Assessment	One examination paper
Length of paper	2 hour 40 minutes
Number of marks	144
Sections	Section A will have between 30 and 40 marks and will comprise more straightforward questions. Section B will have between 104 and 114 marks and will comprise a mixture of more and less straightforward questions.
Percentage of qualification	This mandatory paper counts for 50% of the qualification OCR A Level Further Mathematics B (MEI) (H645).
Use of calculator and other technology	See Section 2b for details about the use of calculators.
Overarching themes	The Overarching Themes (see Section 2b) apply. Some questions may be set in a real world context and require some modelling.
Relationship with other papers	The unshaded sections, labelled (a), comprise the same content as the Core Pure paper (Y410) in the qualification OCR AS Further Mathematics B (MEI) (H635).
Other notes	The content is the same as the detailed content in the DfE document *Further mathematics: AS and A level content: April 2016*, with the addition of Pv7 and Pv8 Vector products. This allows the use of vector products in solving problems involving lines and planes.

Core Pure (Y420)
Contents

Proof (a)	Proof by induction is introduced for formulae for simple sequences, sums of series and powers of matrices.
Proof (b)	Proof by induction is used more generally, including to prove divisibility results
Complex Numbers (a)	Complex numbers and their basic arithmetic are introduced, including in modulus-argument form. They are used to solve polynomial equations with real coefficients and to define loci on the Argand diagram.
Complex Numbers (b)	De Moivre's theorem is used to develop trigonometrical relationships. Roots of complex numbers are used to help to solve geometrical problems using an Argand diagram.
Matrices and transformations (a)	Matrix arithmetic is introduced and applied to linear transformations in 2-D, and some in 3-D. Inverses of matrices (which may be found using a calculator in the 3×3 case) are used to solve matrix equations and related to inverse transformations.
Matrices and transformations (b)	The determinant and inverse of a 3×3 matrix are found without a calculator.
Vectors and 3-D space (a)	Scalar products are introduced, and used to form the equation of a plane. How planes intersect in 3-D space is considered, and matrices are used to find the point(s) of intersection.
Vectors and 3-D space (b)	Vector equations of lines are studied; methods for finding angles and distances between points, lines and planes are developed.
Algebra (a)	Relationships between roots of and coefficients of polynomials are explored.
Series (a)	Standard formulae and the method of differences are used to calculate the sum of given series.
Series (b)	Partial fractions are used to sum series. Maclaurin series are used to approximate functions.
Calculus (b)	Integration techniques are extended to include improper integrals, volumes of revolution, mean values of functions and partial fractions. Inverse trigonometric functions are defined and used for integration.
Polar Coordinates (b)	Curves defined in polar coordinates are explored, including finding the area enclosed by a curve.
Hyperbolic functions (b)	Hyperbolic functions and their inverses are used in integration.
Differential equations (b)	The work in A level Mathematics is extended to include the integrating factor method for first order differential equations. The general 2nd order linear differential equation is solved; applications include SHM, damped oscillations and paired linear 1st order differential equations.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: PROOF (a)					
Proof	*	Be able to prove mathematical results by deduction and exhaustion, and disprove false conjectures by counter-example.	Includes proofs of results used in this specification, where appropriate.		
Induction	Pp4	Be able to construct and present a proof using mathematical induction for given results for a formula for the *n*th term of a sequence, the sum of a series or the *n*th power of a matrix.	The result to be proved will be given. E.g. for the sequence given by $u_1 = 0$, $u_{n+1} = u_n + 2n$ prove that $u_n = n^2 - n$.	u_n, $\sum_{r=1}^{n} r^2$	
CORE PURE: PROOF (b)					
Proof	*	Be able to prove mathematical results by contradiction.			
Induction	Pp5	Be able to construct and present a proof using mathematical induction.	E.g. proofs of divisibility, proof of de Moivre's theorem. The result to be proved will always be given explicitly.		
CORE PURE: COMPLEX NUMBERS (a)					
Language of complex numbers	Pj1	Understand the language of complex numbers.	Real part, imaginary part, complex conjugate, modulus, argument, real axis, imaginary axis.	$z = x + y\mathrm{i}$ $z^* = x - y\mathrm{i}$ $\mathrm{Re}(z) = x$, $\mathrm{Im}(z) = y$	
Complex numbers and polynomial equations with real coefficients	j2	Be able to solve any quadratic equation with real coefficients.		$\mathrm{i}^2 = -1$	
	j3	Know that the complex roots of polynomial equations with real coefficients occur in conjugate pairs. Be able to solve cubic or quartic equations with real coefficients.	Use of the factor theorem once a real root has been determined. Sufficient information will be given to deduce at least one complex root or quadratic factor for quartics.		Equations with degree > 4.
Arithmetic of complex numbers	j4	Be able to add, subtract, multiply and divide complex numbers given in the form $x + y\mathrm{i}$, x and y real.	Division using complex conjugates.		
	j5	Understand that a complex number is zero if and only if both the real and imaginary parts are zero.			

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: COMPLEX NUMBERS (a)					
Modulus-argument form	j6	Be able to use radians in the context of complex numbers.	Use exact values of trigonometric functions for multiples of $\frac{\pi}{4}$ and $\frac{\pi}{6}$.		
	j7	Be able to represent a complex number in modulus-argument form. Be able to convert between the forms $z = x + yi$ and $z = r(\cos\theta + i\sin\theta)$ where r is the modulus and θ is the argument of the complex number.	$zz^* = \|z\|^2$	$\|z\|$ is the modulus of z. $\arg(z)$ for principal argument, where $-\pi < \arg(z) \le \pi$. Radian measure.	
	j8	Be able to multiply and divide complex numbers in modulus-argument form.	$z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$ $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$ The identities for $\sin(\theta \pm \phi)$ and $\cos(\theta \pm \phi)$ may be assumed in the derivation of these results.		
The Argand diagram	j9	Be able to represent and interpret complex numbers and their conjugates on an Argand diagram.			
	j10	Be able to represent the sum, difference, product and quotient of two complex numbers on an Argand diagram.			
	j11	Be able to represent and interpret sets of complex numbers as loci on an Argand diagram.	Circles of the form $\|z - a\| = r$. Half lines of the form $\arg(z - a) = \theta$. Lines of the form $\|z - a\| = \|z - b\|$. Regions defined by inequalities based on the above. E.g. $\|z - a\| > r$. Intersections and unions of these.	For regions defined by inequalities learners must state clearly which regions are included and whether the boundaries are included. No particular shading convention is expected.	$\|z - a\| = k\|z - b\|$ for $k \ne 1$.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: COMPLEX NUMBERS (b)					
De Moivre's theorem and simple applications	Pj12	Understand and use de Moivre's theorem.			
	j13	Be able to apply de Moivre's theorem to finding multiple angle formulae and to summing suitable series.	E.g. the expression of $\tan 4\theta$ as a rational function of $\tan\theta$. E.g. finding $\sum_{r=0}^{n} {}_nC_r \cos r\theta$.		
The form $z = re^{i\theta}$	j14	Understand the definition $e^{i\theta} = \cos\theta + i\sin\theta$ and hence the form $z = re^{i\theta}$.			
The *nth* roots of a complex number	j15	Know that every non-zero complex number has *n* distinct n^{th} roots, and that on an Argand diagram these are the vertices of a regular n-gon.			
	j16	Know that the distinct *n*th roots of $re^{i\theta}$ are: $r^{\frac{1}{n}}\left[\cos\left(\frac{\theta+2k\pi}{n}\right)+i\sin\left(\frac{\theta+2k\pi}{n}\right)\right]$ for $k = 0, 1 \ldots n-1$.			
	j17	Be able to explain why the sum of all the n^{th} roots is zero.			
Applications of complex numbers in geometry	j18	Understand the effect of multiplication by a complex number on an Argand diagram.	Multiplication by $re^{i\theta}$ corresponds to enlargement with scale factor *r* with rotation through θ about the origin. E.g. multiplication by i corresponds to a rotation of $\frac{\pi}{2}$ about the origin.		
	j19	Be able to represent complex roots of unity on an Argand diagram.	'Unity' means 1.		
	j20	Be able to apply complex numbers to geometrical problems.	E.g. relating to the geometry of regular polygons.		

<table>
<tr><th>Specification</th><th>Ref.</th><th>Learning outcomes</th><th>Notes</th><th>Notation</th><th>Exclusions</th></tr>
<tr><td colspan="6">CORE PURE: MATRICES AND TRANSFORMATIONS (a)</td></tr>
<tr><td rowspan="3">Matrix addition and multiplication</td><td>Pm1</td><td>Be able to add, subtract and multiply conformable matrices, and to multiply a matrix by a scalar.</td><td>With and without a calculator for matrices up to 3×3.</td><td>$\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.</td><td></td></tr>
<tr><td>m2</td><td>Understand and use the zero and identity matrices, understand what is meant by equal matrices.</td><td></td><td>$\mathbf{0}$ (zero)
$\mathbf{I}$ (identity).</td><td></td></tr>
<tr><td>m3</td><td>Know that matrix multiplication is associative but not commutative.</td><td></td><td></td><td></td></tr>
<tr><td rowspan="4">Linear transformations and their associated matrices</td><td>m4</td><td>Be able to find the matrix associated with a linear transformation and vice versa.</td><td>2-D transformations include the following.
• Reflection in the x and y axes and in $y = \pm x$.
• Rotation centre the origin through an angle θ (counter clockwise positive).
• Enlargement centre the origin.
• Stretch parallel to x or y axis.
• Shear x or y axis fixed, shear factor[1].
3-D transformations will be confined to reflection in one of $x = 0$, $y = 0$, $z = 0$ or rotation of multiples of 90° about x, y or z axis[2].
Learners should know that any linear transformation may be represented by a matrix.</td><td>Matrices will be shown in bold type, transformations in non-bold type.
The image of the column vector r under the transformation associated with matrix $\mathbf{M}$ is $\mathbf{Mr}$.</td><td></td></tr>
<tr><td colspan="5">[1]A shear may be defined by giving the fixed line and the image of a point. (The fixed line of a shear is a line of invariant points.) The shear factor is the distance moved by a point divided by its perpendicular distance from the fixed line. Learners should know this, but the shear factor should not be used to define a shear as there are different conventions about the sign of a shear factor.
[2]Positive angles counter clockwise when looking towards the origin from the positive side of the axis of rotation.</td></tr>
<tr><td>m5</td><td>Understand successive transformations in two dimensions and the connection with matrix multiplication.</td><td>Describe a transformation as a combination of two of those above.</td><td></td><td>More than 2 dimensions.</td></tr>
<tr><td>*</td><td>Understand the language of vectors in two dimensions and three dimensions.</td><td>Scalar, vector, modulus, magnitude, direction, position vector, unit vector, cartesian components, equal vectors, parallel vectors.</td><td>$\mathbf{i},\ \mathbf{j},\ \mathbf{k},\ \hat{\mathbf{r}},\ \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$</td><td></td></tr>
</table>

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: MATRICES AND TRANSFORMATIONS (a)					
Invariance	Pm6	Know the meaning of, and be able to find, invariant points and invariant lines for a linear transformation.			More than 2 dimensions.
Determinant of a matrix	m7	Be able to calculate the determinant of a 2×2 matrix and a 3×3 matrix. Know the meaning of the terms singular and non-singular as applied to matrices.	With a calculator for 3×3 matrices. A singular square matrix is non-invertible and therefore has determinant zero.	$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ or det $\mathbf{M}$ or $\lvert\mathbf{M}\rvert$.	
	m8	Know that the magnitude of the determinant of a 2×2 matrix gives the area scale factor of the associated transformation, and understand the significance of a zero determinant. Interpret the sign of a determinant in terms of orientation of the image.	E.g. Quadrilateral ABCD is labelled clockwise and transformed in 2-D; a negative determinant for the transformation matrix means that the labelling on the image $\mathrm{A'B'C'D'}$ is anticlockwise.		Proof.
	m9	Know that the magnitude of the determinant of a 3×3 matrix gives the volume scale factor of the associated transformation, and understand the significance of a zero determinant. Interpret the sign of a determinant in terms of orientation of the image.	The sign of the determinant determines whether the associated transformation preserves or reverses orientation ('handedness'). E.g. If a triangle ABC is labelled clockwise when seen from point S, then for a negative determinant, the triangle $\mathrm{A'B'C'}$ is labelled anti-clockwise when seen from $\mathrm{S'}$.		Proof.
	m10	Know that det($\mathbf{MN}$) $=$ det $\mathbf{M} \times$ det $\mathbf{N}$ and the corresponding result for scale factors of transformations.	Scale factors in 2-D only.		Algebraic proof.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
Inverses of square matrices	m11	Understand what is meant by an inverse matrix.	Square matrices of any order.	$\mathbf{M}^{-1}$	
	m12	Be able to calculate the inverse of a non-singular 2×2 matrix or 3×3 matrix.	With a calculator for 3×3 matrices. $\det(\mathbf{A}^{-1}) = \frac{1}{\det \mathbf{A}}$		
	m13	Be able to use the inverse of a non-singular 2×2 or 3×3 matrix. Relate the inverse matrix to the corresponding inverse transformation.	E.g. to solve a matrix equation and interpret in terms of transformations: find the pre-image of a transformation.		
	m14	Understand and use the product rule for inverse matrices.	$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$		
CORE PURE: MATRICES AND TRANSFORMATIONS (b)					
3×3 matrices	m15	Be able to find the determinant and inverse of a 3×3 matrix without a calculator.	May include algebraic terms.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: VECTORS AND 3-D SPACE (a)					
Scalar products and the equations of planes	Pv1	Know how to calculate the scalar product of two vectors, and be able to use the two forms of the scalar product to find the angle between two vectors.	Including test for perpendicular vectors.	$\mathbf{a}.\mathbf{b}$ $= a_1b_1 + a_2b_2 + a_3b_3$ $= \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$	Proof of equivalence of two forms in general case.
	v2	Be able to form and use the vector and cartesian equations of a plane. Convert between vector and cartesian forms for the equation of a plane.	Plane: $(\mathbf{r}-\mathbf{a}).\mathbf{n} = 0$ $n_1x + n_2y + n_3z + d = 0$ where $d = -\mathbf{a}.\mathbf{n}$.		The form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$.
	v3	Know that a vector which is perpendicular to a plane is perpendicular to any vector in the plane.	If a vector is perpendicular to two non-parallel vectors in a plane, it is perpendicular to the plane.		
Intersection of planes	v4	Know the different ways in which three distinct planes can be arranged in 3-D space.	If two planes are parallel the third can be parallel or cut the other two in parallel lines; if no pair is parallel the planes can intersect in a point, form a sheaf or form a prismatic intersection.	A sheaf is where three planes share a common line. A prismatic intersection is where each pair of planes meets in a line; the three lines are parallel.	
	v5	Be able to solve three linear simultaneous equations in three variables by use of the inverse of the corresponding matrix. Interpret the solution or failure of solution geometrically in terms of the arrangement of three planes. Be able to find the intersection of three planes when they meet in a point.	Inverse obtained using a calculator. If the corresponding matrix is singular, learners should know the possible arrangements of the planes; they will be given extra information or guidance if required to distinguish between these arrangements.		Finding equation of lines of intersection of two planes.
	v6	Know that the angle between two planes can be found by considering the angle between their normals.	The angle between two non-perpendicular planes is the acute angle between them.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: VECTORS AND 3-D SPACE (b)					
Vector product	Pv7	Be able to use the vector product in component form to give a vector perpendicular to two given vectors.	Vectors with numerical components only. When a vector perpendicular to two others is required learners should indicate that they are using the vector product but no further working need be shown. Formula will be given; a calculator may be used. $\mathbf{a}\times\mathbf{b}$ $\mathbf{a}\times\mathbf{b}=\begin{pmatrix} a_2b_3-a_3b_2 \\ a_3b_1-a_1b_3 \\ a_1b_2-a_2b_1 \end{pmatrix}=\begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix}$		
Vector products	v8	Be able to use the alternative form for the vector product. Know the significance of $\mathbf{a}\times\mathbf{b}=\mathbf{0}$.	$\mathbf{a}\times\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta\hat{\mathbf{n}}$ where $\mathbf{a}, \mathbf{b}, \hat{\mathbf{n}}$, in that order, form a right-handed triple. Formula will be given.	The vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$, in that order, form a right-handed triple.	
Lines	v9	Be able to form and use the equation of a line in 2-D and 3-D.	In vector and cartesian form. Direction vector.	Line: $\mathbf{r}=\mathbf{a}+t\mathbf{d}$ $\frac{x-a_1}{d_1}=\frac{y-a_2}{d_2}$ $=\frac{z-a_3}{d_3}(=t)$	
	v10	Be able to calculate the angle between two lines.	The angle between two non-perpendicular lines (which may be skew) is the acute angle between their direction vectors.		
	v11	Know the different ways in which two lines can intersect or not in 3-D space.	Two lines intersect at a point or are parallel or skew.		
	v12	Be able to determine whether two lines in three dimensions are parallel, skew or intersect, and to find the point of intersection if there is one.			
	v13	Be able to find the distance between two parallel lines and the shortest distance between two skew lines.	Formula for skew lines will be given, but questions may expect understanding of the underlying principles.		Proof of formula.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: VECTORS AND 3-D SPACE (b)					
Points, lines and planes	Pv14	Be able to find the intersection of a line and a plane.			
	v15	Be able to calculate the angle between a line and a plane.	If they are not perpendicular, the angle between a line and a plane is the acute angle between the line and its orthogonal projection onto the plane.		The language 'orthogonal projection' is not expected.
	v16	Be able to find the distance from a point to a line in 2 or 3 dimensions.	The distance between a point and a line means the shortest distance between them. Formula will be given in 2-D case, but questions may expect understanding of the underlying principles.		Proof of formula.
	v17	Be able to find the distance from a point to a plane.	The distance between a point and a plane means the shortest distance between them. Formula will be given, but questions may expect understanding of the underlying principles.		Proof of formula.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: ALGEBRA (a)					
Relations between the roots and coefficients of polynomial equations	Pa1	Understand and use the relationships between the roots and coefficients of quadratic, cubic and quartic equations.		Roots $\alpha, \beta, \gamma, \delta$.	Equations of degree ≥ 5.
	a2	Be able to form a new equation whose roots are related to the roots of a given equation by a linear transformation.	For a cubic or quartic equation.		Non-linear transformations of roots.
CORE PURE: SERIES (a)					
Summation of series	Ps1	Be able to use standard formulae for Σr , Σr^2 and Σr^3 and the method of differences to sum series.	Formulae for Σr^2 and Σr^3 will be given but proof could be required. E.g. by induction.	$\sum_{r=1}^{n} r^2$	
CORE PURE: SERIES (b)					
Sequences and series	*	Know the difference between a sequence and a series.			
	*	Know the meaning of the word *converge* when applied to either a sequence or a series.			
Summation of series	Ps2	Be able to sum a simple series using partial fractions.			
Maclaurin series. Approximate evaluation of a function	s3	Be able to find the Maclaurin series of a function, including the general term.	Use in evaluating approximate values of a function. Error in approximation = approx value – exact value.	Power series.	
	s4	Know that a Maclaurin series may converge only for a restricted set of values of x.			
	s5	Be able to recognise and use the Maclaurin series of standard functions: e^x, $\ln(1+x)$, $\sin x$, $\cos x$ and $(1+x)^n$.	Identify the set of values of x for which series are valid. Formulae will be given.		Proof of convergence. Complex x.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: CALCULUS (b)					
Improper integrals	Pc1	Evaluate improper integrals where either the integrand is undefined at a value in the interval of integration or the interval of integration extends to infinity.	E.g. $\int_{-1}^{1} x^{-\frac{2}{3}}\,dx = \int_{-1}^{0} x^{-\frac{2}{3}}\,dx + \int_{0}^{1} x^{-\frac{2}{3}}\,dx$. E.g. $\int_{1}^{\infty} e^{-x}\,dx$.		
Volumes of revolution	c2	Be able to derive formulae for and calculate the volumes of the solids generated by rotating a plane region about the x-axis or the y-axis.	$\pi\int y^2\,dx$, $\pi\int x^2\,dy$ and understanding these as the limit of a sum of cylinders.	Volume of revolution.	Axes of rotation other than the x- and y-axes.
Mean value	c3	Understand and evaluate the mean value of a function.	The mean value of $f(x)$ on the interval $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$.		
Partial fractions	c4	Be able to use the method of partial fractions in integration, including where the denominator has a quadratic factor of form $ax^2 + c$ and one linear term.			
The inverse functions of sine, cosine and tangent	*	Understand the definitions of inverse trigonometric functions.	arcsin: $-\frac{\pi}{2} \le \arcsin x \le \frac{\pi}{2}$. arccos: $0 \le \arccos x \le \pi$. arctan: $-\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$.	arcsin, $\sin^{-1}$; arccos, $\cos^{-1}$; arctan, $\tan^{-1}$.	
	c5	Be able to differentiate inverse trigonometric functions.			
Use of trigonometric substitutions in integration	c6	Recognise integrals of functions of the form $(a^2 - x^2)^{-\frac{1}{2}}$ and $(a^2 + x^2)^{-1}$ and be able to integrate related functions by using trigonometric substitutions.	Formulae will be given.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: POLAR COORDINATES (b)					
Polar coordinates in two dimensions	PP1	Understand and use polar coordinates (r, θ) and be able to convert from polar to cartesian coordinates and vice-versa.	θ in radians.	Pole, initial line.	
	P2	Be able to sketch curves with simple polar equations where r is given as a function of θ.	E.g. $r = a(1 + \cos\theta)$, $r = a\cos 2\theta$.	$r > 0$ continuous line. $r < 0$ broken line.	
	P3	Be able to find the area enclosed by a polar curve.	Using $\frac{1}{2}\int r^2 \, d\theta$.		
CORE PURE: HYPERBOLIC FUNCTIONS (b)					
Hyperbolic functions	Pa3	Understand the definitions of hyperbolic functions, know their domains and ranges and be able to sketch their graphs.	$\sinh x = \frac{1}{2}(e^x - e^{-x})$ $\cosh x = \frac{1}{2}(e^x + e^{-x})$ $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\sinh x}{\cosh x}$		
	a4	Understand and use the identity $\cosh^2 x - \sinh^2 x = 1$.			Knowledge of other identities.
	a5	Be able to differentiate and integrate hyperbolic functions.			

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: HYPERBOLIC FUNCTIONS (b)					
Inverse hyperbolic functions	Pa6	Understand and be able to use the definitions of the inverse hyperbolic functions and know their domains and ranges.	$\text{arsinh}\, x$ and $\text{artanh}\, x$ can take any values but $\text{arcosh}\, x \geq 0$.		
	a7	Be able to derive and use the logarithmic forms of the inverse hyperbolic functions.	$\text{arsinh}\, x = \ln[x + \sqrt{(x^2+1)}]$ $\text{arcosh}\, x = \ln[x + \sqrt{(x^2-1)}],\ x \geq 1$ $\text{artanh}\, x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right),\ -1 < x < 1$	$\text{arsinh}, \sinh^{-1}$; $\text{arcosh}, \cosh^{-1}$; $\text{artanh}, \tanh^{-1}$.	
	a8	Recognise integrals of functions of the form $(x^2+a^2)^{-\frac{1}{2}}$ and $(x^2-a^2)^{-\frac{1}{2}}$ and be able to integrate related functions by using substitutions.			
CORE PURE: DIFFERENTIAL EQUATIONS (b)					
Modelling with differential equations	Pp19	Understand how to introduce and define variables to describe a given situation in mathematical terms.			
	p20	Be able to relate 1st and 2nd order derivatives to verbal descriptions and so formulate differential equations.	The differential equations will not be restricted to those which candidates can solve analytically.		
	p21	Know the language of kinematics, and the relationships between the various variables.	Including acceleration $= v\frac{dv}{dx}$.	$v = \frac{dx}{dt} = \dot{x}$ $a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \ddot{x}$	
	*	Know Newton's 2nd law of motion.	In the form $F = ma$.		Variable mass.
	p22	Use differential equations in modelling in kinematics and in other contexts.	Sufficient information will be given about contexts which may be unfamiliar.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: DIFFERENTIAL EQUATIONS (b)					
Solutions of differential equations	Pc7	Know the difference between a general solution and a particular solution. Be able to find both general and particular solutions.			
Integrating factor method	c8	Recognise differential equations where the integrating factor method is appropriate.	Equations which can be rearranged into the form $\frac{dy}{dx} + P(x)y = Q(x)$.		
	c9	Be able to find an integrating factor and understand its significance in the solution of an equation.	Integrating factor, $I(x) = e^{\int P(x)dx}$.		
	c10	Be able to solve an equation using an integrating factor and find both general and particular solutions.	E.g. a particular solution through a given point.		
Second order differential equations	c11	Be able to solve differential equations of the form $y'' + ay' + by = 0$, using the auxiliary equation.	a and b are constants.	Homogeneous. Complementary function.	
	c12	Understand and use the relationship between different cases of the solution and the nature of the roots of the auxiliary equation.	Discriminant > 0. Discriminant = 0. Discriminant < 0.		
	c13	Be able to solve differential equations of the form $y'' + ay' + by = f(x)$, by solving the homogeneous case and adding a particular integral to the complimentary function.	a and b are constants.		
	c14	Be able to find particular integrals in simple cases. Understand the relationship between different cases of the solution and the nature of the roots of the auxiliary equation.	Cases where $f(x)$ is a polynomial, trigonometric or exponential function. Includes cases where the form of the complementary function affects the form required for the particular integral.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: DIFFERENTIAL EQUATIONS (b)					
Simple harmonic motion	Pc15	Be able to solve the equation for simple harmonic motion, $\ddot{x} = -\omega^2 x$, and be able to relate the solution to the motion.	Learners may state that they recognise the differential equation is that for SHM, and quote the solution in an appropriate form (e.g. $p\cos(\omega t) + q\sin(\omega t)$ or $A\cos(\omega t - \phi)$), unless specifically required to solve the equation. E.g. by using the techniques of Pc11.	$\lvert A\rvert = \sqrt{p^2 + q^2} =$ amplitude. $T =$ period $= \dfrac{2\pi}{\omega}$	
Damped oscillations	c16	Be able to model damped oscillations using 2nd order differential equations.			
	c17	Be able to interpret the solutions of equations modelling damped oscillations in words and graphically.	The damping will be described as 'over-', 'critical' or 'under-' according to whether the roots of the auxiliary equation are real distinct, equal or complex.	Where applicable, the amplitude refers to the local maximum distance from the equilibrium position. The amplitude decreases with time.	
Simultaneous differential equations	c18	Analyse and interpret model situations with one independent variable and two dependent variables which lead to coupled 1st order simultaneous linear differential equations and find the solution.	Applications include predator-prey models and other population models. E.g. solve by eliminating one variable to produce a single, 2nd order equation.		

2d. Content of Mechanics Major (Y421) – major option

Description	In this major option, basic principles of forces and their moments, work and energy, impulse and momentum and centres of mass are used to model various situations, including: rigid bodies in equilibrium; particles moving under gravity, on a surface, in a circle, attached to springs; bodies colliding with direct or oblique impact.
Assumed knowledge	Learners are expected to know the content of A Level Mathematics and the Core Pure mandatory paper (Y420). The unshaded sections, labelled (a), can be co-taught with AS Further Mathematics.
Assessment	One examination paper
Length of paper	2 hour 15 minutes
Number of marks	120
Sections	Section A will have between 25 and 35 marks and will comprise more straightforward questions. Section B will have between 85 and 95 marks and will comprise a mixture of more and less straightforward questions.
Percentage of qualification	This optional paper counts for 33⅓% of the qualification OCR A Level Further Mathematics B (MEI) (H645).
Use of calculator and other technology	See Section 2b for details about the use of calculators.
Overarching Themes	The Overarching Themes (see Section 2b) apply. Mechanics is about modelling the real world, so knowledge of the real world appropriate to a learner on this course will be assumed. Examination questions may include, for example, asking learners to suggest an explanation for a discrepancy between the results of a simple class experiment and the theoretical answer they have obtained; learners are expected to comment sensibly about the modelling assumptions in their answer.
Relationship with other papers	The unshaded sections of this content, labelled (a), comprise the same content as Mechanics Minor (Y431). Learners may **not** enter for Y421 and Y431 in the same examination series. The unshaded sections of this content, labelled (a), comprise the same content as Mechanics a (Y411) in the qualification OCR AS Further Mathematics B (MEI) (H635). The shaded sections of this content, labelled (b), comprise the same content as Mechanics b (Y415) in the qualification OCR AS Further Mathematics B (MEI) (H635). Learners may **not** enter for Mechanics Major (Y421) and Mechanics Minor (Y431).
Other notes	The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, learners should use $g = 9.8$. When making calculations within a model, learners are advised to use exact numbers (e.g. fractions or surds) where possible. When interpreting solutions in a practical context, rounding to an appropriate degree of accuracy is expected. † refers to content which overlaps or depends on non-AS content from A Level Mathematics.

Mechanics Major (Y421)
Contents

In this table, and throughout the specification for this optional paper, the unshaded sections, labelled (a), are content items which are in both Mechanics Minor and Mechanics Major; the shaded content, labelled (b), is in Mechanics Major only.	
Dimensional analysis (a)	The dimensions of quantities are analysed in terms of mass, length and time; this allows checking of results and prediction of suitable models in some cases.
Forces (a)	Work on the vector treatment of forces and friction is extended to consider the equilibrium of a particle and of a rigid body.
Work, energy and power (a)	Consideration of kinetic energy, gravitational potential energy and the work done by a force leading to situations which can be modelled using the work-energy principle or conservation of energy. Power is introduced.
Momentum and impulse (a)	Conservation of linear momentum and Newton's experimental law are used to model situations involving direct impact collisions. Mechanical energy lost in a collision is calculated.
Momentum and impulse (b)	The work on collisions is extended to oblique impact.
Circular motion (b)	Circular motion with uniform and non-uniform speed is modelled using Newton's laws in the radial and tangential directions and using energy principles.
Hooke's Law (b)	The motion of a particle attached to a spring or string is modelled using forces and energy conservation.
Centre of mass (a)	The centres of mass of systems of particles and some given shapes are used in situations involving equilibrium of a rigid body.
Centre of mass (b)	The work on centre of mass is extended, using calculus, to include more general laminas and bodies formed by rotating a region about an axis.
Vectors and variable forces (b)	Calculus is applied to situations in more than 1 dimension with constant and non-constant acceleration. Situations involving projectile motion are modelled, including motion up an inclined plane. Differential equation models are considered, including the special case of simple harmonic motion.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MAJOR: DIMENSIONAL ANALYSIS (a)					
Dimensional consistency	Mq1	Be able to find the dimensions of a quantity in terms of M, L, T.	Know the dimensions of angle and frequency. Work out without further guidance the dimensions of density (mass per unit volume), pressure (force per unit area) and other quantities in this specification. Other kinds of density will be referred to as e.g. mass per unit area. Deduce the dimensions of an unfamiliar quantity from a given relationship.	M, L, T, [ ]	
	q2	Understand that some quantities are dimensionless.			
	q3	Be able to determine the units of a quantity by reference to its dimensions.			
	q4	Be able to change the units in which a quantity is given.	E.g. density from kg m^{-3} to g cm^{-3}.		
	q5	Be able to use dimensional analysis to check the consistency of a relationship.			
Formulating and using models by means of dimensional arguments	q6	Use dimensional analysis to determine unknown indices in a proposed formula.	E.g. for the period of a pendulum.		
	q7	Use a model based on dimensional analysis.	E.g. to find the value of a dimensionless constant. E.g. to investigate the effect of a percentage change in some of the variables.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MAJOR: FORCES (a)					
The language of forces	*	Understand the language relating to forces. Understand that the value of the normal reaction depends on the other forces acting and why it cannot be negative.	Weight, tension, thrust (or compression), normal reaction (or normal contact force), frictional force, resistance. Driving force, braking force[1]. NB weight is not considered to be a resistive force.		
	[1]The driving force of a car, bicycle, train engine etc. is modelled as a single external force. Similarly for a braking force. These are actually frictional forces acting at the point(s) of contact with the road or track. The internal processes which cause these forces are not considered.				
Friction	Md1	† Understand that bodies in contact may be subject to a frictional force as well as a normal contact force (normal reaction), and be able to represent the situation in an appropriate force diagram.	Smooth is used to mean frictionless.		
	d2	† Understand that the total contact force between surfaces may be expressed in terms of a frictional force and a normal contact force (normal reaction).			
	d3	† Understand that the frictional force may be modelled by $F \leq \mu R$ and that friction acts in the direction to oppose sliding. Model friction using $F = \mu R$ when sliding occurs.	Limiting friction. The definition of μ as the ratio of the frictional force to the normal contact force.	Coefficient of friction is μ.	The term angle of friction.
	d4	Be able to derive and use the result that a body on a rough slope inclined at an angle α to the horizontal is on the point of slipping if $\mu = \tan\alpha$.			
	d5	† Be able to apply Newton's laws to situations involving friction.			
Vector treatment of forces	d6	† Be able to resolve a force into components and be able to select suitable directions for resolution.	E.g. horizontally and vertically, or parallel and perpendicular to an inclined plane.		
	d7	† Be able to find the resultant of several concurrent forces by vector addition.	Graphically or by adding components.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MAJOR: FORCES (a)					
Equilibrium of a particle	Md8	† Know that a particle is in equilibrium under a set of concurrent forces if and only if their resultant is zero.			
	d9	† Know that a closed figure may be drawn to represent the addition of the forces on an object in equilibrium.	E.g. a triangle of forces.		
	d10	† Be able to formulate and solve equations for equilibrium by resolving forces in suitable directions, or by drawing and using a polygon of forces.	Questions will not be set that require Lami's theorem but learners may quote and use it where appropriate.		
Equilibrium of a rigid body	d11	Be able to draw a force diagram for a rigid body.	In cases where the particle model is not appropriate.		
	d12	Understand that a system of forces can have a turning effect on a rigid body.	E.g. a lever.		
	d13	Know the meaning of the term couple.	A couple is not about a particular axis.		
	d14	Be able to calculate the moments about a fixed axis of forces acting on a body. Be able to calculate the moment of a couple.	Both as the product of force and perpendicular distance of the axis from the line of action of the force, and by first resolving the force into components. Take account of a given couple when taking moments.		Vector treatment.
	d15	Understand and be able to apply the conditions for equilibrium of a rigid body.	The resultant of all the applied forces is zero and the sum of their moments about any axis is zero. Three forces in equilibrium must be concurrent or parallel. Situations may involve uniform 3-D objects, such as a cuboid, whose centre of mass can be written down by considering symmetry. E.g. infer the existence of a couple by consideration of equilibrium and calculate its size.		
	d16	Be able to identify whether equilibrium will be broken by sliding or toppling.	E.g. a cuboid on an inclined plane.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MAJOR: WORK, ENERGY AND POWER (a)					
The language of work, energy and power	Mw1	Understand the language relating to work, energy and power.	Work, energy, mechanical energy, kinetic energy, potential energy, conservative force, dissipative force, driving force, resistive force. Power of a force, power developed by a vehicle[1].		
	[1]In an examination question 'the power developed by a car' (or a bicycle or train engine) means the useful, or available, power. It is the power of the driving force; it is not the power developed by the engine, some of which is lost in the system.				
Concepts of work and energy	w2	Be able to calculate the work done by a force which moves along its line of action.			The use of calculus for variable forces.
	w3	Be able to calculate the work done by a force which moves at an angle to its line of action.	Zero work is done by a force acting perpendicular to displacement.		Use of scalar product $\mathbf{F}.\mathbf{s}$.
	w4	Be able to calculate kinetic energy.		$\text{KE} = \frac{1}{2}mv^2$	
	w5	Be able to calculate gravitational potential energy.	Relative to a defined zero level.	$\text{GPE} = mgh$	
The work-energy principle	w6	Understand when the principle of conservation of energy may be applied and be able to use it appropriately.	E.g. the maximum height of a projectile, a particle sliding down a smooth curved surface, a child swinging on a rope.		
	w7	Understand and use the work-energy principle.	The total work done by all the external forces acting on a body is equal to the increase in the kinetic energy of the body. E.g. a particle sliding down a rough curved surface.		
Power	w8	Understand and use the concept of the power of a force as the rate at which it does work.	Power = (force) × (component of velocity in the direction of the force). The concept of average power as (work done) ÷ (elapsed time). E.g. finding the maximum speed of a vehicle.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MAJOR: MOMENTUM and IMPULSE (a)					
Momentum and impulse treated as vectors	Mi1	Be able to calculate the impulse of a force as a vector and in component form.	Impulse = force × time over which it acts.		The use of calculus for variable forces.
	i2	Understand and use the concept of linear momentum and appreciate that it is a vector quantity.			
	i3	Understand and use the impulse-momentum equation.	The total impulse of all the external forces acting on a body is equal to the change in momentum of the body. Use of relative velocity in one dimension is required.		
Conservation of linear momentum	i4	Understand and use the principle that a system subject to no external force has constant total linear momentum and that this result may be applied in any direction.	The impulse of a finite external force (e.g. friction) acting over a very short period of time (e.g. in a collision) may be regarded as negligible. Application to collisions, coalescence and a body dividing into one or more parts.		
Direct impact	i5	Understand the term direct impact and the assumptions made when modelling direct impact collisions[1].	E.g. a collision between an ice hockey puck and a straight rink barrier: puck moving perpendicular to barrier. E.g. a collision between two spheres moving along their line of centres. E.g. a collision between two railway trucks on a straight track.		Any situation with rotating objects.

[1]Assumptions when modelling direct impact collisions

This note explains the implicit assumptions made in examination questions when modelling direct impact collisions. Learners may be asked about these assumptions.
An *object* means a real-world object. It may be modelled as a *particle* or a *body*.

- If the non-fixed objects involved in collisions may be modelled as particles, then all the motion and any impulses due to the collisions act in the same straight line.
- If the non-fixed objects involved in collisions may be modelled as bodies, then these bodies will be uniform bodies with spherical or circular symmetry.
- The impulse of any collision between such bodies acts on the line joining their centres, and the motion takes place along this line.
  These assumptions ensure that the collision happens at a point and that no angular momentum is created, hence none of the objects starts to rotate.
- The impulse of any collision between such a body, or a particle, and a plane (e.g. a wall or floor) acts in a direction perpendicular to the plane.
  For a direct impact the motion of the object is also in the direction perpendicular to the plane.
- Objects do not rotate before or after the collision. Rotating objects are beyond this specification.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MAJOR: MOMENTUM and IMPULSE (a)					
Direct impact (cont)	Mi6	Be able to apply the principle of conservation of linear momentum to direct impacts within a system of bodies.			
	i7	Know the meanings of Newton's Experimental Law and of coefficient of restitution when applied to a direct impact.	Newton's Experimental Law is: the speed of separation is $e \times$ the speed of approach where e is known as the coefficient of restitution.	Coefficient of restitution is e.	
	i8	Understand the significance of $e = 0$.	The bodies coalesce. The collision is inelastic.		
	i9	Be able to apply Newton's Experimental Law in modelling direct impacts.	E.g. between a particle and a wall. E.g. between two discs.		
	i10	Be able to model situations involving direct impact using both conservation of linear momentum and Newton's Experimental Law.			
	i11	Understand the significance of $e = 1$.	The collision is perfectly elastic. Kinetic energy is conserved.		
	i12	Understand that when $e < 1$ kinetic energy is not conserved during impacts and be able to find the loss of kinetic energy.			

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MAJOR: MOMENTUM AND IMPULSE (b)					
Oblique impact	Mi13	Understand the term oblique impact and the assumptions made when modelling oblique impact collisions[1].	E.g. a collision between a sphere and a surface when the sphere is moving in a direction which is not perpendicular to the surface. E.g. a collision between two discs not moving along their lines of centres.		Any situation with rotating objects.
	i14	Know the meanings of Newton's Experimental Law and of coefficient of restitution when applied to an oblique impact.	The coefficient of restitution is the ratio of the components of the velocities of separation and approach, in the direction of the line of impulse.		
	i15	Be able to model situations involving oblique impact between an object and a smooth plane by considering the components of its motion parallel and perpendicular to the line of the impulse.			
	i16	Be able to model situations involving oblique impact between two bodies by considering the components of their motion in directions parallel and perpendicular to the line of the impulse.			
	i17	Be able to calculate the loss of kinetic energy in an oblique impact.			

[1]Assumptions when modelling oblique impact collisions

This note explains the implicit assumptions made in examination questions when modelling oblique impact collisions. Learners may be asked about these assumptions. When two objects collide obliquely they cannot be modelled as particles; with two particles there is no preferred direction to act as the line of impulse. If an object collides with a plane (e.g. a wall or a floor) then it may be modelled as a particle or as a body, as appropriate.

- If the non-fixed objects involved in collisions may be modelled as bodies, then these bodies will be uniform bodies with spherical or circular symmetry.
- The impulse of any collision between such bodies acts on the line joining their centres.
  These assumptions ensure that the collision happens at a point and that no angular momentum is created, hence none of the objects starts to rotate. An oblique impact collision occurs when the line of relative motion of the bodies is not the same as the line joining their centres at the point of collision.
- The impulse of any collision between such a body, or a particle, and a plane (e.g. a wall or floor) acts in a direction perpendicular to the plane.
  An oblique impact collision in this situation means that the motion of the object is not in the direction perpendicular to the plane.
- The contact between the surfaces in any collision is smooth.
  This is an extra assumption for oblique collisions. It ensures that the linear momentum of each object is conserved in the direction perpendicular to the line of impulse.
- Objects do not rotate before or after the collision. Rotating objects are beyond this specification.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MAJOR: CIRCULAR MOTION (b)					
The language of circular motion	Mr1	Understand the language associated with circular motion.	The terms: tangential, radial and angular velocity; radial component of acceleration.	$\dot{\theta},\ \omega$ for angular velocity. $v = r\dot{\theta}$ or $r\omega$.	Angular velocity as a vector.
Modelling circular motion	r2	Identify the force(s) acting on a body in circular motion.	Learners will be expected to set up equations of motion.		
	r3	Be able to calculate acceleration towards the centre of circular motion.	Using the expressions $\frac{v^2}{r}$ and $r\dot{\theta}^2$.		
Circular motion with uniform speed	r4	Be able to model situations involving circular motion with uniform speed in a horizontal plane.	E.g. a conical pendulum, a car travelling horizontally on a cambered circular track.		
Circular motion with non-uniform speed	r5	Be able to model situations involving circular motion with non-uniform speed.	E.g. rotation in a horizontal circle with non-uniform angular velocity.		
	r6	Be able to calculate tangential acceleration.	Tangential component of acceleration $= r\ddot{\theta}$. Use of Newton's 2nd law, $F = ma$, in the tangential direction.		
	r7	Be able to model situations involving motion in a vertical circle.	The use of conservation of energy, and of $F = ma$ in the radial and tangential directions. E.g. sliding on the interior or exterior surface of a sphere.		
	r8	Identify the conditions under which a particle departs from circular motion.	E.g. when a string becomes slack, when a particle leaves a surface. Questions may ask about the subsequent motion.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MAJOR: HOOKE'S LAW (b)					
The language of elasticity	Mh1	Understand the language associated with elasticity.	Modulus of elasticity, stiffness, natural length, string, spring, equilibrium position.		
	h2	Understand that Hooke's law models the extension/compression of a material as a linear function of tension/thrust.		$T = kx$ where k is the stiffness.	
Extension of an elastic string and extension or compression of a spring.	h3	Be able to calculate the stiffness or modulus of elasticity in a given situation.			
	h4	Be able to calculate the tension in an elastic string or spring.		$T = \frac{\lambda x}{l_0}$ where λ is the modulus of elasticity and l_0 the natural length.	
	h5	Be able to calculate the equilibrium position of a system involving elastic strings or springs.	E.g. a heavy object suspended by a spring.		
	h6	Be able to calculate energy stored in a string or spring.	The proof of this result may include the use of calculus. (This is an exception to the exclusion in Mw2.)	$\frac{1}{2}\frac{\lambda x^2}{l_0}$ or $\frac{1}{2}kx^2$.	
	h7	Be able to use energy principles to model a system involving elastic strings or springs including determining extreme positions.	Applications to maximum extension for given starting conditions in a system, whether horizontal or vertical.		
	h8	Understand when Hooke's law is not applicable.	Hooke's law does not apply when the relationship between extension/compression and tension/thrust for a material is not linear. Many materials obey Hooke's law for a limited range of tensions/thrusts but extend/compress in a non-linear way for high values of tension/thrust.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MAJOR: CENTRE OF MASS (a)					
Locating a centre of mass	MG1	Be able to find the centre of mass of a system of particles of given position and mass.	In 1, 2 and 3 dimensions.	$(\bar{x}\bar{y}, \bar{z})$ $\left(\sum_i m_i\right)\bar{x} = \sum_i m_i x_i$	Non-uniform bodies.
	G2	Know how to locate centre of mass by appeal to symmetry.	E.g. uniform circular lamina, sphere, cuboid.		
	G3	Know the positions of the centres of mass of a uniform rod, a rectangular lamina and a triangular lamina.			
	G4	Be able to find the centre of mass of a composite body by considering each constituent part as a particle at its centre of mass.	Composite bodies may be formed by the addition or subtraction of parts. Where a composite body includes parts whose centre of mass the learner is not expected to know, or be able to find, the centre of mass will be given.		
Applications of the centre of mass	G5	Be able to use the position of the centre of mass in situations involving the equilibrium of a rigid body.	For the purpose of calculating its moment, the weight of a body can be taken as acting through its centre of mass. E.g. a suspended object. E.g. does an object standing on an inclined plane slide or topple?		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MAJOR: CENTRE OF MASS (b)					
Use of calculus to find centre of mass	MG6	† Be able to calculate the volume generated by rotating a plane region about an axis.	Rotation about the x- and y-axes only.		Non-cartesian coordinates.
	G7	Be able to use calculus methods to calculate the centre of mass of solid bodies formed by rotating a plane region about an axis.	E.g. hemisphere, cone.		Variable density. Pappus' theorem.
	G8	Be able to find the centre of mass of a compound body, parts of which are solids of revolution.	By treatment as equivalent to a finite system of particles.		
	G9	Be able to use calculus methods to calculate the centre of mass of a plane lamina.			Pappus' theorem.
	G10	Be able to use the position of the centre of mass in situations involving the equilibrium of a rigid body.	For the purpose of calculating its moment, the weight of a body can be taken as acting through its centre of mass. E.g. a suspended object. E.g. does an object standing on an inclined plane slide or topple?		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MAJOR: VECTORS AND VARIABLE FORCES (b)					
The language of kinematics	Mk1	† Understand the language of kinematics appropriate to motion in 2 and 3 dimensions. Know the distinction between displacement and distance, between velocity and speed, and between acceleration and magnitude of acceleration. Know the distinction between distance from and distance travelled.	Position vector, relative position.	$\mathbf{a} = \dot{\mathbf{v}} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}, \mathbf{v} = \dot{\mathbf{r}} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$ $\mathbf{r} = \int \mathbf{v}\mathrm{d}t, \mathbf{v} = \int \mathbf{a}\mathrm{d}t$ $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ $\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$	Vector form of $v^2 - u^2 = 2as$.
Velocity and position vector	k2	† Be able to extend the scope of techniques from motion in 1 dimension to that in 2 and 3 dimensions by using vectors.	Using calculus and constant acceleration formulae.		
	Mv1	Be able to find the acceleration, velocity and position vector of a particle subject to a constant or variable force in 1, 2 and 3 dimensions.	In terms of time or other parameters of a situation.		
	v2	Be able to use the acceleration, velocity and position vector of a particle to model situations in 1, 2 and 3 dimensions.	Including inferring the force acting.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MAJOR: VECTORS AND VARIABLE FORCES (b)					
The equation of the path of a particle in 2D	Mv3	Be able to eliminate a parameter from the expressions for the position vector of a particle, thereby forming a single equation.	E.g. elimination of time. Questions will be restricted to cases where the elimination is straightforward.		
	v4	Be able to interpret the equation resulting from the elimination of a parameter from the terms of a position vector.	E.g. a bounding parabola. E.g. solving for x or solving for $\tan\alpha$ and interpreting.		
	v5	Derive the cartesian equation of the path of a particle in 2 dimensions from an expression for its position vector.	E.g. the trajectory of a projectile.		
	v6	Be able to find the range of a projectile up or down a uniform slope.	Only cases where the projectile's initial position is on the slope. Appropriate use of coordinates parallel and perpendicular to the slope, or horizontal and vertical. Standard modelling assumptions for projectile motion are as follows. • No air resistance. • The projectile is a particle. • Horizontal distance travelled is small enough to assume that gravity is always in the same direction. • Vertical distance travelled is small enough to assume that gravity is constant.		
	v7	Be able to find the maximum range of a projectile up or down a uniform slope, and the associated angle of projection.	Only cases where the projectile's initial position is on the slope.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MAJOR: VECTORS AND VARIABLE FORCES (b)					
Differential equations	Mv8	Be able to formulate differential equations for motion under variable acceleration in 1 and 2 dimensions.	E.g. use Hooke's law for a particle on a spring or a particle attached to two springs to establish simple harmonic motion (SHM). E.g. establish that SHM applies to part of the motion of a particle. E.g. establish approximate SHM for a simple pendulum. E.g. when resistance is a given function of velocity. Including use of $a = v\frac{\mathrm{d}v}{\mathrm{d}s}$.		
	v9	Be able to verify a general or particular solution of a differential equation for motion under variable acceleration.	In the particular case this requires both showing that the solution is compatible with the equation and also that it conforms to the boundary or initial conditions for the situation when they are known.		Solving a differential equation, other than writing down a solution to SHM as in v12.
	v10	Be able to use the boundary or initial conditions to produce a particular solution from a general solution.			
	v11	Be able to recognise and formulate the simple harmonic motion equation expressed in non-standard forms and to transform it into the standard form by means of substitution.	E.g. $\ddot{x} + cx = 0$, $\ddot{x} = -\omega^2(x+k)$ where x can represent a variable such as an angle.		
	v12	Be able to solve the equation for simple harmonic motion, $\ddot{x} = -\omega^2 x$, and be able to relate the solution to the context.	Learners may state that they recognise the differential equation is that for SHM, and quote the solution in an appropriate form (e.g. $p\cos(\omega t) + q\sin(\omega t)$, $A\sin(\omega t - \varphi)$ or $A\cos(\omega t - \varepsilon)$)	$\|A\| = \sqrt{p^2+q^2} =$ amplitude. $T =$ period $= \frac{2\pi}{\omega}$ $v^2 = \omega^2(A^2 - x^2)$	

2e. Content of Statistics Major (Y422) – major option

Description	In this major option situations are modelled by discrete and continuous random variables; this allows inference about a population in the form of hypothesis testing and point and interval estimates of population parameters. The suitability of models is tested and bivariate data are investigated. Simulation of random variables is introduced, a powerful way of tackling challenging problems.
Assumed knowledge	Learners are expected to know the content of A Level Mathematics and the Core Pure manadatory paper (Y420). The unshaded sections, labelled (a), can be co-taught with AS Further Mathematics.
Assessment	One examination paper
Length of paper	2 hour 15 minutes
Number of marks	120
Sections	Section A will have between 25 and 35 marks and will comprise more straightforward questions. Section B will have between 85 and 95 marks and will comprise a mixture of more and less straightforward questions.
Percentage of qualification	This optional paper counts for 33⅓% of the qualification OCR A Level Further Mathematics B (MEI) (H645).
Use of calculator and other technology	See Section 2b for details about the use of calculators. Calculators used in the examination should be able to calculate means, standard deviations, correlation coefficients, equations of regression lines and probabilities, including cumulative probabilities, from the binomial, Poisson and Normal distributions. It is expected that learners will gain experience of using a spreadsheet or other software for exploring data sets; this should include using software to conduct hypothesis tests and construct confidence intervals. In the examination learners will be assessed on the interpretation of output from such software. It is expected that learners will gain experience of using a spreadsheet for simulating a random variable; in the examination they will be assessed on the interpretation of output from such software.
Overarching Themes	The Overarching Themes (see Section 2b) apply. Statistics is about answering real world problems using data, so knowledge of the real world appropriate to a learner on this course will be assumed. Examination questions may include, for example, asking learners to comment sensibly about the modelling assumptions in their answer.
Relationship with other papers	The unshaded sections of this content, labelled (a), comprise the same content as Statistics Minor (Y432). Learners may not enter for Y422 and Y432 in the same examination series. The unshaded sections of this content, labelled (a), comprise the same content as Statistics a (Y412) in the qualification OCR AS Further Mathematics B (MEI) (H635). The shaded sections of this content, labelled (b), comprise the same content as Statistics b (Y416) in the qualification OCR AS Further Mathematics B (MEI) (H635). Learners may not enter for Statistics Major Y422 and Statistics Minor Y432.
Other notes	When making calculations within a probability model, learners are advised to use exact numbers (e.g. fractions) where possible or decimal numbers to 4 dp. When interpreting solutions in a practical context, rounding to an appropriate (usually lesser) degree of accuracy is expected. Learners are expected to have explored different data sets, using appropriate technology, during the course. No particular data set is expected to be studied, and there will not be any pre-release data.

Statistics major (Y422)

Contents

Sampling (a)	A short section about the importance of sampling methods.
Discrete random variables (a)	The binomial distribution is introduced for modelling discrete univariate data in AS Mathematics. This optional paper extends the range of models available to include the (discrete) uniform, geometric and Poisson distributions. The link between the binomial and Poisson distributions is explored, though the use of the Poisson as an approximation to the binomial distribution for calculation purposes is not included; technology renders it largely obsolete. Some theoretical work on discrete probability distributions, including mean and variance and some of their properties, is introduced and applied to these models.
Bivariate data (a)	Different types of bivariate data are considered. Where appropriate, Pearson's product moment correlation coefficient and Spearman's rank correlation coefficient are used to test for correlation and association, respectively, for bivariate numerical data. The different underlying assumptions are explored. Linear regression as a model for bivariate numerical data is introduced; residuals provide an informal way of looking at the appropriateness of the model.
Chi-squared tests (a)	The hypothesis testing work in AS Level Mathematics - based on the binomial distribution and, informally, on correlation coefficients - is extended to include χ^2 tests and a more formal approach to tests based on correlation coefficients. This gives learners an understanding of a range of tests, including the concept of degrees of freedom, which should allow them to pick up quickly any hypothesis tests they encounter in other subjects. The product moment correlation coefficient is also considered, informally, as an effect size; this serves as an example of a widely-used approach which is complementary to hypothesis testing. The χ^2 test for goodness of fit is used to test whether a particular distribution is appropriate to model a given data set. For bivariate categorical data, the χ^2 test for association, using data given in a contingency table, is introduced.
Continuous random variables (b)	The general work on discrete random variables is now extended to continuous random variables. The Normal distribution work in A level Mathematics is taken further; the Normal probability plot is used to check whether a Normal model may be appropriate; a χ^2 test could be used for this but other tests specific to the Normal distribution are more widely used and are available on a good spreadsheet or other statistical software. The Central Limit Theorem provides another reason for the importance of the Normal distribution. Other continuous models are looked at in less detail.
Inference (b)	The focus here is on constructing and interpreting confidence intervals, rather than on hypothesis testing, which has been addressed in earlier work. The Normal and the *t* distributions are used for single sample, and paired sample confidence intervals. Three hypothesis tests for the average of a population are introduced, a Normal test, a *t* test and a Wilcoxon single sample test; these illustrate when non-parametric methods may be useful.
Simulation (b)	Many of the ideas can be explored using spreadsheet simulation of random variables and results can be obtained for which the theory is technically difficult. Examination questions will include interpreting spreadsheet output from simulations.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MAJOR: SAMPLING (a)					
Sampling	Sx1	Be able to explain the importance of sample size in experimental design.	E.g. an informal explanation of how the size of a sample affects the interpretation of an effect size.		
	x2	Be able to explain why sampling may be necessary in order to obtain information about a population, and give desirable features of a sample.	Population too large or it is too expensive to take a census. Sampling process may be destructive. Sample should be unbiased, representative of the population; data should be relevant, not changed by the act of sampling.	A sample may also be considered as n observations from a random variable.	
	x3	Be able to explain the advantage of using a random sample when inferring properties of a population.	A random sample enables proper inference to be undertaken because the probability basis on which the sample has been selected is known.		
STATISTICS MAJOR: DISCRETE RANDOM VARIABLES (a)					
Probability distributions	SR1	Be able to use probability functions, given algebraically or in tables. Be able to calculate the numerical probabilities for a distribution. Be able to draw and interpret graphs representing probability distributions.	Other than the Poisson and geometric distributions, the underlying random variable will only take a finite number of values. An understanding that probabilities are non-negative and sum to 1 is expected.	$\mathrm{P}(X = x)$	
Expectation and variance	R2	Be able to calculate the expectation (mean), $\mathrm{E}(X)$, and understand its meaning.		$\mathrm{E}(X) = \mu$	
	R3	Be able to calculate the variance, $\mathrm{Var}(X)$, and understand its meaning.	Knowledge of $\mathrm{Var}(X) = \mathrm{E}(X^2) - \mu^2$. Standard deviation $= \sqrt{\mathrm{Var}(X)}$.	$\mathrm{Var}(X) = \mathrm{E}[(X-\mu)^2]$	
	R4	Be able to use the result $\mathrm{E}(a + bX) = a + b\mathrm{E}(X)$ and understand its meaning.			
	R5	Be able to use the result $\mathrm{Var}(a + bX) = b^2\mathrm{Var}(X)$ and understand its meaning.			

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Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MAJOR: DISCRETE RANDOM VARIABLES (a)					
Expectation and variance (cont)	SR6	Be able to find the expectation of any linear combination of independent random variables and the variance of any linear combination of independent random variables.	$\mathrm{E}(X \pm Y) = \mathrm{E}(X) \pm \mathrm{E}(Y)$ $\mathrm{Var}(X \pm Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$ $\mathrm{E}(aX \pm bY) = a\mathrm{E}(X) \pm b\mathrm{E}(Y)$ $\mathrm{Var}(aX \pm bY) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y)$		Proofs.
The discrete uniform distribution	R7	Recognise situations under which the discrete uniform distribution is likely to be an appropriate model.	E.g. X has a uniform distribution over the values $\{4, 5, 6, 7, 8, 9\}$. E.g. a fair spinner with six equally-sized sections, labelled 4, 5, 6, 7, 8, 9.		
	R8	Be able to calculate probabilities using a discrete uniform distribution.			
	R9	Be able to calculate the mean and variance of any given discrete uniform distribution.	If X has a uniform distribution over the values $\{1, 2, \dots n\}$ then $\mathrm{E}(X) = \frac{n+1}{2}$ and $\mathrm{Var}(X) = \frac{1}{12}(n^2 - 1)$. The formulae for this particular uniform distribution will be given but their derivations may be asked for.		
The binomial distribution	R10	Recognise situations under which the binomial distribution is likely to be an appropriate model, and be able to calculate probabilities to use the model. Know and be able to use the mean and variance of a binomial distribution, $\mu = np$ and $\sigma^2 = np(1-p)$. Prove these results in particular cases.	E.g. prove results by considering a binomial random variable as the sum of n independent Bernoulli random variables: $X = X_1 + X_2 + \dots + X_n$ where each X_i takes the value 1 with probability p and 0 with probability $1 - p$. This proof assumes the relationship about variance in SR6.	$X \sim \mathrm{B}(n, p)$	

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MAJOR: DISCRETE RANDOM VARIABLES (a)					
The Poisson distribution	SR11	Recognise situations under which the Poisson distribution is likely to be an appropriate model.	Modelling the number of events occurring in a fixed interval (of time or space) when the events occur randomly at a constant average rate, and independently of each other. It is expected that these conditions can be applied to the particular context. If the mean and variance of the data do not have a similar value then the Poisson model is unlikely to be suitable.	$X \sim \text{Po}(\lambda)$ $X \sim \text{Poisson}(\lambda)$	
	R12	Recognise situations in which both the Poisson distribution and the binomial distribution might be appropriate models.	In a situation where the binomial model is appropriate, if n is large and p is small, then the conditions for a Poisson distribution to be appropriate are approximately satisfied. In the absence of guidance either model can be used.		Formal criteria. Using the Poisson distribution as a numerical approximation for calculating binomial probabilities.
	R13	Be able to calculate probabilities using a Poisson distribution.	Including use of a calculator to access Poisson probabilities and cumulative Poisson probabilities.		
	R14	Know and be able to use the mean and variance of a Poisson distribution.	$\text{E}(X) = \lambda, \ \text{Var}(X) = \lambda$		Proof.
	R15	Know that the sum of two or more independent Poisson distributions is also a Poisson distribution.	$X \sim \text{Po}(\lambda)$ and $Y \sim \text{Po}(\mu)$ $\Rightarrow X + Y \sim \text{Po}(\lambda + \mu)$ when X and Y are independent.		Proof.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MAJOR: DISCRETE RANDOM VARIABLES (a)					
The geometric distribution	SR16	Recognise situations under which the geometric distribution is likely to be an appropriate model.	Link with corresponding binomial distribution.	$X \sim \text{Geo}(p)$, where $X =$ number of Bernoulli trials up to and including the first success.	The alternative definition which counts the number of failures.
	R17	Be able to calculate the probabilities within a geometric distribution, including cumulative probabilities.	$\text{P}(X = r) = (1-p)^{r-1}p$ where $p =$ probability of success and $r \in \{1, 2, \ldots\}$. $\text{P}(X > r) = (1-p)^r$. An understanding of the calculation is expected.		
	R18	Be able to use the mean and variance of a geometric distribution.	$\text{E}(X) = \frac{1}{p}$, $\text{Var}(X) = \frac{1-p}{p^2}$.		Proof.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions

STATISTICS MAJOR: BIVARIATE DATA (a)

There are two kinds of bivariate data considered in A Level Mathematics and Further Mathematics and it is important to distinguish between them when considering correlation and regression. This note explains the reason for the distinction; learners will only be assessed on what appears under a specification reference below.

Case A: Only **one of the variables** may be considered as **a random variable**. Often this occurs when one of the variables, the independent variable, is controlled by an experimenter and the other, the dependent variable, is measured. An example of this would be (weight, extension) in an investigation of Hooke's law for a spring. In this case certain fixed weights are used; this variable is *not* a random variable, any errors in measuring the weights are negligible. The extension *is* a random variable. There will be deviations from the 'true' value that a perfect experimenter would observe from a perfect spring as well as errors in the measurement. This case is referred to as **'random on non-random'**. The points on the scatter diagram are restricted to lie on certain vertical lines corresponding to the values of the controlled variable.

Case B: The **two variables may both** be considered as **random variables**. An example of this would be (height, weight) for a sample from a population of individuals. For any given value of height there is a distribution of weights; for any given value of weight there is a distribution of heights. That is, there is no 'true' weight for a given height or 'true' height for a given weight. This case is referred to as **'random on random'**. The scatter diagram appears as a 'data cloud'.

If a linear relationship between the variables is to be investigated and modelled using correlation and regression techniques then the two cases must be treated differently.

If it is desired to test the significance of Pearson's product moment correlation coefficient then, as with all parametric hypothesis tests, probability calculations have to be performed to calculate the *p*-value or the critical region. These calculations rely on certain assumptions about the underlying distribution – **these assumptions can never be met in the 'random on non-random case'** – because one of the variables does not have a probability distribution – so **such a test is never valid in this case**. In fact the pmcc is not used in this case. In the **'random on random'** case the distributional assumptions **may** be met – see the specification below for details.

If it is desired to calculate the equation of a line of best fit then the least-squares method is often used in both cases. However its interpretation is different in the two cases. In the example of the **'random on non-random'** case, (weight, extension), the line of regression is modelling the 'true' value of the extension for a given weight – the value that a perfect experimenter would observe from a perfect spring. In the example of the **'random on random' case**, (height, weight), the two **lines of regression are modelling the mean value of the distribution of weights for a given height and the mean value of the distribution of heights for a given weight**.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MAJOR: BIVARIATE DATA (a)					
Scatter diagrams	Sb1	Understand what bivariate data are and know the conventions for choice of axis for variables in a scatter diagram.	In the random on non-random case the independent variable is often one which the experimenter controls; the dependent variable is the one which is measured. The independent variable is usually plotted on the horizontal axis. In the random on random case (where both variables are measured), it may be that one is more naturally seen as a function of the other; this determines which variable is plotted on which axis.		
	b2	Be able to use and interpret a scatter diagram.	To look for outliers (by eye). To gain insight into the situation, for example to decide whether a test for correlation or association might be appropriate. Learners may be asked to add to a given scatter diagram in order to interpret a new situation.		
	b3	Interpret a scatter diagram produced by software.	Including where the software draws a trendline and gives a value for pmcc or $(\text{pmcc})^2$.		
Pearson's product moment correlation coefficient (pmcc)	b4	Be able to calculate the pmcc from raw data or summary statistics.	The use of a calculator is expected for calculation from raw data. Summary statistics formulae will be given.	Sample value r.	
	b5	Know when it is appropriate to carry out a hypothesis test using Pearson's product moment correlation coefficient.	The data must be random on random i.e. both variables must be random. There must be a modelling assumption that the data are drawn from a bivariate Normal distribution. This may be recognised on a scatter diagram by an approximately elliptical distribution of points. Learners will not be required to know the formal meaning of bivariate Normality but will be expected to know that where one or both of the distributions is skewed, bimodal, etc., the procedure is likely to be inappropriate. The test is for correlation, a linear relationship, so a scatter diagram is helpful to check that the data cloud does not indicate a non-linear relationship.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MAJOR: BIVARIATE DATA (a)					
Pearson's product moment correlation coefficient (pmcc) (cont)	Sb6	Be able to carry out hypothesis tests using the pmcc and tables of critical values or the *p*-value from software.	Only 'H_0: No correlation in the population' will be tested. Both one-sided and two-sided alternative hypotheses will be tested. Learners should state whether there is sufficient evidence or not to reject H_0 and then give a non-assertive conclusion in context. E.g. 'There is sufficient evidence to suggest that there is positive correlation between … and …'	Null hypothesis, alternative hypothesis H_0, H_1.	
	b7	Use the pmcc as an effect size[1].	Sensible informal comments about effect size are expected, either alongside or instead of a hypothesis test.		Any formal rules for judging effect size will be given.

[1]**Note on effect size for correlation**

For a large set of random on random bivariate data a small non-zero value of the pmcc is likely to lead to a rejection of the null hypothesis of no correlation in the population; the test is uninformative. In some contexts it is more important to consider the size of the correlation rather than test whether the population correlation is non-zero. The phrase 'effect size' is sometimes used in this context for the value of the pmcc. In social sciences, Cohen's guideline is often used: small effect size 0.1, medium effect size 0.3, large effect size 0.5. Learners are not expected to know this rule; this or any other formal rule will be given if necessary.

Effect sizes for other situations, e.g. for the difference of two means, are beyond the scope of this specification.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MAJOR: BIVARIATE DATA (a)					
Spearman's rank correlation coefficient	Sb8	Be able to calculate Spearman's rank correlation coefficient from raw data or summary statistics.	Use of a calculator on the ranked data is expected.	Sample value r_s.	Tied ranks.
	b9	Be able to carry out hypothesis tests using Spearman's rank correlation coefficient and tables of critical values or the output from software.	Hypothesis tests using Spearman's rank correlation coefficient require no modelling assumptions about the underlying distribution. Only 'H_0: No association in the population' will be tested. Both one-sided and two-sided alternative hypotheses will be tested. Learners should state whether there is sufficient evidence or not to reject H_0 and then give a non-assertive conclusion in context. E.g. 'There is insufficient evidence to suggest that there is an association between … and …'.		
Comparison of tests	b10	Decide whether a test based on r or r_s may be more appropriate, or whether neither is appropriate.	Considerations include the appearance of the scatter diagram, the likely validity of underlying assumptions, whether association or correlation is to be tested for. Spearman's test is not appropriate if the scatter diagram shows no evidence of a monotonic relationship i.e. one variable tends to increase (or decrease) as the other increases. Understanding that ranking data loses information, which may affect the outcome of a test.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MAJOR: BIVARIATE DATA (a)					
Regression line for a random variable on a non-random variable	Sb11	Be able to calculate the equation of the least squares regression line using raw data or summary statistics.	The goodness of fit of a regression line may be judged by looking at the scatter diagram. In this case examination questions will be confined to cases in which a random variable, Y, and a non-random variable, X, are modelled by a relationship in which the 'true' value of Y is a linear function of X. The use of a calculator is only expected for calculation from raw data. Summary statistics formulae will be given.		Derivation of the least squares regression line.
	b12	Be able to use the regression line as a model to estimate values and know when it is appropriate to do so. Know the meaning of the term residual and be able to calculate and interpret residuals.	residual $=$ observed value – value from regression line. Informal checking of a model by looking at residuals.	Interpolation. Extrapolation.	
Regression lines for a random variable on a random variable	b13	Be able to calculate the equation of the two least squares regression lines, y on x and x on y, using raw data or summary statistics. Be able to use either regression line to estimate the expected value of one variable for a given value of the other and know when it is appropriate to do so.	In the y on x case, the least squares regression line estimates $\mathrm{E}(Y \mid X = x)$, that is the expected value of Y for a given value of X. Conversely for the x on y case. The use of a calculator is only expected for calculation from raw data.		Derivation of the least squares regression lines.
	b14	Check how well the model fits the data.	Informal checking only of a model by visual inspection of a scatter diagram or consideration of $(\text{pmcc})^2$.		Residuals in this case.
	b15	Know the relationship between the two regression lines and when to use one rather than the other. Be able to use the correct regression line to estimate the expected value of one variable for a given value of the other and know when it is appropriate to do so.	Both lines pass through $(\bar{x}, \bar{y})$. Choice of line to use depends on which variable is to be estimated.	Interpolation. Extrapolation.	

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
		STATISTICS MAJOR: CHI-SQUARED TESTS (a)			
Contingency tables	Sb16	Be able to interpret bivariate categorical data in a contingency table.	Numerical data can be put into categories, but this loses information.		
χ^2 test for a contingency table	SH1	Be able to apply the χ^2 test (chi-squared) to a contingency table.	Only ‘H_0: No association between the factors’ or ‘H_0: variables are independent’ will be tested. Calculating degrees of freedom is expected. Knowing how to calculate observed values and contributions to the test statistic are expected, but repetitive calculations will not be required. Learners should state whether there is sufficient evidence or not to reject H_0 and then give a non-assertive conclusion in context. E.g. ‘There is not sufficient evidence to believe that there is association between … and …’.		Yates’ continuity correction is not expected, though its appropriate use will not be penalised.
	H2	Be able to interpret the results of a χ^2 test using tables of critical values or the output from software.	Output from software may be given as a *p*-value. Interpretation may involve considering the individual cells in the table of contributions to the test statistic.		
χ^2 test for goodness of fit	H3	Be able to carry out a χ^2 test for goodness of fit of a uniform, binomial or Poisson model.	Only ‘H_0: the given model fits the data’ or ‘H_0; the given model is suitable’ will be tested. Calculating degrees of freedom is expected. Knowing how to calculate observed values and contributions to the test statistic is expected, but repetitive calculations will not be required. Learners should be aware that cells are often combined when there are small expected frequencies, but will not have to make such decisions in examination questions. Learners should state whether there is sufficient evidence or not to reject H_0 and then give a non-assertive conclusion in context. E.g.’ It is reasonable to believe that the … model is suitable’.		
	H4	Be able to interpret the results of a χ^2 test using tables of critical values or the output from software.	Output from software may be given as a p-value.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MAJOR: CONTINUOUS RANDOM VARIABLES (b)					
The probability density function (pdf) of a continuous random variable	SR19	Be able to use a simple continuous random variable as a model.	Learners are expected to be familiar with the use of the (continuous) uniform and Normal distributions as models. They should be aware that other distributions underpin some work e.g. t_n, χ_n^2 and that other distributions, such as the exponential distribution, are useful models; knowledge of these is not expected and any necessary details will be provided in the examination.	Continuous uniform distribution also known as rectangular distribution.	Mixed discrete and continuous random variables.
	R20	Understand the meaning of a pdf and be able to use one to find probabilities.	Simple unfamiliar pdf's, including piecewise pdf's, may be given in examination questions. In numerical cases learners are expected to write down the relevant definite integral, and may then use a calculator to evaluate it.	$\mathrm{f}(x)$ or other lower case letter for the function.	Using formula for pdf of Normal distribution.
	R21	Know and use the properties of a pdf. Be able to sketch the graph of a pdf.	$\mathrm{f}(x) \geq 0$ and $\int_{-\infty}^{\infty} \mathrm{f}(x)\,\mathrm{d}x = 1$.		Evaluation of improper integrals.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MAJOR: CONTINUOUS RANDOM VARIABLES (b)					
The probability density function (pdf) of a continuous random variable (cont)	SR22	Be able to find the mean and variance from a given pdf.	Learners are expected to write down the relevant definite integral, and may then use a calculator to evaluate it. In examination questions, any integrations to be performed will be over a finite domain. Standard deviation $= \sqrt{\mathrm{Var}(X)}$. For a continuous uniform distribution over $[a, b]$: $\mathrm{E}(X) = \frac{a+b}{2}$, $\mathrm{Var}(X) = \frac{1}{12}(b-a)^2$. Formulae will be given but derivations may be required.		Deriving mean and variance of the Normal distribution from the pdf. Evaluation of improper integrals.
	R23	Be able to find the mode and median from a given pdf.	Mode only where it exists.		Mode for bimodal distributions.
The cumulative distribution function (cdf)	R24	Understand the meaning of a cdf and be able to obtain one from a given pdf. Be able to sketch a cdf.	$\mathrm{F}(x) = \int_{-\infty}^{x} \mathrm{f}(t)\,\mathrm{d}t$	$\mathrm{F}(x)$ or other upper case letter for the function.	Normal distribution. Evaluation of improper integrals.
	R25	Be able to obtain a pdf from a given cdf.	$\mathrm{f}(x) = \mathrm{F}'(x)$		
	R26	Use a cdf to calculate the median and other percentiles.			
Expectation algebra	R27	Be able to find the mean of any linear combination of random variables and the variance of any linear combination of independent random variables.	$\mathrm{E}(X \pm Y) = \mathrm{E}(X) \pm \mathrm{E}(Y)$ $\mathrm{Var}(X \pm Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$ $\mathrm{E}(aX \pm bY) = a\mathrm{E}(X) \pm b\mathrm{E}(Y)$ $\mathrm{Var}(aX \pm bY) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y)$		Proofs.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MAJOR: CONTINUOUS RANDOM VARIABLES (b)					
The Normal distribution	SR28	†Be able to use the Normal distribution as a model, and to calculate and use probabilities from a Normal distribution.	Calculations of probabilities are to be done using statistical functions on a calculator. Relate calculations of probabilities to the graph of the Normal distribution.		
	R29	Be able to use linear combinations of independent Normal random variables in solving problems.	Use the fact that if $X \sim \text{N}(\mu_X, \sigma_X^2)$ and $Y \sim \text{N}(\mu_Y, \sigma_Y^2)$, with X and Y independent, then $aX + bY \sim \text{N}(a\mu_X + b\mu_Y,\ a^2\sigma_X^2 + b^2\sigma_Y^2)$ Extend to more than two random variables.		Proof.
	R30	Know that the Normal distribution is useful as a model in its own right, and as an approximating distribution in the context of the Central Limit Theorem (CLT).	Includes recognising when the Normal distribution is not appropriate. Details of the CLT are in SI1 to SI6 below.		
	R31	Interpret a Normal probability plot to decide whether a Normal model might be appropriate[1]. Interpret software output, including *p*-value, from the Kolmogorov-Smirnov test, to decide whether a Normal model might be appropriate.	Learners should know that tests other than the χ^2 test of goodness of fit are often applied to the Normal distribution. The null hypothesis for the given test is 'H_0: the Normal distribution fits the data'.		χ^2 test for goodness of fit of Normal distribution. Calculations for Kolmogorov-Smirnov test.
	R32	Be able to use the Normal distribution, when appropriate, in the construction of confidence intervals.	See SI7 to SI14 below for details.		

[1] There are different conventions for how Normal probability plots are drawn, and different features about the underlying distribution, for example skewness, can be inferred from the plot. Learners are only expected to know that the closer the points are to a straight line, the more likely it is that a Normal distribution fits the data; this is to be judged by eye. They are not expected to be able to draw Normal probability plots, nor do any calculations.

In the examination the sample data will be shown on one axis and the other will show expected Normal values.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MAJOR: INFERENCE (b)					
Distribution of sample mean and the Central Limit Theorem (CLT)	SI1	Be able to estimate population mean from sample data.	This is a point estimate; see confidence intervals below for interval estimates.	$\hat{\mu} = \bar{x}$	Proof.
	I2	Be able to estimate population variance using the sample variance.	Sample variance given by $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$.		Proof.
	I3	Understand that the sample mean is a random variable with a probability distribution.	If n independent observations $X_1, X_2 \ldots X_n$ are taken from a distribution with mean μ and variance σ^2 then $\bar{X} = \frac{X_1 + X_2 + \ldots + X_n}{n}$ is a random variable with a probability distribution. Using the results of SR6 and SR27 the mean of this distribution is μ and its variance is $\frac{\sigma^2}{n}$.	Sampling distribution of the mean.	
	I4	Be able to calculate and interpret the standard error of the mean.	The 'standard error of the mean' is the standard deviation of the sampling distribution of the mean. It is equal to $\frac{\sigma}{\sqrt{n}}$. If σ is not known then it may be estimated from a particular sample as $\frac{s}{\sqrt{n}}$; this estimate is also sometimes referred to as the 'standard error of the mean'.		
	I5	Know that if the underlying distribution is Normal then the sample mean is Normally distributed.	Using the results of SR29.		
	I6	Understand how and when the Central Limit Theorem may be applied to the distribution of sample means. Use this result in probability calculations, using a continuity correction where appropriate. Be able to apply the CLT to the sum of n identically distributed independent random variables.	If X has a mean and finite variance, whatever its underlying distribution, the distribution of the sample mean $\bar{X}$ may be approximated by a Normal distribution if n is sufficiently large; $n > 30$ is often used as a rule of thumb for 'sufficiently large'.		Formal statement and derivation of the CLT. Distributions for which the CLT does not apply.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MAJOR: INFERENCE (b)					
Confidence intervals using the Normal and *t* distributions	SI7	Know the meaning of the term confidence interval for a parameter and associated language.	A *confidence interval* is an interval estimate for a population parameter, based on a sample. If the confidence interval is constructed a large number of times, based on independent samples, then the *confidence level* is the long run proportion of confidence intervals which contain the true value of the parameter.		Asymmetric confidence intervals. One-sided confidence intervals.
	I8	Understand the factors which affect the width of a confidence interval.	Sample size, confidence level, population variability.		
	I9	Be able to construct and interpret a confidence interval for a single population mean using the Normal or *t* distributions and know when it is appropriate to do so.	Use the Normal distribution when the sample size is large, using s^2 as an estimate for σ^2 if necessary. For a small sample from an underlying Normal distribution: • if the population variance is known use the Normal distribution; • if the population variance is unknown use the *t* distribution, with s^2 as an estimate for σ^2 with $n-1$ degrees of freedom.		
	I10	Know when samples from two populations should be considered as paired.			
	I11	Be able to construct and interpret a confidence interval for the difference in mean of two paired populations using a paired sample and a Normal or *t* distribution; know when it is appropriate to do so.	Treat the differences as a single distribution and construct a confidence interval for the mean of the differences using the same procedure as for a single mean. It is rarely the case that the population variance for the differences is known; this possibility will not be examined.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MAJOR: INFERENCE (b)					
Confidence intervals using the Normal and *t* distributions (cont)	SI12	Interpret confidence intervals given by software.	May include confidence intervals from distributions with which learners are not familiar; necessary details will be given.		
	I13	Use a confidence interval for a population parameter to make a decision about a hypothesised value of that parameter.	By checking whether the confidence interval contains the hypothesised value.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MAJOR: INFERENCE (b)					
Hypothesis testing for an average using Wilcoxon, Normal or *t* tests	SH5	Be able to carry out a hypothesis test for a single population median using the Wilcoxon signed rank test and know when it is appropriate to do so.	Learners are expected to know that this is an example of a non-parametric (or distribution-free) hypothesis test, and when such tests may be useful. Underlying distribution needs to be symmetrical. H_0: population median is given value. Both one-sided and two-sided alternative hypotheses will be tested. Learners should state whether there is sufficient evidence or not to reject H_0 and then give a non-assertive conclusion in context. E.g. 'There is not sufficient evidence to believe that the median … has changed'.	W_+ the sum of the ranks for positive differences. W_- the sum of the ranks for negative differences.	
	H6	Be able to carry out a hypothesis test for a single population mean using the Normal or *t* distributions and know when it is appropriate to do so.	Use the Normal distribution when the sample size is large, using s^2 as an estimate for σ^2 if necessary. For a small sample from an underlying Normal distribution: • if the population variance is known use the Normal distribution; • if the population variance is unknown use the *t* distribution, with s^2 as an estimate for σ^2 with $n-1$ degrees of freedom. H_0: population mean is given value. Both one-sided and two-sided alternative hypotheses will be tested. Learners should state whether there is sufficient evidence or not to reject H_0 and then give a non-assertive conclusion in context. E.g. 'There is sufficient evidence to believe that the mean… is not …'.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MAJOR: SIMULATION (b)					
Simulation of random variables	SZ1	Know that spreadsheets can be used to simulate probability distributions, and be able to do so for discrete and continuous uniform distributions and Normal distributions.	Learners are expected to appreciate the variation which repeated sampling produces.		
	Z2	Know that simulations can be used to approximate probability distributions and to estimate probabilities, including in situations where the theory may be technically difficult. Be able to interpret output from spreadsheets investigating such situations.	E.g. if X and Y are independent random numbers from $[0, 1]$, estimate $\mathrm{P}(2X - Y > 0)$. E.g. investigate the CLT for the sample mean from a continuous uniform distribution for various values of the sample size, n . E.g. investigate whether the result about linear combinations of independent Normal random variables holds in different cases. E.g. estimate the probability that 10 dice give a total score of greater than 50. E.g. I commute to and from work on trains which run every 15 minutes. If I arrive at the station at a random time between trains, what is the probability that I have to wait for more than 20 minutes in total on any one day? Learners will be expected to interpret output from spreadsheets to investigate such scenarios.		

2f. Content of Mechanics Minor (Y431) – minor option

Description	In this minor option, basic principles of forces and their moments, work and energy, impulse and momentum and centres of mass are used to model various situations, including: rigid bodies in equilibrium; particles moving under gravity or on a surface; bodies colliding with direct impact.
Assumed knowledge	Learners are expected to know the content of A Level Mathematics and the Core Pure mandatory paper (Y420). The content can mostly be co-taught with AS Further Mathematics, though some concepts overlap with A level mathematics content not in AS mathematics.
Assessment	One examination paper
Length of paper	1 hour 15 minutes
Number of marks	60
Sections	The examination paper will not have sections.
Percentage of qualification	This optional paper counts for 16⅔% of the qualification OCR A Level Further Mathematics B (MEI) (H645).
Use of calculator and other technology	See Section 2b for details about the use of calculators.
Overarching Themes	The Overarching Themes (see Section 2b) apply. Mechanics is about modelling the real world, so knowledge of the real world appropriate to a learner on this course will be assumed. Examination questions may include, for example, asking learners to suggest an explanation for a discrepancy between the results of a class experiment and the theoretical answer they have obtained; learners are expected to comment sensibly about the modelling assumptions in their answer.
Relationship with other papers	This is the same content as the unshaded sections, labelled (a), of Mechanics Major (Y421). Learners may not enter for Y431 and Y421 in the same examination series. This is the same content as Mechanics a (Y411) in the qualification OCR AS Further Mathematics B (MEI) (H635). Learners may **not** enter for Mechanics Major (Y421) and Mechanics Minor (Y431).
Other notes	The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, learners should use $g = 9.8$. When making calculations within a model, learners are advised to use exact numbers (e.g. fractions or surds) where possible. When interpreting solutions in a practical context, rounding to an appropriate degree of accuracy is expected. † refers to content which overlaps or depends on non-AS content from A Level Mathematics.

Mechanics minor (Y431)
Contents

Dimensional analysis (a)	The dimensions of quantities are analysed in terms of mass, length and time; this allows checking of results and prediction of suitable models in some cases.
Forces (a)	Work on the vector treatment of forces and friction is extended to consider the equilibrium of a particle and of a rigid body.
Work, energy and power (a)	Consideration of kinetic energy, gravitational potential energy and the work done by a force leading to situations which can be modelled using the work-energy principle or conservation of energy. Power is introduced.
Momentum and impulse (a)	Conservation of linear momentum and Newton's experimental law are used to model situations involving direct impact collisions. Mechanical energy lost in a collision is calculated.
Centre of mass (a)	The centres of mass of systems of particles and some given shapes are used in situations involving equilibrium of a rigid body.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MINOR: DIMENSIONAL ANALYSIS (a)					
Dimensional consistency	Mq1	Be able to find the dimensions of a quantity in terms of M, L, T.	Know the dimensions of angle and frequency. Work out without further guidance the dimensions of density (mass per unit volume), pressure (force per unit area) and other quantities in this specification. Other kinds of density will be referred to as e.g. mass per unit area. Deduce the dimensions of an unfamiliar quantity from a given relationship.	M, L, T, [ ]	
	q2	Understand that some quantities are dimensionless.			
	q3	Be able to determine the units of a quantity by reference to its dimensions.	And vice-versa.		
	q4	Be able to change the units in which a quantity is given.	E.g. density from kg m^{-3} to g cm^{-3}.		
	q5	Be able to use dimensional analysis to check the consistency of a relationship.			
Formulating and using models by means of dimensional arguments	q6	Use dimensional analysis to determine unknown indices in a proposed formula.	E.g. for the period of a pendulum.		
	q7	Use a model based on dimensional analysis.	E.g. to find the value of a dimensionless constant. E.g. to investigate the effect of a percentage change in some of the variables.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MINOR: FORCES (a)					
The language of forces	*	Understand the language relating to forces. Understand that the value of the normal reaction depends on the other forces acting and why it cannot be negative.	Weight, tension, thrust (or compression), normal reaction (or normal contact force), frictional force, resistance. Driving force, braking force[1]. NB weight is not considered to be a resistive force.		
		[1]The driving force of a car, bicycle, train engine etc is modelled as a single external force. Similarly for a braking force. These are actually frictional forces acting at the point(s) of contact with the road or track. The internal processes which cause these forces are not considered.			
Friction	Md1	† Understand that bodies in contact may be subject to a frictional force as well as a normal contact force (normal reaction), and be able to represent the situation in an appropriate force diagram.	Smooth is used to mean frictionless.		
	d2	† Understand that the total contact force between surfaces may be expressed in terms of a frictional force and a normal contact force (normal reaction).			
	d3	† Understand that the frictional force may be modelled by $F \leq \mu R$ and that friction acts in the direction to oppose sliding. Model friction using $F = \mu R$ when sliding occurs.	Limiting friction. The definition of μ as the ratio of the frictional force to the normal contact force.	Coefficient of friction is μ.	The term angle of friction.
	d4	Be able to derive and use the result that a body on a rough slope inclined at an angle α to the horizontal is on the point of slipping if $\mu = \tan\alpha$.			
	d5	† Be able to apply Newton's laws to situations involving friction.			
Vector treatment of forces	d6	† Be able to resolve a force into components and be able to select suitable directions for resolution.	E.g. horizontally and vertically, or parallel and perpendicular to an inclined plane.		
	d7	† Be able to find the resultant of several concurrent forces by vector addition.	Graphically or by adding components.		

Specification	Ref.	Learning Outcomes	Notes	Notation	Exclusions
		MECHANICS MINOR: FORCES (a)			
Equilibrium of a particle	Md8	† Know that a particle is in equilibrium under a set of concurrent forces if and only if their resultant is zero.			
	d9	† Know that a closed figure may be drawn to represent the addition of the forces on an object in equilibrium.	E.g. a triangle of forces.		
	d10	† Be able to formulate and solve equations for equilibrium by resolving forces in suitable directions, or by drawing and using a polygon of forces.	Questions will not be set that require Lami's theorem but learners may quote and use it where appropriate.		
Equilibrium of a rigid body	d11	Be able to draw a force diagram for a rigid body.	In cases where the particle model is not appropriate.		
	d12	Understand that a system of forces can have a turning effect on a rigid body.	E.g. a lever.		
	d13	Know the meaning of the term couple.	A couple is not about a particular axis.		
	d14	Be able to calculate the moments about a fixed axis of forces acting on a body. Be able to calculate the moment of a couple.	Both as the product of force and perpendicular distance of the axis from the line of action of the force, and by first resolving the force into components. Take account of a given couple when taking moments.		Vector treatment.
	d15	Understand and be able to apply the conditions for equilibrium of a rigid body.	The resultant of all the applied forces is zero and the sum of their moments about any axis is zero. Three forces in equilibrium must be concurrent or parallel. Situations may involve simple uniform 3-D objects, such as a cuboid, whose centre of mass can be written down by considering symmetry. E.g. infer the existence of a couple by consideration of equilibrium and calculate its size.		
	d16	Be able to identify whether equilibrium will be broken by sliding or toppling.	E.g. a cuboid on an inclined plane.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MINOR: WORK, ENERGY AND POWER (a)					
The language of work, energy and power	Mw1	Understand the language relating to work, energy and power.	Work, energy, mechanical energy, kinetic energy, potential energy, conservative force, dissipative force, driving force, resistive force. Power of a force, power developed by a vehicle[1].		
	[1]In an examination question 'the power developed by a car' (or a bicycle or train engine) means the useful, or available, power. It is the power of the driving force; it is not the power developed by the engine, some of which is lost in the system.				
Concepts of work and energy	w2	Be able to calculate the work done by a force which moves along its line of action.			The use of calculus for variable forces.
	w3	Be able to calculate the work done by a force which moves at an angle to its line of action.	Zero work is done by a force acting perpendicular to displacement.		Use of scalar product $\mathbf{F.s}$.
	w4	Be able to calculate kinetic energy.		$\mathrm{KE} = \frac{1}{2}mv^2$	
	w5	Be able to calculate gravitational potential energy.	Relative to a defined zero level.	$\mathrm{GPE} = mgh$	
The work-energy principle	w6	Understand when the principle of conservation of energy may be applied and be able to use it appropriately.	E.g. the maximum height of a projectile, a particle sliding down a smooth curved surface, a child swinging on a rope.		
	w7	Understand and use the work-energy principle.	The total work done by all the external forces acting on a body is equal to the increase in the kinetic energy of the body. E.g. a particle sliding down a rough curved surface.		
Power	w8	Understand and use the concept of the power of a force as the rate at which it does work.	Power = (force) × (component of velocity in the direction of the force). The concept of average power as (work done) ÷ (elapsed time). E.g. finding the maximum speed of a vehicle.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MINOR: MOMENTUM and IMPULSE (a)					
Momentum and impulse treated as vectors	Mi1	Be able to calculate the impulse of a force as a vector and in component form.	Impulse = force × time over which it acts.		The use of calculus for variable forces.
	i2	Understand and use the concept of linear momentum and appreciate that it is a vector quantity.			
	i3	Understand and use the impulse-momentum equation.	The total impulse of all the external forces acting on a body is equal to the change in momentum of the body. Use of relative velocity in one dimension is required.		
Conservation of linear momentum	i4	Understand and use the principle that a system subject to no external force has constant total linear momentum and that this result may be applied in any direction.	The impulse of a finite external force (e.g. friction) acting over a very short period of time (e.g. in a collision) may be regarded as negligible. Application to collisions, coalescence and a body dividing into one or more parts.		
Direct impact	i5	Understand the term direct impact and the assumptions made when modelling direct impact collisions[1].	E.g. a collision between an ice hockey puck and a straight rink barrier: puck moving perpendicular to barrier. E.g. a collision between two spheres moving along their line of centres. E.g. a collision between two railway trucks on a straight track.		Any situation with rotating objects.

[1]Assumptions when modelling direct impact collisions

This note explains the implicit assumptions made in examination questions when modelling direct impact collisions. Learners may be asked about these assumptions.
An *object* means a real-world object. It may be modelled as a *particle* or a *body*.

- If the non-fixed objects involved in collisions may be modelled as particles, then all the motion and any impulses due to the collisions act in the same straight line.
- If the non-fixed objects involved in collisions may be modelled as bodies, then these bodies will be uniform bodies with spherical or circular symmetry.
- The impulse of any collision between such bodies acts on the line joining their centres, and the motion takes place along this line.
  These assumptions ensure that the collision happens at a point and that no angular momentum is created, hence none of the objects starts to rotate.
- The impulse of any collision between such a body, or a particle, and a plane (e.g. a wall or floor) acts in a direction perpendicular to the plane.
  For a direct impact the motion of the object is also in the direction perpendicular to the plane.
- Objects do not rotate before or after the collision. Rotating objects are beyond this specification.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MINOR: MOMENTUM and IMPULSE (a)					
Direct impact (cont)	Mi6	Be able to apply the principle of conservation of linear momentum to direct impacts within a system of bodies.			
	i7	Know the meanings of Newton's Experimental Law and of coefficient of restitution when applied to a direct impact.	Newton's Experimental Law is: the speed of separation is $e \times$ the speed of approach where e is known as the coefficient of restitution.	Coefficient of restitution is e.	
	i8	Understand the significance of $e = 0$.	The bodies coalesce. The collision is inelastic.		
	i9	Be able to apply Newton's Experimental Law in modelling direct impacts.	E.g. between a particle and a wall. E.g. between two discs.		
	i10	Be able to model situations involving direct impact using both conservation of linear momentum and Newton's Experimental Law.			
	i11	Understand the significance of $e = 1$.	The collision is perfectly elastic. Kinetic energy is conserved.		
	i12	Understand that when $e < 1$ kinetic energy is not conserved during impacts and be able to find the loss of kinetic energy.			

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MINOR: CENTRE OF MASS (a)					
Locating a centre of mass	MG1	Be able to find the centre of mass of a system of particles of given position and mass.	In 1, 2 and 3 dimensions.	$(\bar{x}, \bar{y}, \bar{z})$ $\left(\sum_i m_i\right)\bar{x} = \sum_i m_i x_i$	Non-uniform bodies.
	G2	Know how to locate centre of mass by appeal to symmetry.	E.g. uniform circular lamina, sphere, cuboid		
	G3	Know the positions of the centres of mass of a uniform rod, a rectangular lamina and a triangular lamina.			
	G4	Be able to find the centre of mass of a composite body by considering each constituent part as a particle at its centre of mass.	Composite bodies may be formed by the addition or subtraction of parts. Where a composite body includes parts whose centre of mass the learner is not expected to know, or be able to find, the centre of mass will be given.		
Applications of the centre of mass	G5	Be able to use the position of the centre of mass in situations involving the equilibrium of a rigid body.	For the purpose of calculating its moment, the weight of a body can be taken as acting through its centre of mass. E.g. a suspended object E.g. does an object standing on an inclined plane slide or topple?		

2g. Content of Statistics Minor (Y432) – minor option

Description	In this minor option situations are modelled by discrete random variables; the suitability of models is tested using χ^2 tests. Bivariate data are investigated, with tests for correlation and association and modelling using regression.
Assumed knowledge	Learners are expected to know the content of A Level Mathematics and the Core Pure mandatory paper (Y420). The content can be co-taught with AS Further Mathematics
Assessment	One examination paper
Length of paper	1 hour 15 minutes
Number of marks	60
Sections	The examination paper will not have sections.
Percentage of qualification	This optional paper counts for 16⅔% of the qualification OCR A Level Further Mathematics B (MEI) (H645).
Use of calculator and other technology	See Section 2b for details about the use of calculators. Calculators used in the examination should be able to calculate means, standard deviations, correlation coefficients, equations of regression lines and probabilities, including cumulative probabilities, from the binomial and Poisson distributions. It is expected that learners will gain experience of using a spreadsheet or other software for exploring data sets; this should include using software to conduct hypothesis tests. In the examination learners will be assessed on the interpretation of output from such software.
Overarching Themes	The Overarching Themes (see Section 2b) apply to this unit. Statistics is about answering real world problems using data, so knowledge of the real world appropriate to a learner on this course will be assumed. Examination questions may include, for example, asking learners to comment sensibly about the modelling assumptions in their answer.
Relationship with other papers	This is the same content as the unshaded sections, labelled (a), of Statistics Major (Y422). Learners may not enter for Y432 and Y422 in the same examination series. This is the same content as Statistics a (Y412) in the qualification OCR AS Further Mathematics B (MEI) (H635). Learners may not enter for Statistics Major (Y422) and Statistics Minor (Y432).
Other notes	When making calculations within a probability model, learners are advised to use exact numbers (e.g. fractions) where possible or decimal numbers to 4 dp. When interpreting solutions in a practical context, rounding to an appropriate (usually lesser) degree of accuracy is expected. Learners are expected to have explored different data sets, using appropriate technology, during the course. No particular data set is expected to be studied, and there will *not* be any pre-release data.

Statistics minor (Y432)
Contents

Sampling (a)	A short section about the importance of sampling methods.
Discrete random variables (a)	The binomial distribution is introduced for modelling discrete univariate data in AS Mathematics. This content extends the range of models available to include the (discrete) uniform, geometric and Poisson distributions. The link between the binomial and Poisson distributions is explored, though the use of the Poisson as an approximation to the binomial distribution for calculation purposes is not included; technology renders it largely obsolete. Some theoretical work on discrete probability distributions, including mean and variance and some of their properties, is introduced and applied to these models.
Bivariate data (a)	Different types of bivariate data are considered. Where appropriate, Pearson's product moment correlation coefficient and Spearman's rank correlation coefficient are used to test for correlation and association, respectively, for bivariate numerical data. The different underlying assumptions are explored. Linear regression as a model for bivariate numerical data is introduced; residuals provide an informal way of looking at the appropriateness of the model.
Chi-squared tests (a)	The hypothesis testing work in AS Level Mathematics - based on the binomial distribution and, informally, on correlation coefficients - is extended to include χ^2 tests and a more formal approach to tests based on correlation coefficients. This gives learners an understanding of a range of tests, including the concept of degrees of freedom, which should allow them to pick up quickly any hypothesis tests they encounter in other subjects. The product moment correlation coefficient is also considered, informally, as an effect size; this serves as an example of a widely-used approach which is complementary to hypothesis testing. The χ^2 test for goodness of fit is used to test whether a particular distribution is appropriate to model a given data set. For bivariate categorical data, the χ^2 test for association, using data given in a contingency table, is introduced.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MINOR: SAMPLING (a)					
Sampling	Sx1	Be able to explain the importance of sample size in experimental design.	E.g. an informal explanation of how the size of a sample affects the interpretation of an effect size.		
	x2	Be able to explain why sampling may be necessary in order to obtain information about a population, and give desirable features of a sample.	Population too large or it is too expensive to take a census. Sampling process may be destructive. Sample should be unbiased, representative of the population; data should be relevant, not changed by the act of sampling.	A sample may also be considered as n observations from a random variable.	
	x3	Be able to explain the advantage of using a random sample when inferring properties of a population.	A random sample enables proper inference to be undertaken because the probability basis on which the sample has been selected is known.		
STATISTICS MINOR: DISCRETE RANDOM VARIABLES (a)					
Probability distributions	SR1	Be able to use probability functions, given algebraically or in tables. Be able to calculate the numerical probabilities for a simple distribution. Be able to draw and interpret graphs representing probability distributions.	Other than the Poisson and geometric distributions, the underlying random variable will only take a finite number of values. An understanding that probabilities are non-negative and sum to 1 is expected.	$\mathrm{P}(X = x)$	
Expectation and variance	R2	Be able to calculate the expectation (mean), $\mathrm{E}(X)$, and understand its meaning.		$\mathrm{E}(X) = \mu$	
	R3	Be able to calculate the variance, $\mathrm{Var}(X)$, and understand its meaning.	Knowledge of $\mathrm{Var}(X) = \mathrm{E}(X^2) - \mu^2$. Standard deviation $= \sqrt{\mathrm{Var}(X)}$.	$\mathrm{Var}(X) = \mathrm{E}[(X - \mu)^2]$	
	R4	Be able to use the result $\mathrm{E}(a + bX) = a + b\mathrm{E}(X)$ and understand its meaning.			
	R5	Be able to use the result $\mathrm{Var}(a + bX) = b^2\mathrm{Var}(X)$ and understand its meaning.			

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MINOR: DISCRETE RANDOM VARIABLES (a)					
Expectation and variance (cont)	SR6	Be able to find the expectation of any linear combination of independent random variables and the variance of any linear combination of independent random variables.	$\mathrm{E}(X \pm Y) = \mathrm{E}(X) \pm \mathrm{E}(Y)$ $\mathrm{Var}(X \pm Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$ $\mathrm{E}(aX \pm bY) = a\mathrm{E}(X) \pm b\mathrm{E}(Y)$ $\mathrm{Var}(aX \pm bY) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y)$		Proofs.
The discrete uniform distribution	R7	Recognise situations under which the discrete uniform distribution is likely to be an appropriate model.	E.g. X has a uniform distribution over the values $\{4, 5, \ldots 9\}$. E.g. a fair spinner with six equally-sized sections, labelled 4, 5, 6, 7, 8, 9.		
	R8	Be able to calculate probabilities using a discrete uniform distribution.			
	R9	Be able to calculate the mean and variance of any given discrete uniform distribution.	If X has a uniform distribution over the values $\{1, 2, \ldots n\}$ then $\mathrm{E}(X) = \frac{n+1}{2}$ and $\mathrm{Var}(X) = \frac{1}{12}(n^2 - 1)$. The formulae for this particular uniform distribution will be given but their derivations may be asked for.		
The binomial distribution	R10	Recognise situations under which the binomial distribution is likely to be an appropriate model, and be able to calculate probabilities to use the model. Know and be able to use the mean and variance of a binomial distribution, $\mu = np$ and $\sigma^2 = np(1-p)$. Prove these results in particular cases.	E.g. prove results by considering a binomial random variable as the sum of n independent Bernoulli random variables: $X = X_1 + X_2 + \ldots + X_n$ where each X_i takes the value 1 with probability p and 0 with probability $1-p$. This proof assumes the relationship about variance in SR6.	$X \sim \mathrm{B}(n, p)$	

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MINOR: DISCRETE RANDOM VARIABLES (a)					
The Poisson distribution	SR11	Recognise situations under which the Poisson distribution is likely to be an appropriate model.	Modelling the number of events occurring in a fixed interval (of time or space) when the events occur randomly at a constant average rate, and independently of each other. It is expected that these conditions can be applied to the particular context. If the mean and variance of the data do not have a similar value then the Poisson model is unlikely to be suitable.	$X \sim \mathrm{Po}(\lambda)$ $X \sim \mathrm{Poisson}(\lambda)$	
	R12	Recognise situations in which both the Poisson distribution and the binomial distribution might be appropriate models.	In a situation where the binomial model is appropriate, if n is large and p is small, then the conditions for a Poisson distribution to be appropriate are approximately satisfied. In the absence of guidance either model can be used.		Formal criteria. Using the Poisson distribution as a numerical approximation for calculating binomial probabilities.
	R13	Be able to calculate probabilities using a Poisson distribution.	Including use of a calculator to access Poisson probabilities and cumulative Poisson probabilities.		
	R14	Know and be able to use the mean and variance of a Poisson distribution.	$\mathrm{E}(X) = \lambda$, $\mathrm{Var}(X) = \lambda$		Proof.
	R15	Know that the sum of two or more independent Poisson distributions is also a Poisson distribution.	$X \sim \mathrm{Po}(\lambda)$ and $Y \sim \mathrm{Po}(\mu)$ $\Rightarrow X + Y \sim \mathrm{Po}(\lambda + \mu)$ when X and Y are independent.		Proof.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MINOR: DISCRETE RANDOM VARIABLES (a)					
The geometric distribution	SR16	Recognise situations under which the geometric distribution is likely to be an appropriate model.	Link with corresponding binomial distribution.	$X \sim \text{Geo}(p)$, where $X =$ number of Bernoulli trials up to and including the first success.	The alternative definition which counts the number of failures.
	R17	Be able to calculate the probabilities within a geometric distribution, including cumulative probabilities.	$\text{P}(X = r) = (1-p)^{r-1}p$ where $p =$ probability of success and $r \in \{1, 2, ...\}$. $\text{P}(X > r) = (1-p)^r$. An understanding of the calculation is expected.		
	R18	Know and be able to use the mean and variance of a geometric distribution.	$\text{E}(X) = \frac{1}{p}$, $\text{Var}(X) = \frac{1-p}{p^2}$.		Proof.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions

STATISTICS MINOR: BIVARIATE DATA (a)

There are two kinds of bivariate data considered in A Level Mathematics and Further Mathematics and it is important to distinguish between them when considering correlation and regression. This note explains the reason for the distinction; learners will only be assessed on what appears under a specification reference below.

Case A: Only **one of the variables** may be considered as **a random variable**. Often this occurs when one of the variables, the independent variable, is controlled by an experimenter and the other, the dependent variable, is measured. An example of this would be (weight, extension) in an investigation of Hooke's law for a spring. In this case certain fixed weights are used; this variable is *not* a random variable, any errors in measuring the weights are negligible. The extension *is* a random variable. There will be deviations from the 'true' value that a perfect experimenter would observe from a perfect spring as well as errors in the measurement. This case is referred to as **'random on non-random'**. The points on the scatter diagram are restricted to lie on certain vertical lines corresponding to the values of the controlled variable.

Case B: The **two variables may both** be considered as **random variables**. An example of this would be (height, weight) for a sample from a population of individuals. For any given value of height there is a distribution of weights; for any given value of weight there is a distribution of heights. That is, there is no 'true' weight for a given height or 'true' height for a given weight. This case is referred to as **'random on random'**. The scatter diagram appears as a 'data cloud'.

If a linear relationship between the variables is to be investigated and modelled using correlation and regression techniques then the two cases must be treated differently.

If it is desired to test the significance of Pearson's product moment correlation coefficient then, as with all parametric hypothesis tests, probability calculations have to be performed to calculate the *p*-value or the critical region. These calculations rely on certain assumptions about the underlying distribution – **these assumptions can never be met in the 'random on non-random case'** – because one of the variables does not have a probability distribution – so **such a test is never valid in this case**. In fact the pmcc is not used in this case. In the **'random on random'** case the distributional assumptions **may** be met – see the specification below for details.

If it is desired to calculate the equation of a line of best fit then the least-squares method is often used in both cases. However its interpretation is different in the two cases. In the example of the **'random on non-random'** case, (weight, extension), the line of regression is modelling the 'true' value of the extension for a given weight – the value that a perfect experimenter would observe from a perfect spring. In the example of the **'random on random' case**, (height, weight), the two **lines of regression are modelling the mean value of the distribution of weights for a given height and the mean value of the distribution of heights for a given weight**.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MINOR: BIVARIATE DATA (a)					
Scatter diagrams	Sb1	Understand what bivariate data are and know the conventions for choice of axis for variables in a scatter diagram.	In the random on non-random case the independent variable is often one which the experimenter controls; the dependent variable is the one which is measured. The independent variable is usually plotted on the horizontal axis. In the random on random case (where both variables are measured), it may be that one is more naturally seen as a function of the other; this determines which variable is plotted on which axis.		
	b2	Be able to use and interpret a scatter diagram.	To look for outliers (by eye). To gain insight into the situation, for example to decide whether a test for correlation or association might be appropriate. Learners may be asked to add to a given scatter diagram in order to interpret a new situation.		
	b3	Interpret a scatter diagram produced by software.	Including where the software draws a trendline and gives a value for pmcc or (pmcc)^2.		
Pearson's product moment correlation coefficient (pmcc)	b4	Be able to calculate the pmcc from raw data or summary statistics.	Use of a calculator expected for calculation from raw data. Summary statistics formulae given.	Sample value r.	
	b5	Know when it is appropriate to carry out a hypothesis test using Pearson's product moment correlation coefficient.	The data must be random on random i.e. both variables must be random. There must be a modelling assumption that the data are drawn from a bivariate Normal distribution. This may be recognised on a scatter diagram by an approximately elliptical distribution of points. Learners will not be required to know the formal meaning of bivariate Normality but will be expected to know that where one or both of the distributions is skewed, bimodal, etc., the procedure is likely to be inappropriate. The test is for correlation, a linear relationship, so a scatter diagram is helpful to check that the data cloud does not indicate a non-linear relationship.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MINOR: BIVARIATE DATA (a)					
Pearson's product moment correlation coefficient (pmcc) (cont)	Sb6	Be able to carry out hypothesis tests using the pmcc and tables of critical values or the *p*-value from software.	Only 'H_0: No correlation in the population' will be tested. Both one-sided and two-sided alternative hypotheses will be tested. Learners should state whether there is sufficient evidence or not to reject H_0 and then give a non-assertive conclusion in context. E.g. 'There is sufficient evidence to suggest that there is positive correlation between ... and ...'	Null hypothesis, alternative hypothesis H_0, H_1.	
	b7	Use the pmcc as an effect size[1].	Sensible informal comments about effect size are expected, either alongside or instead of a hypothesis test.		Any formal rules for judging effect size will be given.

[1]**Note on effect size for correlation**

For a large set of random on random bivariate data a small non-zero value of the pmcc is likely to lead to a rejection of the null hypothesis of no correlation in the population; the test is uninformative. In some contexts it is more important to consider the size of the correlation rather than test whether the population correlation is non-zero. The phrase 'effect size' is sometimes used in this context for the value of the pmcc. In social sciences Cohen's guideline is often used: small effect size 0.1, medium effect size 0.3, large effect size 0.5. Learners are not expected to know this rule; this or any other formal rule will be given if necessary.

Effect sizes for other situations, e.g. for the difference of two means, are beyond the scope of this specification.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MINOR: BIVARIATE DATA (a)					
Spearman's rank correlation coefficient	Sb8	Be able to calculate Spearman's rank correlation coefficient from raw data or summary statistics.	Use of a calculator on the ranked data is expected.	Sample value r_s.	Tied ranks.
	b9	Be able to carry out hypothesis tests using Spearman's rank correlation coefficient and tables of critical values or the output from software.	Hypothesis tests using Spearman's rank correlation coefficient require no modelling assumptions about the underlying distribution. Only 'H_0: No association in the population' will be tested. Both one-sided and two-sided alternative hypotheses will be tested. Learners should state whether there is sufficient evidence or not to reject H_0 and then give a non-assertive conclusion in context. E.g. 'There is insufficient evidence to suggest that there is an association between … and …'.		
Comparison of tests	b10	Decide whether a test based on r or r_s may be more appropriate, or whether neither is appropriate.	Considerations include the appearance of the scatter diagram, the likely validity of underlying assumptions, whether association or correlation is to be tested for. Spearman's test is not appropriate if the scatter diagram shows no evidence of a monotonic relationship i.e. one variable tends to increase (or decrease) as the other increases. Understanding that ranking data loses information, which may affect the outcome of a test.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MINOR: BIVARIATE DATA (a)					
Regression line for a random variable on a non-random variable	Sb11	Be able to calculate the equation of the least squares regression line using raw data or summary statistics.	The goodness of fit of a regression line may be judged by looking at the scatter diagram. In this case examination questions will be confined to cases in which a random variable, Y, and a non-random variable, X, are modelled by a relationship in which the 'true' value of Y is a linear function of X. The use of a calculator is only expected for calculation from raw data. Summary statistics formulae will be given.		Derivation of the least squares regression line.
	b12	Be able to use the regression line as a model to estimate values and know when it is appropriate to do so. Know the meaning of the term residual and be able to calculate and interpret residuals.	residual = observed value – value from regression line Informal checking of a model by looking at residuals.	Interpolation. Extrapolation.	
Regression lines for a random variable on a random variable	b13	Be able to calculate the equation of the two least squares regression lines, y on x and x on y, using raw data or summary statistics. Be able to use either regression line to estimate the expected value of one variable for a given value of the other and know when it is appropriate to do so.	In the y on x case, the least squares regression line estimates $\mathrm{E}(Y \mid X = x)$, that is the expected value of Y for a given value of X. Conversely for the x on y case. Only the use of a calculator is expected for calculation from raw data.		Derivation of the least squares regression lines.
	b14	Check how well the model fits the data.	Informal checking only of a model by visual inspection of a scatter diagram or consideration of (pmcc)^2.		Residuals in this case.
	b15	Know the relationship between the two regression lines and when to use one rather than the other. Be able to use the correct regression line to estimate the expected value of one variable for a given value of the other and know when it is appropriate to do so.	Both lines pass through $(\bar{x}, \bar{y})$. Choice of line to use depends on which variable is to be estimated.	Interpolation. Extrapolation.	

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS MINOR: CHI-SQUARED TESTS (a)					
Contingency tables	Sb16	Be able to interpret bivariate categorical data in a contingency table.	Numerical data can be put into categories, but this loses information.		
χ^2 test for a contingency table	SH1	Be able to apply the χ^2 test (chi-squared) to a contingency table.	Only 'H$_0$: No association between the factors' or 'H$_0$: variables are independent' will be tested. Calculating degrees of freedom is expected. Knowing how to calculate observed values and contributions to the test statistic are expected, but repetitive calculations will not be required. Learners should state whether there is sufficient evidence or not to reject H$_0$ and then give a non-assertive conclusion in context. E.g. 'There is not sufficient evidence to believe that there is association between … and …'.		Yates' continuity correction is not expected, though its appropriate use will not be penalised.
	H2	Be able to interpret the results of a χ^2 test using tables of critical values or the output from software.	Output from software may be given as a p-value. Interpretation may involve considering the individual cells in the table of contributions to the test statistic.		
χ^2 test for goodness of fit	H3	Be able to carry out a χ^2 test for goodness of fit of a uniform, binomial, or Poisson model.	Only 'H$_0$: the given model fits the data' or 'H$_0$: the given model is suitable' will be tested. Calculating degrees of freedom is expected. Knowing how to calculate observed values and contributions to the test statistic is expected, but repetitive calculations will not be required. Learners should be aware that cells are often combined when there are small expected frequencies, but will not have to make such decisions in examination questions. Learners should state whether there is sufficient evidence or not to reject H$_0$ and then give a non-assertive conclusion in context. E.g. 'It is reasonable to believe that the … model is suitable.'		
	H4	Be able to interpret the results of a χ^2 test using tables of critical values or the output from software.	Output from software may be given as a p-value.		

2h. Content of Modelling with Algorithms (Y433) – minor option

Description	Algorithms play a central part in the modern world. This minor option explores algorithms in their own right. Algorithms can be run by hand, but when algorithms are used to model real world problems then technology allows their application to authentic problems. A range of optimisation and network problems are introduced. Many of these can be formulated as linear programming problems, allowing them to be solved using technology.
Assumed knowledge	For the examination learners are expected to know the content of A Level Mathematics and the Core Pure mandatory paper (Y420). The content can be co-taught with AS Further Mathematics.
Assessment	One examination paper
Length of paper	1 hour 15 minutes
Number of marks	60
Sections	The examination paper will not have sections.
Percentage of qualification	This optional paper counts for 16⅔% of the qualification OCR A Level Further Mathematics B (MEI) (H645).
Use of calculator and other technology	See Section 2b for details about the use of calculators. It is expected that learners will gain experience of using a spreadsheet solver or other software for solving linear programs; in the examination they will be assessed on the interpretation of output from such software.
Overarching Themes	The Overarching Themes (see Section 2b) apply. This content includes questions about modelling the real world, so knowledge of the real world appropriate to a learner on this course will be assumed. Examination questions may include, for example, asking learners to suggest why an answer which satisfies the mathematical model may not be acceptable in practice.
Relationship with other papers	This is the same content as Modelling with Algorithms (Y413) in the qualification OCR AS Further Mathematics B (MEI) (H635).
Other notes	When making calculations within a model, learners are advised to use exact numbers (e.g. fractions or surds) where possible. When interpreting solutions in a practical context, rounding to an appropriate degree of accuracy is expected.

Modelling with Algorithms (Y433)
Contents

Algorithms	In covering this section of the specification learners should understand: what an algorithm is; iterative processes; what kind of problems are susceptible to an algorithmic approach; how to compare algorithms, including complexity; the importance of proving that an algorithm works and of the use of heuristic algorithms when this is not possible; the need for an algorithmic approach and computing power to solve problems of the size often met in the real world. Other algorithms are used for modelling in the Networks section; this section emphasises that algorithms can be analysed in their own right.
Networks	Network algorithms are used for modelling a range of real-world problems. Formulating the problems as linear programming (LP) problems allows them to be addressed using technology.
Linear Programming (LP)	This topic introduces constrained optimisation. In some cases LP problems can be interpreted and solved graphically. The simplex method gives an algebraic approach, but using this by hand is limited. The use of a simplex method optimisation routine in a spreadsheet package or other software is introduced, which enables problems of a more realistic size to be tackled. The crucial skills are then setting up the problem in a way suitable for the software and interpreting the output. These are precisely the modelling skills most useful in the real world. Linear programming unifies this content; a wide range of apparently unrelated problems can be formulated as LP problems, and so solved using technology.

This division of the specification is not a recommended division of the material for teaching or assessment. It would be quite sensible, for example, to: introduce a particular network algorithm, using it by hand in suitable cases; where appropriate analyse its complexity and prove that it works; use it for modelling and solving problems; and then formulate it as an LP problem and use technology to tackle authentic problems. The division of the specification is designed to emphasise that this is much more than a collection of algorithms chosen because they are accessible at this level.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MODELLING WITH ALGORITHMS: ALGORITHMS					
Algorithms	AA1	Understand that an algorithm is a finite sequence of operations for carrying out a procedure or solving a problem. Understand that an algorithm can be the basis for a computer program.	Initial state; input; output; variable. 'Finite' means that the procedure terminates.		Algorithms with a random element.
	A2	Be able to interpret and apply algorithms presented in a variety of formats.	Formats include flowcharts; written English; pseudocode. E.g. in pseudocode, Let $i = i + 1$ means that the number in location i is replaced by its current value plus 1. Questions will not be set requiring unduly repetitive calculations.	Loop, pass. 'if … then…' 'Go to step …' Iterative process.	Any particular version of pseudocode or programming language.
	A3	Be able to repair, develop and adapt given algorithms.			
	A4	Understand and be able to use the basic ideas of algorithmic complexity and be able to analyse the complexity of given algorithms. Know that complexity can be used, among other things, to compare algorithms.	Worst case; size of problem; effect on solution time of multiplying the size of a large problem by a given factor and/or repeatedly applying an algorithm.	Order notation e.g. $\mathrm{O}(n^2)$ for quadratic complexity.	Analysis leading to non-polynomial complexity.
	A5	Understand that algorithms can sometimes be proved correct or incorrect.	Proof by exhaustion and disproof by counter-example.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MODELLING WITH ALGORITHMS: ALGORITHMS					
Algorithms (cont)	A6	Understand and know the importance of heuristics.	A heuristic (sometimes called a heuristic algorithm) is a method which finds a solution efficiently, with no guarantee that it is optimal. It is important when classic methods are inefficient or fail.	E.g. packing algorithms. E.g. find a solution to a linear problem which requires an integer solution by exploring around the solution to the corresponding LP.	
Sorting algorithms	A7	Know and be able to use the quick sort algorithm. Be able to apply other sorting algorithms which are specified.		Pivot values. Pass. Ascending, descending.	
	A8	Be able to count the number of comparisons and/or swaps needed in particular applications of sorting algorithms, and relate this to complexity.	Quick sort algorithm has (worst case) complexity $\mathrm{O}(n^2)$.		Average complexity.
	A9	Be able to reason about a given sorting algorithm.	E.g. explain why it will always work.		
Packing algorithms	A10	Know and be able to use first fit and first fit decreasing packing algorithms and full bin strategies.	Know that these are not guaranteed to be optimal.	Bin.	
	A11	Be able to count the number of comparisons needed in particular applications of packing algorithms, and relate this to complexity.	First fit and first fit decreasing algorithms have (worst case) complexity $\mathrm{O}(n^2)$.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MODELLING WITH ALGORITHMS: NETWORKS					
Networks and graphs	AN1	Understand and be able to use graphs and associated language.	Node/vertex; arc/edge; tree; order of a node; simple, complete, connected and bipartite graphs; trees; digraphs.	Incidence matrix.	
	N2	Be able to model problems by using graphs.	E.g. river crossing problems. E.g. matching problems.		
	N3	Understand that a network is a graph with weighted arcs.	Directed and undirected networks.		
	N4	Be able to model problems by using networks.	E.g. shortest path, maximum flow. E.g. allocation and transportation problems.		
Kruskal's, Prim's and Dijkstra's algorithms	N5	Be able to solve minimum connector problems using Kruskal's and Prim's algorithms.	Kruskal's algorithm in graphical form only. Prim's algorithm in graphical or tabular form.	Minimum spanning tree.	
	N6	Model shortest path problems and solve using Dijkstra's algorithm.			
	N7	Know and use the fact that Kruskal's, Prim's and Dijkstra's algorithms have quadratic complexity.	Kruskal's and Prim's have quadratic complexity as a function of the number of edges. Dijkstra's has quadratic complexity as a function of the number of vertices.		
Critical path analysis	N8	Model precedence problems with an activity-on-arc network.			
	N9	Use critical path analysis and be able to interpret outcomes, including implications for criticality. Be able to analyse float (total, independent and interfering), resourcing and scheduling.	E.g. show how to use the minimum number of people to complete a given project in the minimum time.	Critical activities, critical path(s), forward and backward passes, longest path.	

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MODELLING WITH ALGORITHMS: NETWORKS					
Network flows	N10	Be able to use a network to model a transmission system.	Single and super sources and sinks. Flow in = flow out for other nodes.	Source: S. Sink: T.	
	N11	Be able to specify a cut and calculate its capacity.	*Either* split the vertices into two sets, one containing S and the other T, *or* specify the arcs that are cut.		
	N12	Understand and use the maximum flow/minimum cut theorem.	If an established flow is equal to the capacity of an identified cut, then the flow is maximal and the cut is a minimum cut. Exhaustive testing of cuts will not be assessed.		Flow augmentation. Labelling algorithm.
Solving network problems using technology	N13	Understand that network algorithms can be explored, understood and tested in cases in which the algorithm can be run by hand, but for practical problems the algorithm needs to be formulated in a way suitable for computing power to be applied.	Formulations will be restricted to LPs. Questions may be set about the time taken by computer software to implement an algorithm when its complexity is known.		

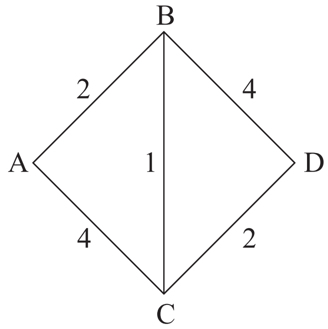
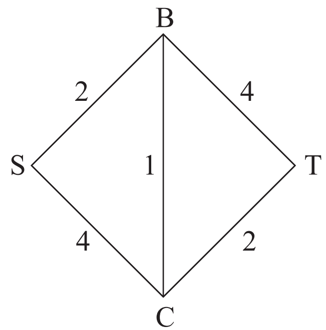
Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MODELLING WITH ALGORITHMS: LINEAR PROGRAMMING					
Formulating a problem	AL1	Understand and use the language associated with linear programming.	Linear programming, objective, maximisation, minimisation, optimisation, constraints.	LP is an abbreviation for linear program.	
	L2	Be able to identify and define variables from a given problem. Be able to formulate a problem as a linear program.	Variables should be clearly identified as representing numerical values. E.g. 'Let x be the number of …'. Problem may be given in context.		
	L3	Be able to recognise when an LP is in standard form.	A linear function to be maximised, constraints with '… $\leq$ constant' and non-negative, continuous variables.		
	L4	Be able to use slack variables to convert an LP in standard form to augmented form.	Also called slack form. As standard form, but using non-negative slack variables to convert inequalities to equalities.	State variables. Slack variables. Basic and non-basic variables.	
	L5	Recognise when an LP requires an integer solution.	E.g. when a variable is discrete. E.g. a shortest path problem, because the variables take the values 1 or 0, depending on whether the corresponding arc is in the path or not. If an LP requires an integer solution this should be stated in the formulation.	ILP is an abbreviation for integer LP.	
	L6	Be able to formulate a range of network problems as LPs.	Shortest path problems; network flows; critical path (longest path) problems; matching, allocation and transportation problems. See after L18 for examples.		

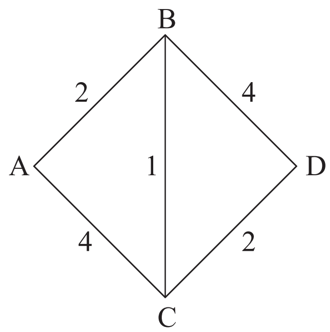
Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MODELLING WITH ALGORITHMS: LINEAR PROGRAMMING					
Graphical solution of an LP	L7	Be able to graph inequalities in 2-D and identify feasible regions. Be able to recognise infeasibility.	No particular shading convention is expected, but learners must make clear which is the feasible region.		Drawing diagrams in more than 2-D.
	L8	Be able to solve a 2-D LP graphically.	By finding at least one optimal feasible point and the value of the objective function at this point. Using the gradient of the objective function or by enumeration.		
	L9	Be able to consider the effect of modifying constraints or the objective function.		Post-optimal analysis.	
	L10	Be able to solve 2-D integer LP problems graphically.	The optimal lattice point may or may not be near the LP solution.		
	L11	Be able to use a visualisation of a 3-D LP to solve it. Be able to reduce a 3-D LP to a 2-D LP when one constraint is an equality.	Diagram will be given. Regions will be defined by an inequality based on the cartesian equation of a plane.		
Simplex method	L12	Be able to use the simplex algorithm on an LP in augmented form.	Setting up an initial tableau, choosing a pivot, transforming the tableau, interpreting a tableau, recognising when a tableau represents an optimal solution. Problems may be infeasible or have multiple solutions (degeneracy).	Initial, intermediate, final tableau. Slack variables. Pivot. Basic/non-basic variables.	Knowledge of complexity of the simplex algorithm.
	L13	Understand the geometric basis for the simplex method.	Interpret a tableau in terms of the vertex and value of the objective function.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MODELLING WITH ALGORITHMS: LINEAR PROGRAMMING					
Simplex and non-standard form	L14	Recognise that if an LP includes $\geq$ constraints then the two-stage simplex method may be used; understand how this method works and be able to set up the initial tableau in such cases.			Big-M method.
	L15	Be able to reformulate an equality constraint as a pair of inequality constraints.	E.g. replace $x = 4$ by $x \geq 4$ and $x \leq 4$.		
	L16	Recognise that if an LP has variables which may take negative values or requires the objective function to be minimised then some initial reformulation is required before the simplex algorithm may be applied.	Learners need only know that such reformulation is possible.		Be able to apply simplex in these situations.
Use of software	L17	Understand that some LPs can be solved using graphical techniques or the simplex method, but for practical problems computing power needs to be applied. Know that a spreadsheet LP solver routine, or other software, can solve an LP given in standard form or, in some cases, in non-standard form.			
	L18	Be able to interpret the output from a spreadsheet optimisation routine, or other software, for the simplex method or ILPs.	Select the appropriate information to solve the original problem. This may lead to further analysis of the problem.		

Examples of reformulating network problems as LPs

These examples show how six types of network problems can be reformulated as LPs. They illustrate the sort of notation that will be used in questions. They do not show the level of difficulty of problems that will be examined.

		Shortest path Find a shortest path from A to D. Variables take the value 1 if the corresponding arc is used in a shortest path, and 0 otherwise.		Minimise $2AB + 4BD + 4AC + 2CD + BC + CB$ subject to $AB + AC = 1$ $AB + CB - BC - BD = 0$ $AC + BC - CB - CD = 0$ $BD + CD = 1$
		Network flow Find a maximum flow from S to T through the network.		Maximise $SB + SC$ subject to $SB + CB - BC - BT = 0$ $SC + BC - CB - CT = 0$ $SB \leq 2$ $BT \leq 4$ $SC \leq 4$ $CT \leq 2$ $BC \leq 1$ $CB \leq 1$

<table>
<tr>
<td></td>
<td></td>
<td>Longest path
Find a longest path from A to D.
Variables take the value 1 if the corresponding arc is used in a shortest path, and 0 otherwise.
This can be used to solve critical path problems on a directed network.</td>
<td></td>
<td>Maximise
$2AB + 4BD + 4AC + 2CD + BC + CB$
subject to
$AB + AC = 1$
$AB + CB - BC - BD = 0$
$AC + BC - CB - CD = 0$
$BD + CD = 1$
$AB \le 1$
$BD \le 1$
$AC \le 1$
$CD \le 1$
$BC \le 1$
$CB \le 1$</td>
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<tr>
<td></td>
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<td>Matching problem
Possible associations between elements of {A, B, C, D} and {1, 2, 3, 4} are shown in the table. In a matching each element of one set is associated with at most one element of the other. The LP tries to find a maximal matching, i.e. a matching with as many associations as possible.
Each variable (e.g. C3) takes the value 1 (if C and 3 are associated) or 0.</td>
<td><table><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>A</td><td>x</td><td></td><td></td><td>x</td></tr><tr><td>B</td><td>x</td><td></td><td>x</td><td></td></tr><tr><td>C</td><td></td><td>x</td><td>x</td><td></td></tr><tr><td>D</td><td></td><td></td><td>x</td><td></td></tr></table>
“x” indicates a possible matching</td>
<td>Maximise
$A1 + A4 + B1 + B3 + C2 + C3 + D3$
subject to
$A1 + A4 \le 1$
$B1 + B3 \le 1$
$C2 + C3 \le 1$
$D3 \le 1$
$A1 + B1 \le 1$
$C2 \le 1$
$B3+C3+D3 \le 1$
$A4 \le 1$</td>
</tr>
</table>

Allocation problem

This is like a matching problem, except that (usually) every association is possible, and each association has a cost. The LP minimises the total cost for a maximal matching.

Each variable (e.g. A1) takes the value 1 or 0, depending on whether A is associated with 1 or not in the matching.

	1	2	3	4
A	5	2	3	6
B	1	7	2	4
C	5	8	3	1
D	4	4	2	6

Minimise

$$5A1 + 2A2 + 3A3 + 6A4 + B1 + 7B2 + 2B3 + 4B4 + 5C1 + 8C2 + 3C3 + C4 + 4D1 + 4D2 + 2D3 + 6D4$$

subject to

$A1 + A2 + A3 + A4 = 1$
$B1 + B2 + B3 + B4 = 1$
$C1 + C2 + C3 + C4 = 1$
$D1 + D2 + D3 + D4 = 1$
$A1 + B1 + C1 + D1 = 1$
$A2 + B2 + C2 + D2 = 1$
$A3 + B3 + C3 + D3 = 1$
$A4 + B4 + C4 + D4 = 1$

Transportation problem

The body of the table shows the costs per item of transporting from one set of locations {A, B, C, D} to another {1, 2, 3, 4}.

The margins show the availability of items at locations A, B, C and D and the demands at 1, 2, 3 and 4.

The LP minimises the total cost of delivering all the required items.

		5	5	5	5
		1	2	3	4
3	A	5	2	3	6
6	B	1	7	2	4
9	C	5	8	3	1
2	D	4	4	2	6

Minimise

$$5A1 + 2A2 + 3A3 + 6A4 + B1 + 7B2 + 2B3 + 4B4 + 5C1 + 8C2 + 3C3 + C4 + 4D1 + 4D2 + 2D3 + 6D4$$

subject to

$A1 + A2 + A3 + A4 = 3$
$B1 + B2 + B3 + B4 = 6$
$C1 + C2 + C3 + C4 = 9$
$D1 + D2 + D3 + D4 = 2$
$A1 + B1 + C1 + D1 = 5$
$A2 + B2 + C2 + D2 = 5$
$A3 + B3 + C3 + D3 = 5$
$A4 + B4 + C4 + D4 = 5$

2i. Content of Numerical Methods (Y434) – minor option

Description	Much of AS/A Level Mathematics and Further Mathematics is restricted to problems which are amenable to exact solution. For many real world problems no exact methods exist, and numerical methods are required to solve them. In this minor option learners apply numerical approaches to four topics from mathematics: solution of equations, differentiation, integration and approximating functions. Learners learn how to use a spreadsheet to implement the methods and learn to analyse the errors associated with numerical methods.
Assumed knowledge	For the examination learners are expected to know the content of A Level Mathematics and the Core Pure mandatory paper (Y420). The content can be co-taught with AS Further Mathematics.
Assessment	One examination paper
Length of paper	1 hour 15 minutes
Number of marks	60
Sections	The examination paper will not have sections.
Percentage of qualification	This optional paper counts for 16⅔% of the qualification OCR A Level Further Mathematics B (MEI) (H645).
Use of calculator and other technology	It is expected that a calculator is used in the examination. In Numerical Methods, candidates are expected to show evidence of working through methods rather than just writing down solutions provided by equation solvers or numerical differential or integration functions on calculators. See Section 2b for details about the use of calculators. It is expected that learners will gain experience of using a spreadsheet for implementing numerical methods; in the examination learners may be given output from spreadsheets and may be asked: to explain what certain cells represent; to explain or give formulae for certain cells; to give solutions and justify their accuracy; to comment on errors, convergence or order.
Overarching Themes	The Overarching Themes (see Section 2b) apply. Questions may be set requiring learners to model a real world situation, and then use numerical methods to solve the mathematics before interpreting the solution; learners may be asked to comment on the error due to the numerical method employed as well as the appropriateness of the model.
Relationship with other papers	This is the same content as Numerical Methods (Y414) in the qualification OCR AS Further Mathematics B (MEI) (H635).
Other notes	N/A

Numerical Methods (Y434)
Contents

Use of technology	This section describes how spreadsheets and calculators are to be used.
Errors	Dealing with errors is tackled in this section; how they arise and propagate and how analysis of errors can lead to improved solutions.
Solution of equations	Five methods for solving equations are studied, with their graphical interpretation: bisection method; false position; secant method; fixed point iteration and Newton-Raphson. Failure and order of convergence is considered. The method of relaxation is applied to fixed point iteration.
Numerical differentiation	Two methods are studied and compared: forward difference method and central difference method.
Numerical integration	Three methods are studied and the relationships between them exploited: midpoint, trapezium and Simpson's methods.
Approximation to functions	Two methods are studied, including the circumstances in which each is appropriate: Newton's forward difference interpolation method and Lagrange's form of the interpolating polynomial.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
NUMERICAL METHODS: USE OF TECHNOLOGY					
Use of spreadsheets and calculators	NQ1	Be able to use a spreadsheet to implement the methods and to explore associated ideas. Be able to interpret the output from a spreadsheet.	Learners are expected to be familiar with a spreadsheet; no particular one is expected. In the examination the spreadsheet facility available on some calculators may be used, but this is not expected. Learners will be given output from a spreadsheet and may be asked to explain what certain cells represent, to explain or give formulae for certain cells, to give solutions and justify their accuracy, to comment on errors, convergence or order.	Cell B4 will mean the cell in column B, row 4. = IF(condition, value_if_true, value_if_false). Learners may give formulae from any spreadsheet with which they are familiar.	Use of a computer in the examination.
	Q2	Be able to use the iterative capability of a calculator.	In the examination learners are expected to use the iterative capabilities of their calculators (e.g. the ANS button) to generate values of iterative sequences. Any permitted calculator may be used, but capabilities such as numerical differentiation, numerical integration and equation solvers should not be used in the examination; learners must show sufficient working to make their method clear. Lengthy calculations will not be required.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
NUMERICAL METHODS: ERRORS					
Absolute and relative error	NU1	Know how to calculate errors in sums, differences, products and quotients. Know the meaning of absolute and relative error.	Exact value: x Approximate value: X Absolute error: $X-x$ Relative error: $\frac{X-x}{x}$.	Absolute error will be used as a signed quantity. Another convention defines absolute error to be the magnitude of this quantity; this usage will not be penalised.	
Error propagation by arithmetical operations and by functions	U2	Know how to calculate the error in $\mathrm{f}(x)$ when there is an error in x.			Functions of more than one variable.
	U3	Understand the effects on errors of changing the order of a sequence of operations.			
Errors in the representation of numbers: rounding; chopping	U4	Understand that computers represent numbers to limited precision.			
	U5	Understand the consequences of subtracting nearly equal quantities.	The subtraction might be embedded within a more complicated calculation e.g. in a fraction or in solving simultaneous equations.		
	U6	Understand rounding and chopping and their consequences, including for calculations.	E.g. 7.86 rounded to 1 d.p. is 7.9; 7.86 chopped to 1 d.p. is 7.8; 5.7 chopped to the nearest integer is 5. E.g. 200 numbers are each expressed to 1 d.p. Each number is chopped to the nearest integer, and then they are added. Maximum error in any one number is 0.9; maximum error in sum is $200 \times 0.9 = 180$. Average error in one number is 0.45, so expected error for sum is 90.	Maximum, average and expected error.	

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
NUMERICAL METHODS: ERRORS					
Order of convergence and order of method	NU7	Understand convergence and divergence when applied to sequences. Understand the order of convergence of an iterative sequence and the order of a method. Be able to comment on these given output from a spreadsheet.	An iterative sequence (e.g. a sequence produced by the Newton-Raphson method) has kth order convergence if the sequence of errors ε_n satisfy the approximate relationship $\varepsilon_{n+1} \propto \varepsilon_n^k$. For a method with a 'step-length' h, (e.g. central difference method), the order of the method is the value k such that, approximately, $\text{error} \propto h^k$. (For such a method a sequence of approximations can be produced by using a sequence of values of h; the sequence of errors will have an order of convergence, but this is **not**, in general, the order of the method.)		Formal analysis e.g. using Taylor expansions.
Improving a solution	U8	Be able to use error analysis to produce an improved solution.	Learners may be expected to calculate or identify the ratio of differences of a sequence of approximations to, for example, a definite integral. This may be presented as part of a spreadsheet output. They should be able to use an appropriate value for the ratio of differences to obtain an improved approximation by extrapolation – including to infinity and should be able to quote and justify an appropriate level of precision in their final answer.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
NUMERICAL METHODS: SOLUTION OF EQUATIONS					
Bisection method; False Position (linear interpolation); Secant method; Fixed point iteration; Newton-Raphson method	Ne1	Understand the graphical interpretations of these methods.	Including staircase and cobweb diagrams. Learners should be able to comment on suitability of starting point.		Proofs of orders of convergence.
	e2	Be able to solve equations to any required degree of accuracy using these methods.	Justify the accuracy claimed.		
	e3	Understand the relative computational merits and possible failure of these methods.	Learners should recognise situations in which fixed point iteration and Newton-Raphson methods will fail.		
	e4	Know that fixed point iteration generally has first order convergence, Newton-Raphson generally has second order convergence.	Learners should be able to comment on failure of the method or lower-order convergence from graphical considerations or from spreadsheet output: e.g. the relationship between the order of convergence and the gradient of $\mathrm{g}(x)$ at the root in the iteration $x_{n+1} = \mathrm{g}(x_n)$.		Formal proofs of convergence. Formal analysis of failure or lower-order convergence.
	e5	Understand and be able to apply relaxation to a fixed point iteration: to accelerate convergence; to convert a divergent sequence to a convergent sequence.	For the iteration $x_{n+1} = \mathrm{g}(x_n)$ the relaxed iteration is $x_{n+1} = (1-\lambda)x_n + \lambda \mathrm{g}(x_n)$. Formula will be given. Different values of λ have different effects on convergence.		Calculus to find optimal choice for λ.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
NUMERICAL METHODS: NUMERICAL DIFFERENTIATION					
Forward difference method; Central difference method	Nc1	Be able to estimate a derivative using the forward and central difference methods with a suitable value (or sequence of values) of h.	Use a suitable sequence of values of h to observe when the limitation of a spreadsheet's accuracy is reached, to analyse errors and to justify the accuracy of a solution.	$\mathrm{f}'(x) \approx \frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$ $\mathrm{f}'(x) \approx \frac{\mathrm{f}(x+h)-\mathrm{f}(x-h)}{2h}$	Second derivatives.
	c2	Have an empirical and graphical appreciation of the greater accuracy of the central difference method. Know that the forward difference method is generally a first order method and that the central difference method is generally a second order method.			Proofs of order of method.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusion
NUMERICAL METHODS: NUMERICAL INTEGRATION					
Midpoint rule; trapezium rule; Simpson's rule	Nc3	Be able to evaluate a given definite integral to any desired degree of accuracy using these methods.	To estimate $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$: $M_n = h(y_{\frac{1}{2}} + y_{\frac{3}{2}} + \ldots + y_{n-\frac{3}{2}} + y_{n-\frac{1}{2}})$ where $h = \frac{b-a}{n}$ $T_n = \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$ where $h = \frac{b-a}{n}$ $S_{2n} = \frac{1}{3}h\{(y_0 + y_{2n}) + 4(y_1 + y_3 + \ldots + y_{2n-1}) + 2(y_2 + y_4 + \ldots + y_{2n-2})\}$ where $h = \frac{b-a}{2n}$. These formulae will be given. Lengthy calculations will not be required in the examination. Any of the rules may be applied more than once, e.g. with h halving each time. Learners are expected to be able to consider properties of the function – e.g. the graph is concave upwards over the given interval – to determine whether the rule over- or under-estimates.	N.B. The commonly used notation for Simpson's rule, S_{2n}, shown in the formula, leads to an inconsistent definition of h. M_n will be referred to as the midpoint rule based on n strips; T_n will be referred to as the trapezium rule based on n strips; the concept of strips will not be applied to Simpson's rule.	
The relationship between methods	c4	Know that, generally, the midpoint and trapezium rules are second order methods and Simpson's rule is a fourth order method. Understand the development of Simpson's rule from the midpoint and trapezium rules.	$T_{2n} = \frac{1}{2}(M_n + T_n)$ $S_{2n} = \frac{1}{3}(2M_n + T_n) = \frac{1}{3}(4T_{2n} - T_n)$ Formulae will be given.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusion
NUMERICAL METHODS: APPROXIMATIONS TO FUNCTIONS					
Newton's forward difference interpolation method	Nf1	Be able to use Newton's forward difference interpolation formula to reconstruct polynomials and to approximate functions.	Functions tabulated at equal intervals; learners should recognise when this is not the case and the method is not suitable. Formula will be given. Learners should be able to construct and use a difference table and know that nth differences are constant for an nth degree polynomial.	$\Delta f(x) = f(x+h) - f(x)$ $\Delta^2 f(x) = \Delta f(x+h) - \Delta f(x)$	
Lagrange's form of the interpolating polynomial	f2	Be able to construct the interpolating polynomial of degree n given a set of $n+1$ data points.	Formula will be given.		

2j. Content of Extra Pure (Y435) – minor option

Description	In this minor option learners extend their understanding of matrices by studying eigenvectors and eigenvalues, study first and second order recurrence relations, are introduced to finite and infinite groups and extend their understanding of calculus to functions of two variables. The topics are chosen because they introduce concepts which are foundational in undergraduate mathematics; reveal unexpected connections between different parts of mathematics and underlying structure; or provide a suitable end point for topics within Further Mathematics. Some topics are from continuous mathematics and others from discrete mathematics.
Assumed knowledge	Learners are expected to know the content of A Level Mathematics and the Core Pure mandatory paper (Y420). The content of some of the topics can be taught concurrently with AS Further Mathematics.
Assessment	One examination paper
Length of paper	1 hour 15 minutes
Number of marks	60
Sections	The examination paper will not have sections.
Percentage of qualification	This optional paper counts for 16⅔% of the qualification OCR A Level Further Mathematics B (MEI) (H645).
Use of calculator and other technology	See Section 2b for details about the use of calculators.
Overarching Themes	The Overarching Themes (see Section 2b) apply. Contexts will sometimes be used to help understanding of a problem.
Relationship with other papers	N/A
Other notes	N/A

Extra Pure (Y435)
Contents

Recurrence relations	Recurrence relations are defined and investigated, including their long-term behaviour. Methods for solving first and second order linear recurrence relations are studied.
Groups	Group axioms and examples of finite groups of small order and infinite groups are studied. Lagrange's theorem is applied to the order of subgroups of finite groups and the concept of an isomorphism is introduced.
Matrices	The work in Core Pure is extended to include eigenvalues and eigenvectors; this is applied to transformations. The Cayley-Hamilton theorem is introduced.
Multivariable calculus	Partial differentiation is introduced and used to explore features of surfaces in 3-D: contours, sections, stationary points, normal lines and tangent planes.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
EXTRA PURE: RECURRENCE RELATIONS					
Construction of models	Xs1	Be able to model appropriate problems by using recurrence relations.	Recurrence relations are sometimes called difference equations.	u_n Sequences may start with u_0 or u_1.	
Investigation of recurrence relations	s2	Understand and use the language of recurrence relations.	Limit, convergent, divergent, periodic, oscillating, linear, non-linear, homogeneous, non-homogeneous, general solution, particular solution.	E.g. $u_{n+1} = 2u_n$ is a recurrence relation. For the initial condition $u_1 = 5$ the particular solution of the recurrence relation is (the sequence given by) $u_n = 5 \times 2^{n-1}$.	
	s3	Be able to investigate and comment on the behaviour of recurrence relations, including long-run behaviour.	Spreadsheet output or a (partial) solution may be given, or sufficient information to examine the behaviour. Recurrence relations may be non-linear.		Rates of convergence.
Solution of recurrence relations	s4	Be able to verify a given solution of a recurrence relation.	Check that the given solution satisfies the iterative relationship and any initial, or other, conditions.		
	s5	Be able to solve first order linear homogeneous recurrence relations with constant coefficients.	$u_{n+1} = au_n$		Use of generating functions.
	s6	Be able to solve first order linear non-homogeneous recurrence relations with constant coefficients.	$u_{n+1} = au_n + \mathrm{f}(n)$ $\mathrm{f}(n)$ may be constant, linear, quadratic or of the form dk^n.		
	s7	Be able to solve second order linear homogeneous recurrence relations with constant coefficients.	$u_{n+2} = au_{n+1} + bu_n$ The roots of the auxiliary equation may be real distinct, real repeated or complex. General or particular solutions may be required.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
EXTRA PURE: RECURRENCE RELATIONS					
Solution of recurrence relations (cont)	s8	Be able to solve second order linear non-homogenous recurrence relations with constant coefficients.	$u_{n+2} = au_{n+1} + bu_n + \mathrm{f}(n)$ $\mathrm{f}(n)$ may be constant, linear, quadratic or of the form dk^n. The roots of the auxiliary equation may be real distinct, real repeated or complex. General or particular solutions may be required.		Use of generating functions.
	s9	Be able to investigate and comment on properties of solutions of recurrence relations.	E.g. investigate associated sequences. E.g. prove the value of a limit.	$\lim_{n \to \infty} u_n$	

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
EXTRA PURE: GROUPS					
Sets	XS1	Understand and use the language and notation of sets.	For all non-empty sets A , $A \subseteq A$ and $\varnothing \subset A$.	$\in \notin \subset \subseteq \not\subset \varnothing \cup \cap$ A' $A \backslash B$ $\left\{\frac{a}{b}: a, b \in \mathbb{Z} \text{ and } b \neq 0\right\}$ $n(A) = \|A\| =$ number of elements in a finite set A.	
	S2	Understand and use the common notation for sets of numbers. Know and use properties of these numbers.	$\mathrm{N} = \{1, 2, 3...\}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$ E.g. determine whether $a+b$ is rational or irrational for different cases.	$\mathbb{Q}^+ = \{q \in \mathbb{Q}: q > 0\}$ $\mathbb{N}_0 = \{0, 1, 2, 3...\}$ $\mathbb{R}_0^+ = \{x \in \mathbb{R}: x \geq 0\}$	
The axioms of a group	Xa1	Understand the group axioms and the associated language.	The terms binary operation, closed, associative, identity, inverse, commutative, abelian. Composition table. $< x, y >$ is the group generated by the elements x and y. Groups may be finite or infinite.	$(G, \circ)$ is the set G under the binary operation $\circ$. G may refer to the set or the group. e for identity, or 0 or 1 if appropriate. x^{-1} for inverse, or $-x$ if 0 used for identity.	
Illustrations of groups	a2	Be familiar with examples of groups, and of the use of group tables for finite groups.	E.g. based on symmetries of geometric figures, modulo arithmetic, matrices, complex numbers; integers under addition. It may be assumed that function composition is associative, hence so is (e.g.) matrix multiplication and composition of transformations.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
EXTRA PURE: GROUPS					
Cyclic groups	a3	For finite groups, understand the meaning of the term cyclic group, and how such a group is generated by a single element.		$< x >$ generator	Infinite cyclic groups.
The order of a finite group; the order of an element of a group	a4	Understand the terms order of a finite group, order of an element.	The order of an element may be infinite.		
Subgroups	a5	Understand and work with subgroups. Know the conditions for a subset of a group to be a subgroup.	A proper subgroup of G is any subgroup of G other than G itself. A non-trivial subgroup of G is any subgroup of G other than $\{e\}$. Infinite groups may have finite and infinite subgroups.		
Lagrange's theorem	a6	Know and be able to use Lagrange's theorem.	In a finite group, the order of a subgroup divides the order of the group. The corollary that the order of an element divides the order of the group.		Proof of Lagrange's theorem.
Isomorphism	a7	Understand that different situations can give rise to the same underlying structure.	Concept and illustrations only.		
	a8	Be able to specify an isomorphism between two groups. Know that two cyclic groups of the same order are isomorphic and be able to specify an isomorphism.	E.g. be able to decide whether two groups of order 6 are isomorphic. E.g. between a set of linear transformations in 2-D under composition and a set of 2×2 matrices under matrix multiplication.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
EXTRA PURE: MATRICES					
Eigenvalues and eigenvectors	Xm1	Understand the meaning of eigenvalue and eigenvector, and be able to find these for 2×2 and 3×3 matrices.	Learners will only be required to find eigenvectors where the corresponding eigenvalue is real.		
	m2	Be able to form and solve the characteristic equation of a 2×2 or 3×3 matrix.	$\det(\mathbf{M} - \lambda\mathbf{I}) = 0$		
Diagonalisation and powers of matrices	m3	Be able to form the matrix of eigenvectors and use this to reduce a matrix to diagonal form.	Distinct real eigenvalues only.		
	m4	Be able to find powers of a 2×2 or 3×3 matrix using diagonal form.			
Cayley-Hamilton Theorem	m5	Understand and be able to use the Cayley-Hamilton theorem.	Every matrix satisfies its own characteristic equation. E.g. to find relationships between the powers of a matrix or to find the inverse matrix.		Proof.
Applications of eigenvalues and eigenvectors to transformations	m6	Understand the significance of eigenvalues and eigenvectors in 2-D and 3-D transformations.	E.g. The given 3×3 matrix $\mathbf{M}$ represents a reflection in 3-D; find the equation of the plane of reflection. (A normal vector of the plane will be an eigenvector associated with the eigenvalue -1.)		Finding a matrix for a given 3-D transformation is not expected, except for those which are covered by the assumed knowledge.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
EXTRA PURE: MULTIVARIABLE CALCULUS					
$z = \mathrm{f}(x, y)$ and its interpretation as a surface	Xc1	Know that the relation $z = \mathrm{f}(x, y)$ defines a surface in 3-D.	E.g. $z = xy^2 - 4x^2y + 20$		
	c2	Be able to sketch contours and sections, and know how these are related to the surface.	Sections of the form $z = \mathrm{f}(a, y)$ or $z = \mathrm{f}(x, b)$ E.g. $z = ay^2 - 4a^2y + 20$		
Partial derivatives and stationary points	c3	Be able to find first order partial derivatives.			Second and higher order partial derivatives.
	c4	Be able to use the conditions $\frac{\partial z}{\partial x} = 0$ and $\frac{\partial z}{\partial y} = 0$ to find the coordinates of stationary points on a surface.	If investigation of the nature of the stationary point is required, the method will be given.	A saddle point is any stationary point which is neither a maximum nor a minimum.	
Surfaces defined as $\mathrm{g}(x, y, z) = c$	c5	Know that the relation $\mathrm{g}(x, y, z) = c$ defines a surface in 3-D.	Surfaces may be defined by $z = \mathrm{f}(x, y)$ or $\mathrm{g}(x, y, z) = c$.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
EXTRA PURE: MULTIVARIABLE CALCULUS					
Partial derivatives: the normal line and tangent plane at a point	c6	Be able to find **grad** g, and to evaluate this at a point on the surface to give a normal vector.	$\mathbf{grad}\ \mathrm{g} = \begin{pmatrix} \frac{\partial \mathrm{g}}{\partial x} \\ \frac{\partial \mathrm{g}}{\partial y} \\ \frac{\partial \mathrm{g}}{\partial z} \end{pmatrix}$. If $\mathrm{g}(x, y, z)$ can be written as $z = \mathrm{f}(x, y)$ then $\mathbf{grad}\ \mathrm{g} = \begin{pmatrix} \frac{\partial \mathrm{f}}{\partial x} \\ \frac{\partial \mathrm{f}}{\partial y} \\ -1 \end{pmatrix}$ Formulae will be given.	$\nabla \mathrm{g}$ for **grad** g.	
	c7	Understand the concepts of, and be able to find and use, the equations of the normal line and tangent plane at a point on the surface.			

2k. Content of Further Pure with Technology (Y436) – minor option

Description	In this minor option learners use a computer algebra system, a spreadsheet, a graph plotter and a programming language to investigate curves, explore topics in number theory and explore the solutions to families of differential equations analytically and numerically.
Assumed knowledge	Learners are expected to know the content of A Level Mathematics and the Core Pure mandatory paper (Y420). The content of some of the topics can be taught concurrently with AS Further Mathematics.
Assessment	One examination paper
Length of paper	1 hour 45 minutes
Number of marks	60
Sections	Each of the three topics will be assessed by one question worth about 20 marks.
Percentage of qualification	This optional paper counts for 16⅔% of the qualification OCR A Level Further Mathematics B (MEI) (H645).
Use of calculator and other technology	See Section 2b for details about the **Use of calculators**, **Use of technology**, and **Use of a computer in Further Pure with Technology**. Learners need to have access to a calculator or computer with a computer algebra system, a spreadsheet, a graph plotter and a programming language in the examination. The graphing software should be able to plot families of curves in cartesian, polar and parametric forms, with sliders (or equivalent) for parameters. The graphing software should be able to draw tangent fields for families of differential equations given in the form $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$. The computer algebra system should be able to perform at least all the algebraic requirements of A Level Further Mathematics, and should be able to differentiate and integrate analytically functions where solutions are known to exist in terms of elementary functions. The spreadsheet should feature the ability to enter formulae based on cell references using the notation $\mathrm{A}, \mathrm{B}, \mathrm{C}$ ... for columns and $1, 2, 3$ ... for rows. The programming language must have capacity for: checking conditions; looping through values; local/global variables; inputting/outputting variables. The language should have a prime-testing routine, or learners should be able to write one. The lists of approved software and programming languages can be found in Appendix 5e. If there are any updates, these will be communicated to centres annually prior to first teaching each year, and will be available at www.ocr.org.uk. Centres wishing to use alternative software should submit details to OCR for approval prior to first teaching, on the appropriate form available at www.ocr.org.uk. If further examples are considered for approval these will be updated on the list above on an annual basis, before first teaching each year. Centres may seek advice about software from www.ocr.org.uk or by contacting OCR at maths@ocr.org.uk. The conduct of examinations including the use of computers is covered by the JCQ Instructions for Conducting Examinations (http://www.jcq.org.uk/).
Overarching Themes	The Overarching Themes (see Section 2b) apply. Questions involving modelling will usually be restricted to modelling mathematical situations using a spreadsheet or program.
Relationship with other papers	N/A
Other notes	Learners' answers are handwritten in a Printed Answer Booklet. This entails transcribing from a computer screen, so the length of this paper is 1 hour 45 minutes, 30 minutes longer than comparable papers.

Further Pure with Technology (Y436)
Contents

Investigation of curves	In this topic learners develop skills associated with curves. They learn to look for and recognise important properties of curves, making appropriate use of graphing software and a Computer Algebra System (CAS). They are expected to be able to generalise their findings; at times this will require analytical techniques. Examination questions will use a variety of curves but learners will not be expected to know their particular properties. Instead, the questions will test learners' ability to select and apply the skills to investigate them. It is, however, anticipated that while studying this topic, learners will meet a wide selection of curves including curves expressed as cartesian equations, parametric equations and polar curves.
Exploring differential equations	In this topic learners explore first order differential equations. They use technology to explore exact solutions to differential equations when this is possible, but appreciate that many differential equations cannot be solved analytically. They use technology to produce tangent fields in order to gain insights into the behaviour of solutions to (families of) differential equations. They learn some numerical techniques for estimating solutions to differential equations.
Number theory	In this topic learners use a programming language to explore results in number theory. Examination questions will feature short programs which produce the solutions to problems in Number Theory. Learners may be expected to write their own programs as well as understanding a program and suggesting limitations and refinements to it.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
FURTHER PURE WITH TECHNOLOGY: INVESTIGATION OF CURVES					
Use of software	TC1	Be able to plot a family of curves in graphing software, in cartesian, polar and parametric forms.	Learners are expected to be able to use sliders for parameters.		
	C2	Be able to use CAS to work with equations of curves.	E.g. solve equations, evaluate derivatives, find limits.		
Properties of curves	C3	Know and use the vocabulary associated with curves	Asymptote, cusp, loop; bounded; terms relating to symmetry.		
	C4	Be able to find, describe and generalise properties of curves.	Generalisation may involve exploratory use of graphing software. Any algebraic work may involve the use of CAS. Curves may be given in cartesian, polar or parametric form.		Knowledge of properties of particular curves.
	C5	Be able to convert equations between cartesian and polar forms in all cases. Be able to convert equations from polar to parametric form, and parametric to cartesian form.			
Derivatives of curves	C6	Be able to find the gradient of the tangent to a curve at a point.	Tangents to curves in cartesian, polar and parametric form.		
	C7	Be able to find and work with equations of chords, tangents and normals.			
Arc lengths and envelopes	C8	Be able to calculate arc length using cartesian, parametric and polar coordinates.	Learners are expected to be able to give expressions in terms of integrals. Formulae will be given.		Formal definition of partial derivatives.
	C9	Understand the meaning of an envelope of a family of curves. Be able to find an envelope of a family of curves.	A curve which every member of a family of curves touches tangentially is called an envelope of the family. By eliminating p between $\mathrm{f}(x, y, p) = 0$ and $\frac{\partial \mathrm{f}}{\partial p}(x, y, p) = 0$.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
FURTHER PURE WITH TECHNOLOGY: INVESTIGATION OF CURVES					
Limiting behaviour	C10	Be able to use the limit of $\mathrm{f}(x)$ as $x \to a$ or $x \to \infty$ to investigate and deduce properties of a curve.	Limits may be found with CAS.		
	C11	Be able to determine asymptotes.	Including oblique asymptotes and asymptotic approach to curves.		
	C12	Be able to identify cusps by examining the behaviour nearby.	By examining the limit of the gradient as the curve approaches the cusp along both branches. A cusp is a point where two branches of a curve meet, with the branches having a common tangent at the point. E.g. $(0, 0)$ on $y^2 = x^5$ or $(0, 0)$ on $y^5 = x^2$.		Formal definition of branches; they are observed from a sketch or plot. Alternative definitions of a cusp.
FURTHER PURE WITH TECHNOLOGY: EXPLORING DIFFERENTIAL EQUATIONS					
Use of software	Tc1	Be able to use software to produce analytical solutions to (families of) first order differential equations, when this is possible.	Use of a slider.		
	c2	Be able to use software to produce a tangent field for a differential equation.	Differential equation of the form $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$.		
	c3	Be able to construct, adapt or interpret a spreadsheet to solve first order differential equations numerically.	Using the techniques in Tc7 to Tc10. Learners may be asked to state the spreadsheet formulae they have used in constructing their spreadsheets. They should show sufficient formulae to indicate the design of the spreadsheet. E.g. giving one formula from each column.		
Analytical solutions of differential equations	c4	Be able to verify a given solution of a differential equation.	May be a general or particular solution.		
	c5	Be able to work with particular solutions and families of particular solutions.			
Tangent fields	c6	Be able to sketch a tangent field for a first order differential equation and be able to interpret it.	E.g. sketch the curve of a particular solution. E.g. work with isoclines.	Tangent field, slope field, direction field, isocline.	

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
FURTHER PURE WITH TECHNOLOGY: EXPLORING DIFFERENTIAL EQUATIONS					
The Euler method. Modified Euler method (Runge-Kutta order 2).	c7	Know how to solve a given first order differential equation $\frac{dy}{dx} = f(x, y)$ with initial conditions to any required degree of accuracy by repeated application of the Euler method or a modified Euler method.	Euler method $y_{n+1} = y_n + hf(x_n, y_n)$ A modified Euler method $k_1 = hf(x_n, y_n)$ $k_2 = hf(x_n + h, y_n + k_1)$ $y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2)$ Formulae will be given.	Step length. This modified Euler method is a Runge-Kutta method of order 2.	
	c8	Understand that a smaller step length usually gives a more accurate answer. Understand that a modified Euler method usually gives a more accurate solution than an Euler method for a given step length.	Using two different step lengths may give information about the accuracy of a solution. E.g. if they agree to 3 d.p.		Formal treatment of errors. Convergence.
Runge-Kutta methods	c9	Understand the concepts underlying Runge-Kutta methods.	Learners will be expected to be familiar with the standard Runge-Kutta method of order 4. $k_1 = hf(x_n, y_n)$ $k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$ $k_3 = hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2})$ $k_4 = hf(x_n + h, y_n + k_3)$ $y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ Formulae will be given.		
	c10	Be able to solve first order differential equations using Runge-Kutta methods.	Formulae for variations of these methods may be given.		Methods of order > 4.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
FURTHER PURE WITH TECHNOLOGY: NUMBER THEORY					
Programming	TT1	Be able to write, adapt and interpret programs to solve number theory problems.	Programming language must have capacity for: checking conditions; looping through values; local/global variables; inputting/outputting variables. The language should have a prime-testing routine, or learners should be able to write one.		
	T2	Be able to identify the limitations of a short program and suggest refinements to it.	Learners will be expected to discuss, informally, the limitations of a program and offer a small number of changes to it.		
Prime Factorisation	T3	Know and use the unique prime factorisation of natural numbers.			Proof of the fundamental theorem of arithmetic.
Congruences and modular arithmetic	T4	Be able to solve problems using modular arithmetic.	Use of the term congruent. When working modulo n final answers should be given as integers, m, where $0 \leq m < n$.	$16 \equiv 2 (\bmod 7)$ An answer of 16 should be written as 2.	
	T5	Know and use Fermat's little theorem.	Use of the terms co-prime and GCD/HCF.		
	T6	Know and use Euler's totient function, $\varphi(n)$, Euler's theorem.	$\varphi(n) =$ the number of positive integers less than n that are co-prime with n.		
	T7	Know and use Wilson's theorem.	$(p-1)! \equiv -1 (\bmod p)$ $\Leftrightarrow p$ prime		
Diophantine Equations	T8	Be able to find Pythagorean triples and use related equations.	Related equations may include different indices, such as $x^2 + y^2 = z^3$.		
	T9	Be able to solve Pell's equation and use solutions to solve related problems.	$x^2 - ny^2 = 1$		
	T10	Be able to solve other Diophantine equations and use solutions to solve related problems.	All the required information will be given in the question.		

2l. Prior knowledge, learning and progression

It is assumed that learners are familiar with the whole content of GCSE(9–1) Mathematics for first teaching from 2015 and A Level Mathematics for first teaching from 2017. There is no requirement that learners have completed these qualifications.

OCR A Level in Further Mathematics B (MEI) is designed for students with an enthusiasm for mathematics, many of whom will go on to degrees in mathematics, engineering, the sciences and economics.

OCR A Level in Further Mathematics B (MEI) is both deeper and broader than A Level mathematics. AS and A Level further mathematics build from GCSE Level and AS and A Level mathematics. As well as building on algebra and calculus introduced in A Level mathematics, the A Level further mathematics core content introduces complex numbers and matrices, fundamental mathematical ideas with wide applications in mathematics, engineering, physical sciences and computing. The non-core content includes different options that can enable students to specialise in areas of mathematics that are particularly relevant to their interests and future aspirations. A level further mathematics prepares students for further study and employment in highly mathematical disciplines that require knowledge and understanding of sophisticated mathematical ideas and techniques.

The co-teachability of AS Further Mathematics B (MEI) and A Level Further Mathematics B (MEI) is made possible by the following links:

- For each of the minor options Mechanics minor, Statistics minor, Modelling with algorithms and Numerical methods there is a corresponding AS option with the same content.
- For each of the major options Mechanics Major and Statistics Major there are two AS options, one covering the first half and the other the second half of the A Level major option; the content for the major options is clearly labelled (a) or (b) to make the links clear
- One third of the Core Pure content is labelled (a), and is the same content as the AS Core Pure.

There are a number of Mathematics specifications at OCR. Find out more at www.ocr.org.uk

3 Assessment of A Level in Further Mathematics B (MEI)

3a. Forms of assessment

OCR A Level in Further Mathematics B (MEI) is externally assessed by written examination. All the papers may contain some synoptic assessment, some extended response questions and some stretch and challenge questions.

Stretch and challenge questions are designed to allow the most able learners the opportunity to demonstrate the full extent of their knowledge and skills.

Stretch and challenge questions will support the awarding of A* grade at A Level, addressing the need for greater differentiation between the most able learners.

OCR's A Level in Further Mathematics B is a linear qualification in which all papers must be taken in the same examination series.

The details of the examination papers are in the table below

Code	Paper	Raw marks	Proportion of A level after scaling	Sections/questions	Rubric
Y420	Core Pure	144	50%	A: 30–40 marks[1] B: 104–114 marks	Answer all questions
Y421	Mechanics Major	120	33⅓%	A: 25–35 marks[1] B: 85–95 marks	Answer all questions
Y422	Statistics Major	120	33⅓%	A: 25–35 marks[1] B: 85–95 marks	Answer all questions
Y431	Mechanics Minor	60	16⅔%	no: there is a gradient of demand across the paper	Answer all questions
Y432	Statistics Minor	60	16⅔%	no: there is a gradient of demand across the paper	Answer all questions
Y433	Modelling with Algorithms	60	16⅔%	no: there is a gradient of demand across the paper	Answer all questions
Y434	Numerical Methods	60	16⅔%	no: there is a gradient of demand across the paper	Answer all questions
Y435	Extra Pure	60	16⅔%	no: there is a gradient of demand across the paper	Answer all questions
Y436	Further Pure with Technology	60	16⅔%	one question, about 20 marks, on each of three topics	Answer all questions

[1] Section A consists of shorter questions with minimal reading and interpretation; the aim of this is to ensure that all students feel as though they can do some of the questions on the paper. Section B includes longer questions and more problem solving. Section B has a gradient of difficulty.

3b. Assessment objectives (AO)

There are three Assessment Objectives in OCR A Level in Further Mathematics B (MEI). These are detailed in the table below.

Learners are expected to demonstrate their ability to:

	Assessment Objective
AO1	**Use and apply standard techniques** Learners should be able to: • select and correctly carry out routine procedures; and • accurately recall facts, terminology and definitions.
AO2	**Reason, interpret and communicate mathematically** Learners should be able to: • construct rigorous mathematical arguments (including proofs); • make deductions and inferences; • assess the validity of mathematical arguments; • explain their reasoning; and • use mathematical language and notation correctly. *Where questions/tasks targeting this assessment objective will also credit Learners for the ability to 'use and apply standard techniques' (AO1) and/or to 'solve problems within mathematics and in other contexts' (AO3) an appropriate proportion of the marks for the question/task will be attributed to the corresponding assessment objective(s).*
AO3	**Solve problems within mathematics and in other contexts** Learners should be able to: • translate problems in mathematical and non-mathematical contexts into mathematical processes; • interpret solutions to problems in their original context, and, where appropriate, evaluate their accuracy and limitations; • translate situations in context into mathematical models; • use mathematical models; and • evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them. *Where questions/tasks targeting this assessment objective will also credit Learners for the ability to 'use and apply standard techniques' (AO1) and/or to 'reason, interpret and communicate mathematically' (AO2) an appropriate proportion of the marks for the question/task will be attributed to the corresponding assessment objective(s).*

AO weightings in A Level in Further Mathematics B (MEI)

The relationship between the assessment objectives and the examination papers are shown in the following table:

Paper	% weighting of paper in qualification	% weighting of AOs for each paper		
		AO1	AO2	AO3
Core Pure	50	50	30	20
Mechanics Major	33⅓	50	15	35
Statistics Major	33⅓	50	20	30
Mechanics Minor	16⅔	50	15	35
Statistics Minor	16⅔	50	20	30
Modelling with Algorithms	16⅔	50	20	30
Numerical Methods	16⅔	50	30	20
Extra Pure	16⅔	50	30	20
Further Pure with Technology	16⅔	50	30	20

This gives the following mark weighting of each AO for each paper in the qualification.

Paper	AO marks per paper		
	AO1	AO2	AO3
Core Pure	70–74 marks	40–47 marks	25–32 marks
Mechanics Major	58–62 marks	16–21 marks	39–44 marks
Statistics Major	58–62 marks	21–27 marks	33–39 marks
Mechanics Minor	29–31 marks	8–11 marks	19–22 marks
Statistics Minor	29–31 marks	10–14 marks	16–20 marks
Modelling with Algorithms	29–31 marks	10–14 marks	16–20 marks
Numerical Methods	29–31 marks	16–20 marks	10–14 marks
Extra Pure	29–31 marks	16–20 marks	10–14 marks
Further Pure with Technology	29–31 marks	16–20 marks	10–14 marks

Across all chosen papers combined in any given series, AO totals will fall within the following percentages for the qualification:
AO1 – 50% ± 2%
AO2 and AO3 – at least 15%

3c. Assessment availability

There will be one examination series available each year in May/June to **all** learners.

All examined papers must be taken in the same examination series at the end of the course.

This specification will be certificated from the June 2019 examination series onwards.

3d. Retaking the qualification

Learners can retake the qualification as many times as they wish. They must retake a complete valid combination of examination paper for the qualification as detailed in Section 2a.

3e. Assessment of extended response

The assessment materials for this qualification provide learners with the opportunity to demonstrate their ability to construct and develop a sustained and coherent line of reasoning.

3f. Synoptic assessment

Synoptic assessment allows learners to demonstrate the understanding they have acquired from the course as a whole and their ability to integrate and apply that understanding. This level of understanding is needed for successful use of the knowledge and skills from this course in future life, work and study.

Learners are required to know and understand all the content of A Level Mathematics, and the assessment will reflect this when appropriate.

Learners are required to be able to apply the overarching themes from this specification, along with associated mathematical thinking and understanding.

In each optional examination paper learners are required to know and understand all the content of the Core Pure of this specification, and the assessment will reflect this when appropriate.

In each optional examination paper learners are required to draw together knowledge, skills and understanding from different parts of that content area.

In all the examination papers, learners will be required to integrate and apply their understanding in order to address problems which require both breadth and depth of understanding in order to reach a satisfactory solution.

Learners will be expected to reflect on and interpret solutions, drawing on their understanding of different aspects of the course.

3g. Calculating qualification results

If a learner enters for a major option then the overall qualification grade for A Level in Further Mathematics B (MEI) will be calculated by adding together their scaled mark from the Core Pure paper, their major optional paper and their minor optional paper to give their total weighted mark.

If a learner does not enter for a major option then the overall qualification grade for A Level in Further Mathematics B (MEI) will be calculated by adding together their scaled mark from the Core Pure paper, and the marks from the three minor option papers to give their total weighted mark.

Their total weighted mark will then be compared to the qualification level grade boundaries that apply for the combination of papers taken by the learner and for the relevant exam series to determine the learner's overall qualification grade.

Where learners take more than the required number of papers, the combination of papers that result in the best **grade** will be used.

Note: this may NOT be the combination with the highest number of raw marks.

4 Admin: what you need to know

The information in this section is designed to give an overview of the processes involved in administering this qualification so that you can speak to your exams officer. All of the following processes require you to submit something to OCR by a specific deadline.

More information about the processes and deadlines involved at each stage of the assessment cycle can be found in the Administration area of the OCR website. OCR's *Admin overview* is available on the OCR website at http://www.ocr.org.uk/administration.

4a. Pre-assessment

Estimated entries

Estimated entries are your best projection of the number of learners who will be entered for a qualification in a particular series. Estimated entries should be submitted to OCR by the specified deadline. They are free and do not commit your centre in any way.

Final entries

Final entries provide OCR with detailed data for each learner, showing each assessment to be taken. It is essential that you use the correct entry code, considering the relevant entry rules.

Final entries must be submitted to OCR by the published deadlines or late entry fees will apply.

All learners taking A Level in Further Mathematics B (MEI) must be entered for H645.

Entry code	Title	Code	Title	Type
H645	Further Mathematics B (MEI)	Y420	Core Pure	Mandatory
		Y421	Mechanics Major	Major option
		Y422	Statistics Major	Major option
		Y431	Mechanics Minor	Minor option
		Y432	Statistics Minor	Minor option
		Y433	Modelling with Algorithms	Minor option
		Y434	Numerical Methods	Minor option
		Y435	Extra Pure	Minor option
		Y436	Further Pure with Technology	Minor option
		All assessments are by written examination.		

A Learner must be entered for the qualification entry code H645 and a combination of papers satisfying the following entry rules:

Learners must take **one** of three routes through the qualification, Route A, Route B or Route C. The qualification comprises of one mandatory Core Pure paper taken by all learners and then a combination of optional papers.

Route A: Candidates must take the mandatory Core Pure and Mechanics Major papers and then one further optional minor paper. This paper **must not** be Mechanics Minor.
Route B: Candidates must take the mandatory Core Pure and Statistics Major papers and then one further optional minor paper. This paper **must not** be Statistics Minor.
Route C: Candidates must take the mandatory Core Pure paper and then three further minor optional papers.

Learners may **not** enter for Mechanics Major (Y421) and Mechanics Minor (Y431), Statistics Major (Y422) and Statistics Minor (Y432) or Mechanics Major (Y421) and Statistics Major (Y422).

Where learners enter additional papers, the combination of optional papers that results in the best grade will count towards the award. **Note: this may NOT be the combination with the highest number of raw marks.**

4b. Special consideration

Special consideration is a post–assessment adjustment to marks or grades to reflect temporary injury, illness or other indisposition at the time the assessment was taken.

Detailed information about eligibility for special consideration can be found in the JCQ publication *A guide to the special consideration process.*

4c. External assessment arrangements

Regulations governing examination arrangements are contained in the JCQ *Instructions for conducting examinations*.

Head of Centre annual declaration

The Head of Centre is required to provide a declaration to the JCQ as part of the annual NCN update, conducted in the autumn term, to confirm that the centre is meeting all of the requirements detailed in the specification. Any failure by your centre to provide the Head of Centre Annual Declaration will result in your centre status being suspended and could lead to the withdrawal of our approval for you to operate as a centre.

Private candidates

Private candidates may enter for OCR assessments.

A private candidate is someone who pursues a course of study independently but takes an examination or assessment at an approved examination centre. A private candidate may be a part-time student, someone taking a distance learning course, or someone being tutored privately. They must be based in the UK.

Private candidates need to contact OCR approved centres to establish whether they are prepared to host them as a private candidate. The centre may charge for this facility and OCR recommends that the arrangement is made early in the course.

Further guidance for private candidates may be found on the OCR website: http://www.ocr.org.uk

4d. Results and certificates

Grade Scale

A level qualifications are graded on the scale: A*, A, B, C, D, E, where A* is the highest. Learners who fail to reach the minimum standard for E will be Unclassified (U). Only subjects in which grades A* to E are attained will be recorded on certificates.

Papers are graded on the scale a*, a, b, c, d, e, where a* is the highest. Learners who fail to reach the minimum standard for e will be Unclassified (u). Individual paper results will not be recorded on certificates.

Results

Results are released to centres and learners for information and to allow any queries to be resolved before certificates are issued.

Centres will have access to the following results information for each learner:

- the grade for the qualification
- the raw mark and grade for each paper
- the total weighted mark for the qualification.

The following supporting information will be available:

- raw mark grade boundaries for each paper
- weighted mark grade boundaries for the combinations of papers taken by their learners.

Until certificates are issued, results are deemed to be provisional and may be subject to amendment.

A learner's final results will be recorded on an OCR certificate. The qualification title will be shown on the certificate as 'OCR Level 3 Advanced GCE in Further Mathematics B (MEI)'.

4e. Post-results services

A number of post-results services are available:

- **Review of marking** – If you are not happy with the outcome of a learner's results, centres may request a review of marking. Full details of the post-results services are provided on the OCR website.
- **Missing and incomplete results** – This service should be used if an individual subject result for a learner is missing, or the learner has been omitted entirely from the results supplied.
- **Access to scripts** – Centres can request access to marked scripts.

4f. Malpractice

Any breach of the regulations for the conduct of examinations and non-exam assessment work may constitute malpractice (which includes maladministration) and must be reported to OCR as soon as it is detected.

Detailed information on malpractice can be found in the JCQ publication *Suspected Malpractice in Examinations and Assessments: Policies and Procedures*.

5 Appendices

5a. Overlap with other qualifications

The content of this specification overlaps with AS Further Mathematics B (MEI) and with other specifications in A Level Further Mathematics and AS Further Mathematics.

The overlap with AS Further Mathematics B (MEI) allows co-teachability with that qualification. The details are as follows:

A Level Further Mathematics …	**has same content as AS Level Further Mathematics …**
Y420 Core Pure section (a)	Y410 Core Pure
Y421 Mechanics Major section (a)	Y411 Mechanics a
Y421 Mechanics Major section (b)	Y415 Mechanics b
Y422 Statistics Major section (a)	Y412 Statistics a
Y422 Statistics Major section (b)	Y416 Statistics b
Y431 Mechanics Minor	Y411 Mechanics a
Y432 Statistics Minor	Y412 Statistics a
Y433 Modelling with Algorithms	Y413 Modelling with Algorithms
Y434 Numerical Methods	Y414 Numerical Methods
Y435 Extra Pure	No equivalent
Y436 Further Pure with Technology	No equivalent

5b. Accessibility

Reasonable adjustments and access arrangements allow learners with special educational needs, disabilities or temporary injuries to access the assessment and show what they know and can do, without changing the demands of the assessment. Applications for these should be made before the examination series. Detailed information about eligibility for access arrangements can be found in the JCQ *Access Arrangements and Reasonable Adjustments.*

The A level qualification and subject criteria have been reviewed in order to identify any feature which could disadvantage learners who share a protected Characteristic as defined by the Equality Act 2010. All reasonable steps have been taken to minimise any such disadvantage.

5c. Mathematical notation

The tables below set out the notation that must be used by AS and A Level Mathematics and Further Mathematics specifications. Students will be expected to understand this notation without need for further explanation. Any additional notation is listed in section 2 of the specification.

1	**Set Notation**	
1.1	$\in$	is an element of
1.2	$\notin$	is not an element of
1.3	$\subseteq$	is a subset of
1.4	$\subset$	is a proper subset of
1.5	$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
1.6	$\{x : \ldots\}$	the set of all x such that $\ldots$
1.7	$\mathrm{n}(A)$	the number of elements in set A
1.8	$\varnothing$	the empty set
1.9	ε	the universal set
1.10	A'	the complement of the set A
1.11	$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \ldots\}$
1.12	$\mathbb{Z}$	the set of integers, $\{0, \pm1, \pm2, \pm3, \ldots\}$
1.13	$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
1.14	$\mathbb{Z}_0^+$	the set of non-negative integers, $\{0, 1, 2, 3, \ldots\}$
1.15	$\mathbb{R}$	the set of real numbers
1.16	$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z},\ q \in \mathbb{Z}^+\right\}$
1.17	$\cup$	union

1.18	$\cap$	intersection
1.19	$(x,\ y)$	the ordered pair x, y
1.20	$[a,\ b]$	the closed interval $\{x \in \mathbb{R} : a \le x \le b\}$
1.21	$[a,\ b)$	the interval $\{x \in \mathbb{R} : a \le x < b\}$
1.22	$(a,\ b]$	the interval $\{x \in \mathbb{R} : a < x \le b\}$
1.23	$(a,\ b)$	the open interval $\{x \in \mathbb{R} : a < x < b\}$
1	**Set Notation (Further Mathematics only)**	
1.24	$\mathbb{C}$	the set of complex numbers
2	**Miscellaneous Symbols**	
2.1	$=$	is equal to
2.2	$\neq$	is not equal to
2.3	$\equiv$	is identical to or is congruent to
2.4	$\approx$	is approximately equal to
2.5	∞	infinity
2.6	$\propto$	is proportional to
2.7	$\therefore$	therefore
2.8	$\because$	because
2.9	$<$	is less than
2.10	$\leqslant, \le$	is less than or equal to, is not greater than
2.11	$>$	is greater than
2.12	$\geqslant, \ge$	is greater than or equal to, is not less than
2.13	$p \Rightarrow q$	p implies q (if p then q)
2.14	$p \Leftarrow q$	p is implied by q (if q then p)
2.15	$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
2.16	a	first term for an arithmetic or geometric sequence
2.17	l	last term for an arithmetic sequence
2.18	d	common difference for an arithmetic sequence
2.19	r	common ratio for a geometric sequence
2.20	S_n	sum to n terms of a sequence
2.21	S_∞	sum to infinity of a sequence

3	Operations	
3.1	$a+b$	a plus b
3.2	$a-b$	a minus b
3.3	$a\times b,\ ab,\ a.b$	a multiplied by b
3.4	$a \div b,\ \dfrac{a}{b}$	a divided by b
3.5	$\sum_{i=1}^{n} a_i$	$a_1+a_2+\ldots+a_n$
3.6	$\prod_{i=1}^{n} a_i$	$a_1\times a_2\times\ldots\times a_n$
3.7	$\sqrt{a}$	the non-negative square root of a
3.8	$\lvert a\rvert$	the modulus of a
3.9	$n!$	n factorial: $n!=n\times(n-1)\times\ldots\times2\times1,\ n\in\mathbb{N};\ 0!=1$
3.10	$\binom{n}{r}, {}^nC_r, {}_nC_r$	the binomial coefficient $\dfrac{n!}{r!(n-r)!}$ for $n,\ r\in\mathbb{Z}_0^+, r\leqslant n$ or $\dfrac{n(n-1)\ldots(n-r+1)}{r!}$ for $n\in\mathbb{Q}, r\in\mathbb{Z}_0^+$
4	**Functions**	
4.1	$\mathrm{f}(x)$	the value of the function f at x
4.2	$\mathrm{f}: x\mapsto y$	the function f maps the element x to the element y
4.3	f^{-1}	the inverse function of the function f
4.4	gf	the composite function of f and g which is defined by $\mathrm{gf}(x)=\mathrm{g}(\mathrm{f}(x))$
4.5	$\lim_{x\to a}\mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
4.6	$\Delta x,\ \delta x$	an increment of x
4.7	$\dfrac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
4.8	$\dfrac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
4.9	$\mathrm{f}'(x),\ \mathrm{f}''(x),\ \ldots,\ \mathrm{f}^{(n)}(x)$	the first, second, ..., n^{th} derivatives of $\mathrm{f}(x)$ with respect to x
4.10	$\dot{x},\ \ddot{x},\ \ldots$	the first, second, ... derivatives of x with respect to t
4.11	$\int y\,\mathrm{d}x$	the indefinite integral of y with respect to x
4.12	$\int_a^b y\,\mathrm{d}x$	the definite integral of y with respect to x between the limits $x=a$ and $x=b$

5	**Exponential and Logarithmic Functions**	
5.1	e	base of natural logarithms
5.2	e^x, $\exp x$	exponential function of x
5.3	$\log_a x$	logarithm to the base a of x
5.4	$\ln x$, $\log_e x$	natural logarithm of x
6	**Trigonometric Functions**	
6.1	$\left.\begin{array}{l}\sin, \cos, \tan\\ \mathrm{cosec}, \sec, \cot\end{array}\right\}$	the trigonometric functions
6.2	$\left.\begin{array}{l}\sin^{-1}, \cos^{-1}, \tan^{-1}\\ \arcsin, \arccos, \arctan\end{array}\right\}$	the inverse trigonometric functions
6.3	$^{\circ}$	degrees
6.4	rad	radians
6	**Trigonometric and Hyperbolic Functions (Further Mathematics only)**	
6.5	$\left.\begin{array}{l}\mathrm{cosec}^{-1}, \sec^{-1}, \cot^{-1}\\ \mathrm{arccosec}, \mathrm{arcsec}, \mathrm{arccot}\end{array}\right\}$	the inverse trigonometric functions
6.6	$\left.\begin{array}{l}\sinh, \cosh, \tanh,\\ \mathrm{cosech}, \mathrm{sech}, \coth\end{array}\right\}$	the hyperbolic functions
6.7	$\left.\begin{array}{l}\sinh^{-1}, \cosh^{-1}, \tanh^{-1}\\ \mathrm{cosech}^{-1}, \mathrm{sech}^{-1}, \coth^{-1}\end{array}\right\}$ $\left.\begin{array}{l}\mathrm{arsinh}, \mathrm{arcosh}, \mathrm{artanh},\\ \mathrm{arcosech}, \mathrm{arsech}, \mathrm{arcoth}\end{array}\right\}$	the inverse hyperbolic functions
7	**Complex Numbers (Further Mathematics only)**	
7.1	i, j	square root of -1
7.2	$x + \mathrm{i}y$	complex number with real part x and imaginary part y
7.3	$r(\cos\theta + \mathrm{i}\sin\theta)$	modulus argument form of a complex number with modulus r and argument θ
7.4	z	a complex number, $z = x + \mathrm{i}y = r(\cos\theta + \mathrm{i}\sin\theta)$
7.5	$\mathrm{Re}(z)$	the real part of z, $\mathrm{Re}(z) = x$
7.6	$\mathrm{Im}(z)$	the imaginary part of z, $\mathrm{Im}(z) = y$
7.7	$\lvert z \rvert$	the modulus of z, $\lvert z \rvert = \sqrt{x^2 + y^2}$
7.8	$\arg(z)$	the argument of z, $\arg(z) = \theta$, $-\pi < \theta \leq \pi$
7.9	z^*	the complex conjugate of z, $x - \mathrm{i}y$

8	**Matrices (Further Mathematics only)**	
8.1	$\mathbf{M}$	a matrix $\mathbf{M}$
8.2	$\mathbf{0}$	zero matrix
8.3	$\mathbf{I}$	identity matrix
8.4	$\mathbf{M}^{-1}$	the inverse of the matrix $\mathbf{M}$
8.5	$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
8.6	Δ, det $\mathbf{M}$ or $\lvert\mathbf{M}\rvert$	the determinant of the square matrix $\mathbf{M}$
8.7	$\mathbf{Mr}$	Image of column vector $\mathbf{r}$ under the transformation associated with the matrix $\mathbf{M}$
9	**Vectors**	
9.1	$\mathbf{a}$, $\underline{a}$, $\underset{\sim}{a}$	the vector $\mathbf{a}$, $\underline{a}$, $\underset{\sim}{a}$; these alternatives apply throughout section 9
9.2	$\overrightarrow{\mathrm{AB}}$	the vector represented in magnitude and direction by the directed line segment AB
9.3	$\hat{\mathbf{a}}$	a unit vector in the direction of $\mathbf{a}$
9.4	$\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
9.5	$\lvert\mathbf{a}\rvert$, a	the magnitude of $\mathbf{a}$
9.6	$\lvert\overrightarrow{\mathrm{AB}}\rvert$, AB	the magnitude of $\overrightarrow{\mathrm{AB}}$
9.7	$\begin{pmatrix} a \\ b \end{pmatrix}$, $a\mathbf{i} + b\mathbf{j}$	column vector and corresponding unit vector notation
9.8	$\mathbf{r}$	position vector
9.9	$\mathbf{s}$	displacement vector
9.10	$\mathbf{v}$	velocity vector
9.11	$\mathbf{a}$	acceleration vector
9	**Vectors (Further Mathematics only)**	
9.12	$\mathbf{a.b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
10	**Differential Equations (Further Mathematics only)**	
10.1	ω	angular speed
11	**Probability and Statistics**	
11.1	$A, B, C,$ etc.	events
11.2	$A \cup B$	union of the events A and B
11.3	$A \cap B$	intersection of the events A and B
11.4	$\mathrm{P}(A)$	probability of the event A

11.5	A'	complement of the event A
11.6	$\mathrm{P}(A\|B)$	probability of the event A conditional on the event B
11.7	X, Y, R, etc.	random variables
11.8	x, y, r, etc.	values of the random variables X, Y, R etc.
11.9	$x_1, x_2, \ldots$	observations
11.10	$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
11.11	$\mathrm{p}(x)$, $\mathrm{P}(X=x)$	probability function of the discrete random variable X
11.12	$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
11.13	$\mathrm{E}(X)$	expectation of the random variable X
11.14	$\mathrm{Var}(X)$	variance of the random variable X
11.15	$\sim$	has the distribution
11.16	$\mathrm{B}(n,\ p)$	binomial distribution with parameters n and p, where n is the number of trials and p is the probability of success in a trial
11.17	q	$q = 1 - p$ for binomial distribution
11.18	$\mathrm{N}(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2
11.19	$Z \sim N(0, 1)$	standard Normal distribution
11.20	ϕ	probability density function of the standardised Normal variable with distribution $\mathrm{N}(0,\ 1)$
11.21	Φ	corresponding cumulative distribution function
11.22	μ	population mean
11.23	σ^2	population variance
11.24	σ	population standard deviation
11.25	$\bar{x}$	sample mean
11.26	s^2	sample variance
11.27	s	sample standard deviation
11.28	H_0	Null hypothesis
11.29	H_1	Alternative hypothesis
11.30	r	product moment correlation coefficient for a sample
11.31	ρ	product moment correlation coefficient for a population
12	**Mechanics**	
12.1	kg	kilograms
12.2	m	metres
12.3	km	kilometres
12.4	m/s, m s^{-1}	metres per second (velocity)
12.5	m/s^2, m s^{-2}	metres per second per second (acceleration)

12.6	F	force or resultant force
12.7	N	newton
12.8	N m	newton metre (moment of a force)
12.9	t	time
12.10	s	displacement
12.11	u	initial velocity
12.12	v	velocity or final velocity
12.13	a	acceleration
12.14	g	acceleration due to gravity
12.15	μ	coefficient of friction

5d. Mathematical formulae, identities and statistical tables

Learners must be able to use the following formulae and identities for AS and A Level Further Mathematics, without these formulae and identities being provided, either in these forms or in equivalent forms. These formulae and identities may only be provided where they are the starting point for a proof or as a result to be proved.

Pure Mathematics

Quadratic Equations

$ax^2 + bx + c = 0$ has roots $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Laws of Indices

$$a^x a^y \equiv a^{x+y}$$
$$a^x \div a^y \equiv a^{x-y}$$
$$(a^x)^y \equiv a^{xy}$$

Laws of Logarithms

$x = a^n \Leftrightarrow n = \log_a x$ for $a > 0$ and $x > 0$

$$\log_a x + \log_a y \equiv \log_a (xy)$$
$$\log_a x - \log_a y \equiv \log_a \left(\frac{x}{y}\right)$$
$$k \log_a x \equiv \log_a (x^k)$$

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Coordinate Geometry

A straight line graph, gradient m passing through (x_1, y_1) has equation

$y - y_1 = m(x - x_1)$

Straight lines with gradients m_1 and m_2 are perpendicular when $m_1 m_2 = -1$

Sequences

General term of an arithmetic progression:

$u_n = a + (n-1)d$

General term of a geometric progression:

$u_n = ar^{n-1}$

Trigonometry

In the triangle ABC

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area $= \dfrac{1}{2}ab\sin C$

$\cos^2 A + \sin^2 A \equiv 1$

$\sec^2 A \equiv 1 + \tan^2 A$

$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2\sin A\cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \dfrac{2\tan A}{1 - \tan^2 A}$

Mensuration

Circumference and Area of circle, radius r and diameter d:

$C = 2\pi r = \pi d \qquad A = \pi r^2$

Pythagoras' Theorem: In any right-angled triangle where a, b and c are the lengths of the sides and c is the hypotenuse:

$c^2 = a^2 + b^2$

Area of a trapezium $= \dfrac{1}{2}(a+b)h$, where a and b are the lengths of the parallel sides and h is their perpendicular separation.

Volume of a prism = area of cross section × length

For a circle of radius r, where an angle at the centre of θ radians subtends an arc of length l and encloses an associated sector of area a:

$l = r\theta \qquad a = \dfrac{1}{2}r^2\theta$

Complex Numbers

For two complex numbers $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$:

$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$

$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$

Loci in the Argand diagram:

$|z - a| = r$ is a circle radius r centred at a

$\arg(z - a) = \theta$ is a half line drawn from a at angle θ to a line parallel to the positive real axis

Exponential Form:

$e^{i\theta} = \cos\theta + i\sin\theta$

Matrices

For a 2 by 2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the determinant $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

the inverse is $\frac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

The transformation represented by matrix **AB** is the transformation represented by matrix **B** followed by the transformation represented by matrix **A**.

For matrices **A**, **B**:

$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

Algebra

$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$

For $ax^2 + bx + c = 0$ with roots α and β:

$\alpha + \beta = \frac{-b}{a} \qquad \alpha\beta = \frac{c}{a}$

For $ax^3 + bx^2 + cx + d = 0$ with roots α, β and γ:

$\sum \alpha = \frac{-b}{a} \qquad \sum \alpha\beta = \frac{c}{a} \quad \alpha\beta\gamma = \frac{-d}{a}$

Hyperbolic Functions

$\cosh x \equiv \frac{1}{2}(e^x + e^{-x})$

$\sinh x \equiv \frac{1}{2}(e^x - e^{-x})$

$\tanh x \equiv \frac{\sinh x}{\cosh x}$

Calculus and Differential Equations

Differentiation

Function	Derivative
x^n	nx^{n-1}
$\sin kx$	$k\cos kx$
$\cos kx$	$-k\sin kx$
$\sinh kx$	$k\cosh kx$
$\cosh kx$	$k\sinh kx$
e^{kx}	ke^{kx}
$\ln x$	$\frac{1}{x}$
$f(x)+g(x)$	$f'(x)+g'(x)$
$f(x)g(x)$	$f'(x)g(x)+f(x)g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$

Integration

Function	Integral
x^n	$\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\cos kx$	$\frac{1}{k}\sin kx+c$
$\sin kx$	$-\frac{1}{k}\cos kx+c$
$\cosh kx$	$\frac{1}{k}\sinh kx+c$
$\sinh kx$	$\frac{1}{k}\cosh kx+c$
e^{kx}	$\frac{1}{k}e^{kx}+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c,\ x\neq 0$
$f'(x)+g'(x)$	$f(x)+g(x)+c$
$f'(g(x))g'(x)$	$f(g(x))+c$

Area under a curve $= \int_a^b y\,dx\,(y \geq 0)$

Volumes of revolution about the x and y axes:

$$V_x = \pi\int_a^b y^2\,dx \qquad V_y = \pi\int_c^d x^2\,dy$$

Simple Harmonic Motion:

$$\ddot{x} = -\omega^2 x$$

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Vectors

$|x\mathbf{i}+y\mathbf{j}+z\mathbf{k}|=\sqrt{(x^2+y^2+z^2)}$

Scalar product of two vectors $\mathbf{a}=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$ and $\mathbf{b}=\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$ is

$$\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}.\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}=a_1b_1+a_2b_2+a_3b_3=|\mathbf{a}||\mathbf{b}|\cos\theta$$

where θ is the acute angle between the vectors **a** and **b**

The equation of the line through the point with position vector **a** parallel to vector **b** is:

$\mathbf{r}=\mathbf{a}+t\mathbf{b}$

The equation of the plane containing the point with position vector **a** and perpendicular to vector **n** is:

$(\mathbf{r}-\mathbf{a}).\mathbf{n}=0$

<u>Mechanics</u>

Forces and Equilibrium

Weight = mass $\times g$

Friction: $F \le \mu R$

Newton's second law in the form: $F=ma$

Kinematics

For motion in a straight line with variable acceleration:

$$v=\frac{\mathrm{d}r}{\mathrm{d}t} \qquad a=\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{\mathrm{d}^2r}{\mathrm{d}t^2}$$

$$r=\int v\,\mathrm{d}t \qquad v=\int a\,\mathrm{d}t$$

Statistics

The mean of a set of data: $\bar{x}=\frac{\sum x}{n}=\frac{\sum fx}{\sum f}$

The standard Normal variable: $Z=\frac{X-\mu}{\sigma}$ where $X \sim N(\mu, \sigma^2)$

Learners will be provided with the following formulae and statistical tables in each examination for A Level Further Mathematics B (MEI). Please note that the same formula booklet will be used for AS and A Level Further Mathematics B (MEI).

Contents

A Level Mathematics

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N}),$$

where $^nC_r = {}^nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

f(x)	f'(x)
tan kx	k sec² kx
sec x	sec x tan x
cot x	–cosec² x
cosec x	–cosec x cot x

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)}dx=\ln|f(x)|+c$$

$$\int f'(x)(f(x))^n dx=\frac{1}{n+1}(f(x))^{n+1}+c$$

Integration by parts $\int u\frac{dv}{dx}dx=uv-\int v\frac{du}{dx}dx$

Small Angle Approximations

$\sin\theta\approx\theta$, $\cos\theta\approx 1-\frac{1}{2}\theta^2$, $\tan\theta\approx\theta$ where θ is measured in radians

Trigonometric identities

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}\qquad (A\pm B\neq(k+\tfrac{1}{2})\pi)$$

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Numerical Methods

Trapezium rule: $\int_a^b y\,dx\approx\frac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+\ldots+y_{n-1})\}$, where $h=\frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x)=0$: $x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$P(A\cap B)=P(A)P(B\mid A)=P(B)P(A\mid B)$ **or** $P(A\mid B)=\frac{P(A\cap B)}{P(B)}$

Sample Variance

$$s^2=\frac{1}{n-1}S_{xx}\text{ where }S_{xx}=\sum(x_i-\bar{x})^2=\sum x_i^2-\frac{\left(\sum x_i\right)^2}{n}=\sum x_i^2-n\bar{x}^2$$

Standard deviation, $s=\sqrt{\text{variance}}$

The Binomial Distribution

If $X\sim B(n,p)$ then $P(X=r)={}^nC_r p^r q^{n-r}$ where $q=1-p$

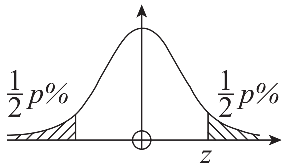
Mean of X is np

Hypothesis test for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two and three dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

-

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

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Core Pure

Complex Numbers

De Moivre's theorem:

$\{r(\cos\theta + \mathrm{i}\sin\theta)\}^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta)$

Roots of unity:

The roots of $z^n = 1$ are given by $z = \exp\left(\frac{2\pi k}{n}\mathrm{i}\right)$ for $k = 0, 1, 2, ..., n-1$

Vectors and 3-D geometry

Cartesian equation of a plane is

$n_1 x + n_2 y + n_3 z + d = 0$

Cartesian equation of a line in 3-D is

$$\frac{x-a_1}{d_1} = \frac{y-a_2}{d_2} = \frac{z-a_3}{d_3}$$

Vector product $\mathbf{a}\times\mathbf{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

$\mathbf{a}\times\mathbf{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}$ where $\mathbf{a}, \mathbf{b}, \hat{\mathbf{n}}$, in that order, form a right-handed triple.

Distance between skew lines is $\left|\frac{\mathbf{d}_1\times\mathbf{d}_2}{|\mathbf{d}_1\times\mathbf{d}_2|}.(\mathbf{a}_1 - \mathbf{a}_2)\right|$ where $\mathbf{a}_1$ is the position vector of a point on the first line and $\mathbf{d}_1$ is parallel to the first line, similarly for the second line.

Distance between point (x_1, y_1) and line $ax + by + c = 0$ is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

Distance between point (x_1, y_1, z_1) and plane $n_1x + n_2y + n_3z + d = 0$ is $\frac{|n_1x_1 + n_2y_1 + n_3z_1 + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$

Hyperbolic functions

$\cosh^2 x - \sinh^2 x = 1$

$\operatorname{arsinh} x = \ln[x + \sqrt{(x^2+1)}]$

$\operatorname{arcosh} x = \ln[x + \sqrt{(x^2-1)}],\ x \geq 1$

$\operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right),\ -1 < x < 1$

5

Calculus

$f(x)$	$f'(x)$
$\arcsin x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\dfrac{1}{1+x^2}$

$f(x)$	$\int f(x)\,dx$
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\arcsin\left(\dfrac{x}{a}\right)+c \quad (\lvert x\rvert < a)$
$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a}\arctan\left(\dfrac{x}{a}\right)+c$
$\dfrac{1}{\sqrt{a^2+x^2}}$	$\operatorname{arsinh}\left(\dfrac{x}{a}\right)+c$ or $\ln(x+\sqrt{x^2+a^2})+c$
$\dfrac{1}{\sqrt{x^2-a^2}}$	$\operatorname{arcosh}\left(\dfrac{x}{a}\right)+c$ or $\ln(x+\sqrt{x^2-a^2})+c \quad (x > a)$

The mean value of $f(x)$ on the interval $[a, b]$ is $\dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$

Area of sector enclosed by polar curve is $\dfrac{1}{2}\displaystyle\int r^2\,d\theta$

Series

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1), \quad \sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(r)}(0)}{r!}x^r + \ldots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^2}{r!} + \ldots \text{ for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots \; (-1 < x \le 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r\frac{x^{2r+1}}{(2r+1)!} + \ldots \text{ for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r\frac{x^{2r}}{(2r)!} + \ldots \text{ for all } x$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad (\lvert x\rvert < 1, n \in \mathbb{R})$$

Mechanics

Motion in a circle

For motion in a circle,

tangential velocity is $v = r\dot{\theta}$

radial acceleration is $\frac{v^2}{r}$ or $r\dot{\theta}^2$ towards the centre

tangential acceleration is $r\ddot{\theta}$

Further Pure with Technology

Numerical solution of differential equations

For $\frac{dy}{dx} = f(x, y)$:

Euler's method: $x_{n+1} = x_n + hx_n,\ y_{n+1} = y_n + hf(x_n, y_n)$

Modified Euler method (A Runge-Kutta method of order 2):

$k_1 = hf(x_n, y_n)$

$k_2 = hf(x_n + h, y_n + k_1)$

$x_{n+1} = x_n + h,\ y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2)$

Runge-Kutta method of order 4:

$k_1 = hf(x_n, y_n)$

$k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$

$k_3 = hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2})$

$k_4 = hf(x_n + h, y_n + k_3)$

$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

Gradient of tangent to a polar curve

For a curve $r = f(\theta)$, $\frac{dy}{dx} = \dfrac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$

5

Extra Pure

Multivariable calculus

$\nabla g = \textbf{grad } g = \begin{pmatrix} \frac{\partial g}{\partial x} \\ \frac{\partial g}{\partial y} \\ \frac{\partial g}{\partial z} \end{pmatrix}$. If $g(x, y, z)$ can be written as $z = f(x, y)$ then $\textbf{grad } g = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ -1 \end{pmatrix}$

Numerical Methods

Solution of equations

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

For the iteration $x_{n+1} = g(x_n)$ the relaxed iteration is $x_{n+1} = (1-\lambda)x_n + \lambda g(x_n)$.

Numerical integration

To estimate $\int_a^b f(x)\,dx$:

The midpoint rule:

$M_n = h(y_{\frac{1}{2}} + y_{\frac{3}{2}} + \ldots + y_{n-\frac{3}{2}} + y_{n-\frac{1}{2}})$ where $h = \dfrac{b-a}{n}$

The trapezium rule:

$T_n = \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$ where $h = \dfrac{b-a}{n}$

Simpson's rule

$S_{2n} = \frac{1}{3}h\{(y_0 + y_{2n}) + 4(y_1 + y_3 + \ldots + y_{2n-1}) + 2(y_2 + y_4 + \ldots + y_{2n-2})\}$

where $h = \dfrac{b-a}{2n}$.

These are related as follows:

$T_{2n} = \frac{1}{2}(M_n + T_n)$

$S_{2n} = \frac{1}{3}(2M_n + T_n) = \frac{1}{3}(4T_{2n} - T_n)$

Interpolation

Newton's forward difference interpolation formula:

$$f(x) = f(x_0) + \frac{(x-x_0)}{h}\Delta f(x_0) + \frac{(x-x_0)(x-x_1)}{2!h^2}\Delta^2 f(x_0) + \ldots$$

Lagrange's polynomial:

$$P_n(x) = \sum L_r(x)\,f(x_r) \text{ where } L_r(x) = \prod_{\substack{i=0 \\ i\neq r}}^{n} \frac{x - x_i}{x_r - x_i}$$

Statistics

Discrete distributions

X is a random variable taking values x_i in a discrete distribution with $P(X = x_i) = p_i$

Expectation: $\mu = E(X) = \sum x_i p_i$

Variance: $\sigma^2 = \text{Var}(X) = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2$

	Probability	$E(X)$	$\text{Var}(X)$
Uniform distribution over 1, 2, …, n	$P(X = r) = \frac{1}{n}$	$\frac{n+1}{2}$	$\frac{1}{12}(n^2 - 1)$
Geometric distribution	$P(X = r) = q^{r-1}p$ $q = 1 - p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson distribution	$P(X = r) = e^{-\lambda}\frac{\lambda^r}{r!}$		

Correlation and regression

For a sample of n pairs of observations (x_i, y_i)

$$S_{xx} = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n},\ S_{yy} = \sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n},\ S_{xy} = \sum x_i y_i - \frac{\sum x_i \sum y_i}{n}$$

product moment correlation coefficient: $r = \dfrac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \dfrac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sqrt{\left[\left(\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}\right)\left(\sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}\right)\right]}}$

least squares regression line of y on x is $y - \bar{y} = b(x - \bar{x})$ where $b = \dfrac{S_{xy}}{S_{xx}} = \dfrac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}}$

least squares regression line of x on y is $x - \bar{x} = b'(y - \bar{y})$ where $b' = \dfrac{S_{xy}}{S_{yy}} = \dfrac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}}$

Spearman's coefficient of rank correlation:

$$r_s = 1 - \frac{6\sum d_1^2}{n(n^2 - 1)}$$

5

5

Confidence intervals

To calculate a confidence interval for a mean or difference in mean in different circumstances, use the given distribution to calculate the critical value, *k*.

To estimate…	Confidence interval	Distribution
a mean	$\bar{x} \pm k\frac{\sigma}{\sqrt{n}}$	N(0, 1)
a mean	$\bar{x} \pm k\frac{s}{\sqrt{n}}$	t_{n-1}
difference in mean of paired populations	treat differences as a single distribution	

Hypothesis tests

Description	Test statistic	Distribution
Pearson's product moment correlation test	$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sqrt{\left[\left(\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}\right)\left(\sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}\right)\right]}}$	
Spearman's rank correlation test	$r_s = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$	
χ^2 test	$\sum \frac{(f_o - f_e)^2}{f_e}$	χ^2_v
Normal test for a mean	$\frac{\bar{x} - \mu}{\left(\sigma/\sqrt{n}\right)}$	N(0, 1)
t-test for a mean	$\frac{\bar{x} - \mu}{\left(s/\sqrt{n}\right)}$	t_{n-1}
Wilcoxon single sample test	A statistic *T* is calculated from the ranked data	

Continuous distributions

X is a continuous random variable with probability density function (pdf) $\mathrm{f}(x)$

Expectation: $\mu = \mathrm{E}(X) = \int x\mathrm{f}(x)\,\mathrm{d}x$

Variance: $\sigma^2 = \mathrm{Var}(X) = \int (x-\mu)^2 \mathrm{f}(x)\,\mathrm{d}x = \int x^2 \mathrm{f}(x)\,\mathrm{d}x - \mu^2$

Cumulative distribution function $\mathrm{F}(x) = \mathrm{P}(X \le x) = \int_{-\infty}^{x} \mathrm{f}(t)\,\mathrm{d}t$

	$\mathrm{E}(X)$	$\mathrm{Var}(X)$
Continuous uniform distribution over $[a, b]$	$\frac{a+b}{2}$	$\frac{1}{12}(b-a)^2$

5

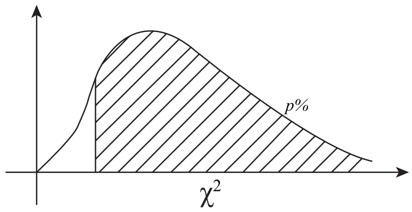
Critical values for the product moment correlation coefficient, r

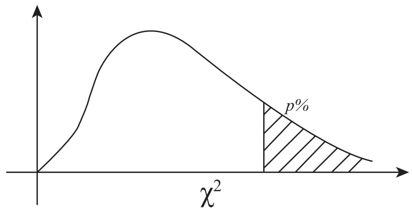
1-Tail Test	5%	2½%	1%	½%	1-Tail Test	5%	2½%	1%	½%
2-Tail Test	**10%**	**5%**	**2%**	**1%**	**2-Tail Test**	**10%**	**5%**	**2%**	**1%**
n					*n*				
1	–	–	–	–	31	0.3009	0.3550	0.4158	0.4556
2	–	–	–	–	32	0.2960	0.3494	0.4093	0.4487
3	0.9877	0.9969	0.9995	0.9999	33	0.2913	0.3440	0.4032	0.4421
4	0.9000	0.9500	0.9800	0.9900	34	0.2869	0.3388	0.3972	0.4357
5	0.8054	0.8783	0.9343	0.9587	35	0.2826	0.3338	0.3916	0.4296
6	0.7293	0.8114	0.8822	0.9172	36	0.2785	0.3291	0.3862	0.4238
7	0.6694	0.7545	0.8329	0.8745	37	0.2746	0.3246	0.3810	0.4182
8	0.6215	0.7067	0.7887	0.8343	38	0.2709	0.3202	0.3760	0.4128
9	0.5822	0.6664	0.7498	0.7977	39	0.2673	0.3160	0.3712	0.4076
10	0.5494	0.6319	0.7155	0.7646	40	0.2638	0.3120	0.3665	0.4026
11	0.5214	0.6021	0.6851	0.7348	41	0.2605	0.3081	0.3621	0.3978
12	0.4973	0.5760	0.6581	0.7079	42	0.2573	0.3044	0.3578	0.3932
13	0.4762	0.5529	0.6339	0.6835	43	0.2542	0.3008	0.3536	0.3887
14	0.4575	0.5324	0.6120	0.6614	44	0.2512	0.2973	0.3496	0.3843
15	0.4409	0.5140	0.5923	0.6411	45	0.2483	0.2940	0.3457	0.3801
16	0.4259	0.4973	0.5742	0.6226	46	0.2455	0.2907	0.3420	0.3761
17	0.4124	0.4821	0.5577	0.6055	47	0.2429	0.2876	0.3384	0.3721
18	0.4000	0.4683	0.5425	0.5897	48	0.2403	0.2845	0.3348	0.3683
19	0.3887	0.4555	0.5285	0.5751	49	0.2377	0.2816	0.3314	0.3646
20	0.3783	0.4438	0.5155	0.5614	50	0.2353	0.2787	0.3281	0.3610
21	0.3687	0.4329	0.5034	0.5487	51	0.2329	0.2759	0.3249	0.3575
22	0.3598	0.4227	0.4921	0.5368	52	0.2306	0.2732	0.3218	0.3542
23	0.3515	0.4132	0.4815	0.5256	53	0.2284	0.2706	0.3188	0.3509
24	0.3438	0.4044	0.4716	0.5151	54	0.2262	0.2681	0.3158	0.3477
25	0.3365	0.3961	0.4622	0.5052	55	0.2241	0.2656	0.3129	0.3445
26	0.3297	0.3882	0.4534	0.4958	56	0.2221	0.2632	0.3102	0.3415
27	0.3233	0.3809	0.4451	0.4869	57	0.2201	0.2609	0.3074	0.3385
28	0.3172	0.3739	0.4372	0.4785	58	0.2181	0.2586	0.3048	0.3357
29	0.3115	0.3673	0.4297	0.4705	59	0.2162	0.2564	0.3022	0.3328
30	0.3061	0.3610	0.4226	0.4629	60	0.2144	0.2542	0.2997	0.3301

Critical values for Spearman's rank correlation coefficient, r_s

1-Tail Test	5%	2½%	1%	½%	1-Tail Test	5%	2½%	1%	½%
2-Tail Test	**10%**	**5%**	**2%**	**1%**	**2-Tail Test**	**10%**	**5%**	**2%**	**1%**
n					*n*				
1	–	–	–	–	31	0.3012	0.3560	0.4185	0.4593
2	–	–	–	–	32	0.2962	0.3504	0.4117	0.4523
3	–	–	–	–	33	0.2914	0.3449	0.4054	0.4455
4	1.0000	–	–	–	34	0.2871	0.3396	0.3995	0.4390
5	0.9000	1.0000	1.0000	–	35	0.2829	0.3347	0.3936	0.4328
6	0.8286	0.8857	0.9429	1.0000	36	0.2788	0.3300	0.3882	0.4268
7	0.7143	0.7857	0.8929	0.9286	37	0.2748	0.3253	0.3829	0.4211
8	0.6429	0.7381	0.8333	0.8810	38	0.2710	0.3209	0.3778	0.4155
9	0.6000	0.7000	0.7833	0.8333	39	0.2674	0.3168	0.3729	0.4103
10	0.5636	0.6485	0.7455	0.7939	40	0.2640	0.3128	0.3681	0.4051
11	0.5364	0.6182	0.7091	0.7545	41	0.2606	0.3087	0.3636	0.4002
12	0.5035	0.5874	0.6783	0.7273	42	0.2574	0.3051	0.3594	0.3955
13	0.4835	0.5604	0.6484	0.7033	43	0.2543	0.3014	0.3550	0.3908
14	0.4637	0.5385	0.6264	0.6791	44	0.2513	0.2978	0.3511	0.3865
15	0.4464	0.5214	0.6036	0.6536	45	0.2484	0.2945	0.3470	0.3822
16	0.4294	0.5029	0.5824	0.6353	46	0.2456	0.2913	0.3433	0.3781
17	0.4142	0.4877	0.5662	0.6176	47	0.2429	0.2880	0.3396	0.3741
18	0.4014	0.4716	0.5501	0.5996	48	0.2403	0.2850	0.3361	0.3702
19	0.3912	0.4596	0.5351	0.5842	49	0.2378	0.2820	0.3326	0.3664
20	0.3805	0.4466	0.5218	0.5699	50	0.2353	0.2791	0.3293	0.3628
21	0.3701	0.4364	0.5091	0.5558	51	0.2329	0.2764	0.3260	0.3592
22	0.3608	0.4252	0.4975	0.5438	52	0.2307	0.2736	0.3228	0.3558
23	0.3528	0.4160	0.4862	0.5316	53	0.2284	0.2710	0.3198	0.3524
24	0.3443	0.4070	0.4757	0.5209	54	0.2262	0.2685	0.3168	0.3492
25	0.3369	0.3977	0.4662	0.5108	55	0.2242	0.2659	0.3139	0.3460
26	0.3306	0.3901	0.4571	0.5009	56	0.2221	0.2636	0.3111	0.3429
27	0.3242	0.3828	0.4487	0.4915	57	0.2201	0.2612	0.3083	0.3400
28	0.3180	0.3755	0.4401	0.4828	58	0.2181	0.2589	0.3057	0.3370
29	0.3118	0.3685	0.4325	0.4749	59	0.2162	0.2567	0.3030	0.3342
30	0.3063	0.3624	0.4251	0.4670	60	0.2144	0.2545	0.3005	0.3314

Percentage points of the χ^2(chi-squared) distribution

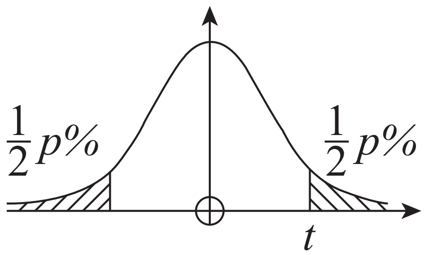




p%	99	97.5	95	90	10	5	2.5	1	0.5
ν = 1	.0001	.0010	.0039	.0158	2.706	3.841	5.024	6.635	7.879
2	.0201	.0506	0.103	0.211	4.605	5.991	7.378	9.210	10.60
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34	12.84
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28	14.86
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09	16.75
6	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81	18.55
7	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48	20.28
8	1.646	2.180	2.733	3.490	13.36	15.51	17.53	20.09	21.95
9	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67	23.59
10	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21	25.19
11	3.053	3.816	4.575	5.578	17.28	19.68	21.92	24.72	26.76
12	3.571	4.404	5.226	6.304	18.55	21.03	23.34	26.22	28.30
13	4.107	5.009	5.892	7.042	19.81	22.36	24.74	27.69	29.82
14	4.660	5.629	6.571	7.790	21.06	23.68	26.12	29.14	31.32
15	5.229	6.262	7.261	8.547	22.31	25.00	27.49	30.58	32.80
16	5.812	6.908	7.962	9.312	23.54	26.30	28.85	32.00	34.27
17	6.408	7.564	8.672	10.09	24.77	27.59	30.19	33.41	35.72
18	7.015	8.231	9.390	10.86	25.99	28.87	31.53	34.81	37.16
19	7.633	8.907	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	8.260	9.591	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.897	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	9.542	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
35	18.51	20.57	22.47	24.80	46.06	49.80	53.20	57.34	60.27
40	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
100	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

5

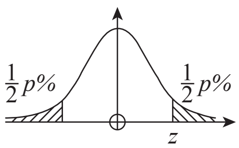
Percentage points of the *t* distribution



v \ $p\%$	10	5	2	1
1	6.314	12.71	31.82	63.66
2	2.920	4.303	6.965	9.925
3	2.353	3.182	4.541	5.841
4	2.132	2.776	3.747	4.604
5	2.015	2.571	3.365	4.032
6	1.943	2.447	3.143	3.707
7	1.895	2.365	2.998	3.499
8	1.860	2.306	2.896	3.355
9	1.833	2.262	2.821	3.250
10	1.812	2.228	2.764	3.169
11	1.796	2.201	2.718	3.106
12	1.782	2.179	2.681	3.055
13	1.771	2.160	2.650	3.012
14	1.761	2.145	2.624	2.977
15	1.753	2.131	2.602	2.947
20	1.725	2.086	2.528	2.845
30	1.697	2.042	2.457	2.750
50	1.676	2.009	2.403	2.678
100	1.660	1.984	2.364	2.626
∞	1.645	1.960	2.326	2.576

= percentage points of the Normal distribution N(0, 1)

Percentage points of the Normal distribution



p	10	5	2	1
z	1.645	1.960	2.326	2.576

Critical values for the Wilcoxon Single Sample test									
1-tail 2-tail	5% 10%	2½% 5%	1% 2%	½% 1%	1-tail 2-tail	5% 10%	2½% 5%	1% 2%	½% 1%
n					*n*				
					26	110	98	84	75
2	–	–	–	–	27	119	107	92	83
3	–	–	–	–	28	130	116	101	91
4	–	–	–	–	29	140	126	110	100
5	0	–	–	–	30	151	137	120	109
6	2	0	–	–	31	163	147	130	118
7	3	2	0	–	32	175	159	140	128
8	5	3	1	0	33	187	170	151	138
9	8	5	3	1	34	200	182	162	148
10	10	8	5	3	35	213	195	173	159
11	13	10	7	5	36	227	208	185	171
12	17	13	9	7	37	241	221	198	182
13	21	17	12	9	38	256	235	211	194
14	25	21	15	12	39	271	249	224	207
15	30	25	19	15	40	286	264	238	220
16	35	29	23	19	41	302	279	252	233
17	41	34	27	23	42	319	294	266	247
18	47	40	32	27	43	336	310	281	261
19	53	46	37	32	44	353	327	296	276
20	60	52	43	37	45	371	343	312	291
21	67	58	49	42	46	389	361	328	307
22	75	65	55	48	47	407	378	345	322
23	83	73	62	54	48	426	396	362	339
24	91	81	69	61	49	446	415	379	355
25	100	89	76	68	50	466	434	397	373

5e. Software for Further Pure with Technology (Y436)

Learners require access to a computer and/or calculator with suitable software in the examination for Further Pure with Technology (Y436). The software must include a computer algebra system, a graph plotter and a spreadsheet. The software used must be from the list of software approved by OCR – see below.

The computer algebra system should be able to perform at least all the algebraic requirements of A level Further Mathematics, and should be able to differentiate and integrate analytically functions where solutions are known to exist in terms of elementary functions.

The graphing software should be able to plot families of curves in cartesian, polar and parametric forms, with sliders (or equivalent) for parameters. The graphing software should be able to draw tangent fields for families of differential equations given in the form $\frac{dy}{dx} = f(x, y)$.

The spreadsheet should feature the ability to enter formulae based on cell references using the notation A, B, C ... for columns and 1,2,3 ... for rows.

The software approved for use in the examination is as follows.

Approved software for use in the examination for Further Pure with Technology (Y436) – this version updated March 2017

Geogebra (v5 or later)

TI-Nspire software (any model with CAS);

CASIO ClassPad software (any model with CAS)

Mathematica (v11.0 or later)

Maple (v2016 or later)

Excel (any version)

Gnumeric (any version)

Apache OpenOffice spreadsheet (v4 or later)

Learners are also required to use a programming language in the examination. The programming language must have capacity for: checking conditions; looping through values; local/global variables; inputting/outputting variables.

Any examination question which requires a learner to create, adapt or interpret a program must be answered using a programming language from the following list.

Approved programming language(s) for use in the examination for Further Pure with Technology (Y436) – this version updated March 2017

Python (v 3.6 or later)

The lists of approved software and programming languages may be updated. If there are any updates, these will be communicated to centres annually prior to first teaching each year, and will be available at www.ocr.org.uk.

Summary of updates

Date	Version	Section	Title of section	Change
June 2018	1.1	Front cover	Disclaimer	Addition of Disclaimer
September 2019	1.2	Multiple		Correction of minor typographical errors

YOUR CHECKLIST

Our aim is to provide you with all the information and support you need to deliver our specifications.

- ☐ Bookmark **ocr.org.uk/alevelfurthermathsmei** for all the latest resources, information and news on A Level Further Mathematics B (MEI)
- ☐ Be among the first to hear about support materials and resources as they become available – register for AS/A Level GCE Maths updates at **ocr.org.uk/updates**
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- ☐ Learn more about Active Results at **ocr.org.uk/activeresults**
- ☐ Join our Maths social network community for teachers at **social.ocr.org.uk**
- ☐ In addition to the support available from OCR, MEI provides advice and professional development relating to all the curriculum and teaching aspects of the course. It also provides teaching and learning resources, which can be found at **mei.org.uk**

Download high-quality, exciting and innovative A Level Further Mathematics B (MEI) resources from ocr.org.uk/alevelfurthermathsmei

Resources and support for our A Level Further Maths qualifications, developed through collaboration between our Maths Subject Advisors, teachers and other subject experts, are available from our website. You can also contact our Maths Subject Advisors who can give you specialist advice, individual service, guidance and support.

Contact the team at:
01223 553998
maths@ocr.org.uk
@OCR_Maths

To stay up to date with all the relevant news about our qualifications, register for email updates at **ocr.org.uk/updates**

Maths Community

The social network is a free platform where teachers can engage with each other – and with us – to find and offer guidance, discover and share ideas, best practice and a range of further maths support materials. To sign up, go to **social.ocr.org.uk**

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