GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

Report on the Units

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REPORT ON THE UNITS

4751: Introduction to Advanced Mathematics (C1)

General Comments

A full spread of marks was seen, with candidates usually attempting all parts of all questions, although occasionally question 13 petered out, perhaps because of time spent earlier on long methods.

The examiners were concerned, as last year, about the long tail of very weak candidates entered for this examination. In contrast, there were excellent scripts seen from those who had learnt the algebraic techniques in the specification and were able to apply them with confidence.

Comments on Individual Questions

Section A

1 Many gained full marks here, although some weak candidates did not have a good strategy for solving inequalities. Those who collected terms to obtain $-3 < 5x$ were usually more successful than those who chose $-5x < 3$ and then often failed to reverse the inequality when dividing by a negative number.

2 There were many correct answers to this rearrangement, although some were careless in their positioning of the square root sign, not making it clear that it included the numerator as well as denominator. Candidates who obtained $\sqrt{\dfrac{s}{\frac{1}{2}a}}$ were expected to simplify this. Weak candidates often made errors in the first step of the rearrangement – for instance those who found the square root as their first step rarely did so correctly.

3 This question exposed a limited understanding of $\Rightarrow$ and $\Leftarrow$, although the wording of the question helped many to write the converse. Most candidates gained the second mark though citing a counter-example rather than using the argument that an even number multiplied by 2 is also even.

4 Some did not appreciate the significance of F(0) = 6 and so were unable to find c. A common error was to get as far as f(2) = 8 + 2k + 6 but then not equate this to 0, often using 6 instead. As usual in this type of question, some evaluated f(−2) instead of f(2).

5 The first part of this question on indices was often well done. The main errors were $4x^9y$ instead of $4x^4y$ or problems in finding $\sqrt[3]{64}$. In part (ii), many candidates did not cope well with the negative index. Few used $\left(\dfrac{2}{1}\right)^5$ and a common wrong answer was $\dfrac{1}{32}$ instead of 32. Some weaker candidates used decimals and spent a while calculating 0.03125, for which they received no credit.

6 A few candidates did not attempt to solve this expansion, and some spent unnecessary time in calculating all the terms rather than just the requested term in x^3. As expected with this topic, the most common error was to forget to cube the coefficient of x, although coping correctly with the other constituent parts, so that an answer of (−)180 instead of −720 was common.

7 This question produced a good spread of marks. Those who wrongly 'cancelled' $4x$ and $2x$ at the start did not gain any credit. After a correct first couple of steps, many made errors in proceeding from $10x + 5 = 0$, with 0.5 and ±2 being common wrong answers.

8 In the first part, most candidates realised that they had to break down the 98 and 50 into factors, but some became muddled with their roots, giving $2\sqrt{5}$ instead of $5\sqrt{2}$, for example. A common wrong answer from weaker candidates was $\sqrt{48}$.

In the second part, many realised the need to multiply both numerator and denominator by $2 - \sqrt{5}$, but errors in doing so were frequent.

9 The first part was often correct, but many candidates ignored the hint and gave only one root of $x^2 = 25$. The second part was poorly done, with most candidates not giving the correct equation of the translated curve. Common errors were $y = x^2 - 2$, $y = x^2 - 6$, $y = (x^2 - 2) - 4$.

10 A surprising number of candidates did not know where to start in part (i), in spite of the diagram given. Those who did correctly start with the area of a triangle being ½ base × height were usually able to proceed correctly to gain their two marks.

In part (ii), some candidates used the quadratic formula, in spite of the instruction to solve by factorising. However, most managed to factorise correctly and realised that only the positive value of x yielded a practical result.

Section B

11 (i) Most candidates worked confidently towards the given answer and gained full marks. Some of the weaker candidates just showed that (3, 7) was on the given line; others tried a bit of working backwards and were sometimes then successful in appreciating they needed to show independently that the gradient of AC was 2 and the gradient of the tangent therefore $-\frac{1}{2}$. This part was one of the best done questions in section B.

(ii) Those who chose to eliminate rather than substitute for y often made errors, particularly those who found both equations in terms of y and did not cope with the resulting fractions. However, many gained full marks here.

(iii) Many candidates knew what to do, but errors in substitution into the equation of the circle and subsequent simplification were frequent. As a result, many candidates gained only the method marks in this part. Of those who successfully found the intersection as (5, 1), some did not mention that the equal roots showed that the line was a tangent.

12 This was probably the best-attempted section B question as a whole, with the more able candidates finding it straightforward and weaker candidates doing various of the parts independently of each other.

(i) This part caused most problems, with many not able to cope with the coefficient of x^2 not being 1. Common wrong answers $4(x - 3)^2 - 9/4$, $4(x - 12)^2 - 117$ and $4(x - 3)^2 + 18$. In contrast were those who gave the correct answer with little or no working.

(ii) Most candidates used part (i) and were allowed full marks follow through. Some started again and were able to gain full credit by using calculus or the line of symmetry of the graph.

(iii) Those who followed part (i) had problems if they had $4(x - 3)^2 + 18$, but others were able to proceed, although some did not cope with the 4 when taking the square root. The large number of candidates who used the quadratic formula had some hefty arithmetic to do without a calculator and many made errors in doing so. Most successful were those who used factorisation correctly.

(iv) Many candidates now had conflicting information for their graph, but most ignored any that did not appear to fit a parabola-shaped curve. Some candidates started again and constructed a table of values. Where there were two roots given in part (iii), a follow-through was allowed for the intersections with the x-axis. Those who used graph paper often omitted the y intercept due to their scale.

13 (i) Most candidates made a correct attempt to multiply out; some showed the given result successfully by long division. However, not many candidates fully showed $x = 3$ to be the only real root. Some showed that $f(3) = 0$, and others that the discriminant of the quadratic factor was negative, but few did both. A number of candidates said that the root was $(x - 3)$.

(ii) Most candidates started this part by substituting to show that f(2) = −22 or that f(2) + 22 = 0. For weaker candidates, this was often as far as they got. Better candidates often went back to long division of f(x) + 22 and successfully obtained the quadratic factor and hence the roots, using factorisation or the formula.

(iii) Some good graphs of y = f(x) were seen which gained all 3 marks, but frequently, candidates used the results in part (ii) as if they were roots of y = f(x) not y = f(x) + 22, drawing a graph that gained only the first mark. Despite this, a few of these candidates realised that the graph should cross the axes at (3, 0) and (0, −12) and gained another mark for showing this.

4752: Concepts for Advanced Mathematics (C2)

General Comments

There was a full range of achievement, and there were many excellent scripts. However, a significant minority of candidates were evidently not ready for the examination, and scored very few marks. Section B was generally better received than section A. Most candidates set their work out clearly. Nevertheless, many marks were lost by failing to show sufficient detail of the method – simply providing a statement of an answer to a question worth 3 or 4 marks generally scores zero.

Comments on Individual Questions

Section A

1 The majority of candidates struggled with part (i) – most wrote down as many decimal places as they could from their calculator, and scored zero. However, some weak candidates did gain the first mark – evidently because they had a calculator which would deal with surds.
Part (ii) was generally done well, although a number of candidates lost marks by failing to cancel the fraction down to its lowest terms.

2 This question was very well done, with the majority scoring full marks. Some candidates lost the final mark because they failed to show the intermediate step.

3 Both parts defeated many candidates. A good number lost marks through poor notation, such as omitting "y =". In most of the better scripts full marks was awarded.

4 Many candidates were evidently not familiar with the inductive definition in part (i), and treated it as an algebraic definition. In part (ii), some candidates simply added together the three terms from part (i) (scoring zero), and a good number apparently had no idea what to do. Some candidates correctly identified 2, 6 and 12, but neglected to find the sum.

5 Most candidates used the correct formula to find θ = 0.72 correctly, but often spoiled their answer by writing it as 0.72π, or multiplying by $\frac{180}{\pi}$. Many then recovered to score full marks in the second part. A significant minority made poor use of the formula book and tried to use $\int \frac{1}{2}r^2 d\theta$.

6 Most scored full marks in part (i), although some candidates decided to write their own question, and evaluate (for example) $\log_{10}1$ and $\log_{10}10$. Only those who made it clear that they understood the results for the general case scored full marks here.
Part (ii) defeated the majority. Most scored one mark for correctly applying the third law of logarithms, but were then unable to apply the second law correctly because of the minus sign, or the 2, or both in $-2\log_a \frac{x^3}{4}$.

7 Marks were thrown away in part (i) by only showing one quadrant or failing to show the y-intercept clearly. Many candidates unnecessarily produced a large table of results and an accurate plot on graph paper, thus wasting valuable time. Full marks were awarded for the correct shape in both quadrants and a clear indication that the curve passes through (0,1).
Most scored full marks in part (ii), a small number of candidates scored zero by finding $\sqrt[3]{20}$.

8 Many candidates attempted to manipulate the given expression without using Pythagoras, scoring zero. Those who did use Pythagoras often used poor notation such as $\cos^2 + \sin^2 = 1$. In the second part a significant minority elected to use the quadratic formula, thus throwing away the first two marks. These candidates often stopped at $\sin\theta = ½$, $\sin\theta = 3$, scoring a total of 0. However, many were able to score full marks here, although some made the basic error of leaving their calculator in radian mode and losing the final two marks.

Section B

9 (i) Most candidates found $\frac{dy}{dx}$ correctly, and then the gradient of the curve at x = 3. Many went on to score full marks, but a common error was to find the equation of the line through (-1,-41) with gradient 12, and then show that this passes through (-1,-41).

(ii) Most candidates did very well here, although there were occasional errors in finding the appropriate y-values.

(iii) A good number of candidates did not see the relevance of the work they had done in part (ii), and started again by producing a table of results and an accurate plot on graph paper. This was unnecessary for full marks – a cubic with correct intercepts and turning points indicated scored full marks.

10 (i) Most candidates scored full marks for this part. In a small number of cases candidates lost marks by failing to use the table, and reading values incorrectly from the curve, or by omitting a pair of brackets, or by using the wrong value for h. A few candidates substituted x-values in, all the way through, scoring zero.

(ii) Those who produced a sketch generally obtained the first mark, but using appropriate rectangles defeated many. A common approach was to use values which were not taken from the curve.

(iii) Most obtained the value 19.5 correctly, but then failed to obtain the second mark because they evaluated $\frac{0.5}{19.5} \times 100$. Some candidates obtained t = 10, speed = 16.5 and then seemed to think that the difference of 2.5 was less than 3% of 19. It was surprising that there was usually no attempt to go back and check their work.

(iv) This was done very well, with some candidates scoring full marks here when the rest of the question was inaccessible to them. There were problems with the first term, however, with many writing 28x or 28^2.

11 (a) The vast majority of candidates scored full marks on this part of the question.

(b) Parts (i) and (ii) were very well done, with most scoring full marks. However, part (iii) defeated most – only the best were able to use logarithms correctly, and very few changed the inequality at the appropriate place. Surprisingly few candidates obtained the correct value of n. Many wrote n > 29.06, scoring zero.

4753: Concepts in Pure Mathematics (C3)

General Comments

This paper proved to be accessible to the full range of candidates, many of whom scored over 50 marks. There were plenty of relatively straightforward marks available, and scores of less than 20 were uncommon. Candidates rarely had trouble completing the paper. In general, the core calculus topics appear to be well known by candidates. The topics which seem to cause problems are modulus, inverse trig functions, proof and disproof, implicit differentiation, and function language (domain and range etc). This paper seems to have had relatively accessible questions on most of these topics.

The general standard of presentation was mixed, and the usual issues of poor notation and algebra are still evident in some scripts. In particular, omitting brackets in expressions like the quotient rule remains common, and the conventions of calculus are quite often inaccurately applied.

One particular issue with regard to this paper is the use of graph paper. We have considered removing this from the rubric: it is almost always the case that using graph paper in this paper for sketches is less efficient than not. However, we realise that some candidates like to be able to draw accurate sketches. We recommend, however, that centres do not automatically issue graph paper to all candidates, as this encourages its use when it is unnecessary.

Comments on Individual Questions

Section A

1 Part (i) was very well answered, with the majority of candidates scoring three marks. Weaker candidates often achieved the 'B' mark for ½ $u^{-1/2}$ without multiplying this by 2.

Part (ii) proved to be a harder test, with many candidates tempted into 'fudges' to get from $1 - e^{-x}$ to $e^x - 1$ in the denominator. Again, weaker candidates scored the 'B' mark for differentiating $\ln(1 - e^{-x})$ to get $\frac{1}{1-e^{-x}}$. Only the better candidates achieved the final 'E' mark for showing that $\frac{e^{-x}}{1-e^{-x}} = \frac{1}{e^x - 1}$ (although this statement without working was allowed).

2 This question was well done generally. The composite function was usually correct, though some candidates did 'f' and 'g' in the wrong order to get $1 - |x|$. We needed some evidence of the values of the intercepts with the axes for the first sketch, though condoned their omission from the sketch of gf(x). Many candidates used graph paper here – we would encourage candidates not to do this for sketches, and therefore discourage centres from routinely handing out graph paper for this paper.

3 Most candidates achieved full marks for part (i). There were instances of sloppy notation, for example $\frac{dy}{dx} = 4y\frac{dy}{dx} + \frac{dy}{dx} = 18x$, which were usually condoned. However, implicit differentiation does of course require accurate deployment of d*y*'s and d*x*'s.

Part (ii) was less successful, with weaker candidates unable to derive or solve the quadratic in *y*. Nevertheless, most candidates scored the first five marks, and many scored full marks.

4 This was another well answered question. There were some generous 'M' marks here, and even poor solutions often achieved 4 or 5 marks out of 8. It is pleasing to note that the large majority of candidates handled the use of logarithms to solve the equations successfully, and the modal mark was 8 out of 8.

5 'Proof' questions have been found difficult in recent papers, but this two-marker proved to be very accessible, with most understanding the concept of counter-example. Some arithmetic errors were made in evaluating the quadratic expression. The most common counter-example was naturally $n = 6$, but $n = 11$ also proved quite popular. Occasionally, candidates went off the rails by trying to do some algebra with the quadratic expression.

6 This proved to be the most demanding of the section A questions – in general, candidates do not find questions on inverse trigonometric functions easy. The first part, in particular, was not well done – many could not find the range of arctan *x* and, even if successful with this, failed to handle arctan ½ *x*. Some then lost marks for using ≤ rather than < in the inequality.

In part (ii), there are still candidates who mix up $\tan^{-1} x$ with $1/\tan x$. In a substantial number of solutions we found $f^{-1}(x) = 2 \tan x$ instead of tan 2*x* – we awarded an M1 mark for this for reversing the arctan. The derivative of tan 2*x* was often not known, with many candidates starting from scratch using a quotient rule on sin 2*x* /cos 2*x*.

In awarding the mark in part (iii), we allowed where possible follow-through on their answers from part (ii). Many changed the sign of the reciprocal, confusing this situation with the condition for perpendicularity of gradients.

Section B

7 There were plenty of easy marks available here, and many weaker candidates scored reasonably on this question.

(i) Most candidates realised that the asymptote occurs when the denominator is zero, and there were many fully correct solutions. However, there was some carelessness with signs, either in re-arranging the equation $1 + 2x^3 = 0$, or in dropping the minus sign subsequently.

(ii) This was a straightforward quotient rule, made easier by the answer being given. We condoned omitted brackets in the numerator (but reserve the right not to do so in future!). However, quotient rules starting $u\,\mathrm{d}v/\mathrm{d}x - v\,\mathrm{d}u/\mathrm{d}x$ achieved M0.

There were plenty of algebraic errors in finding the turning points. The most common mistake was to find an extra turning point, either by setting the denominator to zero, or from $x^3 = 1 \Rightarrow x = \pm 1$. Other errors were $2x - 2x^4 = 2x(1 - x^2)$ or $2x - 2x^4 = 2x(1 - 2x^3)$.

(iii) This proved to be an easy 5 marks for most candidates, either by inspection or substituting $u = 1 + 2x^3$. However, there was quite a lot of sloppy notation, including omitting dx or du, limits incompatible with variable, etc. We would encourage teachers to advise students that this can cause marks to be penalised, although on this paper the mark scheme was generous.

A minority of candidates tried integration by parts here, losing time in the process.

8 This was less well done overall than question 7. Weak candidates still managed to 'cherry-pick' a few marks here and there, but completely correct solutions were relatively few.

(i) This was not as well done as might have been predicted. Using degrees as the default instead of radians was quite common – 45° scored M1M1A0.

(ii) Most scored one mark for $f(-x) = -x\cos(-2x)$. To achieve the 'E' mark required clearly equating this to an expression for $-f(x)$. Attempts based on individual points achieved no marks. The 'B' mark was better done, though quite a few candidates took 'odd' to mean 'not even'.

(iii) The product rule was well done, though some candidates omitted the '2' from the derivative of cos 2x.

(iv) Spotting the connection $\tan 2x = \dfrac{\sin 2x}{\cos 2x}$ was common, and many candidates got the algebra correct.

(v) Most candidates substituted $x = 0$ into their derivative to achieve the first mark – follow-through was allowed on product rule expressions here. However, many missed the further application of the product rule required for the second derivative, or made errors in expanding the bracket, and thereby lost the final 'E' mark, which required them to use the correct second derivative.

(vi) Integration by parts proved to be familiar territory for candidates, even if they had struggled with the differentiation earlier. Sign errors in integrating sin and cos were quite common, and only the better candidates managed to obtain a correct, exact answer. The interpretation of the result as an area, either described verbally or by sketch, was well done, though some candidates tried to link this to the 'oddness' of the function.

4754: Applications of Advanced Mathematics (C4)

General Comments

This summer the questions in Paper A proved to be accessible to almost all students with correct solutions to all questions seen. Some candidates failed to complete question 8 so there may have been a shortage of time. All candidates scored well on Section A and found parts of the Section B questions more challenging. The Comprehension was the least well answered question and scored low marks in general compared with the rest of the paper.

It was pleasing to see that candidates did not make some of the mistakes that they did in earlier papers. The correct formula for the trapezium rule was used most of the time, unlike in the January 2007 paper, and efficient methods were used generally except in 8(ii). The most disappointing factors were:-

- failing to put a constant of integration in indefinite integrals
- failing to give full stages of verification in order to establish given answers
- poor algebraic skills, including sign errors and the absence of brackets.

Comments on Individual Questions

Paper A

Section A

1 The first part- using the 'R' method- was well answered. Most errors arose from attempts to quote results rather than working from first principles.
In the second part most candidates used the correct method but most only found the solution 90° and omitted the final solution. Those that realised an extra solution was expected often gave 270° instead of 233.1°. It was disappointing to note that in the second part some candidates still incorrectly expanded sin(A+B) as sinA+sinB.

2 Most candidates wrote down the expected normal vectors - although some used them without clearly stating them. The scalar product was usually well answered with full working shown. The most common error was in not stating that they had shown the planes were in fact perpendicular. There were a large proportion of fully correct solutions to this question.

3 (i) Candidates did not always give clear reasons why the given formula was the required volume of revolution. Stages were too often missed out and this caused marks to be lost as the answer was given in the question. For example, some did not clearly state that they were starting from $\int \pi x^2 dy$ and others omitted the limits.

(ii) This part was more successful although candidates sometimes multiplied by two rather than dividing when integrating. Others failed to give their answer in exact form or failed to evaluate e^0.

4 Most candidates rearranged the equation to give *t* in terms of *x* and then substituted this in the equation for *y*. The first part was usually completed successfully but there were some confused attempts at eliminating the subsidiary fractions. Their basic algebra was disappointing. For example 2+1/(*x*+1) was too often changed to 2+*x*+1.Other methods were possible and successful but much less common.

5 The verification was attempted in a variety of ways and often successfully. The most common involved finding the values of *λ* and *μ* first and then substituting to show that these satisfied all the coordinates. The most common mistake was to not show the full verification of these coordinates.
Although the method for the second part was well understood many candidates chose the incorrect vectors- usually the position vectors-in order to find the required angle. Another common error was to omit the negative sign in the vector 1i+0j-2k. Surprisingly many candidates found only the obtuse angle-either ignoring the need for the acute angle or not realising what was required.

6 In part (i) most candidates used the correct trapezium rule formula and the correct answer was usually found and given with appropriate accuracy.
In part (ii), the binomial expansion was well known and was almost always used correctly.
Part (iii) was less successful. A surprisingly large number failed to integrate the expression and substituted the limits into the integrand. Others made errors in the integration, often $\int -1/8e^{-2x}$ =1/4e ^{2x} , or in their evaluation of the terms in e.

Section B

7 (a) The first part was usually correct with most candidates realising that *sin t* took maximum and minimum values at ±1. The most frequent error was the substitution of *t* = 0 and *t* = 6 since 0 < *t* < 6.
The second part was approached in a variety of ways. Use of the quotient rule for differentiation was quite common and was often successful. There were sign errors and many made the error (*2-sin t).0*= *2-sin t*. For those that progressed beyond this stage, explanations of the substitution for *P* were not always clear and they had been asked to verify.
Some candidates used the chain rule. There were often sign errors in the differentiation of *2-sin t*. *Cos t* was often given as the differentiation - possibly influenced by the given answer.
Perhaps the most common approach was to separate the variables and integrate. Again there were sign errors but the most common mistake was to omit the constant of integration and thus not be able to achieve the required result.
A less common, but often successful approach - particularly from good candidates - was to use implicit differentiation. Different starting points were seen but *2/P* = *2-sin t* leading to $-2/P^2 dP/dt$ = *-cos t* and then rearranging was an efficient way of achieving the result directly.

(b) Almost all candidates were able to gain all the marks for the partial fractions.
In part (ii) most candidates separated the variables correctly and integrated both sides. There were, however, some poor attempts at the separation with $2P^2 - P$ appearing as a numerator. For those that did integrate correctly - and there were many - it was once again disappointing to note the frequent absence of the constant of integration. In some cases it seemed that candidates had tried to work backwards from the given answer.
In the third part good marks were scored although some candidates prematurely approximated their working and achieved inaccurate answers.

8 (i) Many candidates scored well in this part. Some made the error of quoting $dy/d\theta = \cos\theta + \cos 2\theta$ and similarly for x – possibly working backwards from the given answer. Others made sign errors or differentiated $5\cos 2\theta$ as $5\sin 2\theta$. When verifying $dy/dx = 0$ in the next part it was necessary to see the evaluation of each part of $\cos \pi/3$ and $\cos 2\pi/3$. This was often omitted. The answer, zero, was given and so it needed to be established. The final part, finding the coordinates of A, was usually correct.

(ii) Although most candidates realised that they needed to square both the expressions in x and y and add them together, the squaring of the terms was often incorrect. Many omitted the middle term completely or wrote incorrectly that $\cos\theta \,.\, \cos 2\theta = \cos 3\theta$ or that $\cos 2\theta \,.\, \cos 2\theta = \cos 4\theta$. There were also some very long methods in this part. Some of them were successful. Full marks usually followed substitution for $\sin 2\theta$ and $\cos 2\theta$.

(iii) & (iv) A number of candidates did not attempt this or the final part and were perhaps short of time at this stage. The common error here was to fail to square root at the final stage.
Some felt that this quadratic equation could be factorised. Others used the quadratic equation formula correctly to solve for $\cos\theta$ but then used this value as θ in the final stage. Some candidates found the distance OB^2, thinking it was OB, as they forgot to square root their final value.

Paper B
The Comprehension

1 Although there were some completely correct solutions many candidates found this question difficult. Common incorrect answers were M(aπ/2-a, a) and N (2πa,0). There also seemed to be a lack of understanding of when to use radians.

2 This question was the most successful in the Comprehension and most candidates found both the wavelength and height correctly.

3 Many found the value of a correctly although a common mistake was to cancel 20/2π to 10π. The value for b was usually correct. In the second part most candidates found the values of 12 and 8 from the given figure although 12 and 4 were often seen. In the final part many did not realise they needed to substitute in the formula $\pi a+2b:\pi a-2b$ but those that did were usually successful.

4 Answers to question 4 were often wordy and missed the point.
Among the best explanations in the first part involved comparing the ratios of the lengths of the troughs and crests in both graphs. Some did not fully answer the question either indicating which was which without justification or explaining without indicating which was which. Explanations were often insufficient. For example, saying that the sine curve was symmetrical.
In the second part counting the squares on the graph was often given as the explanation of why $d = b$. Others started by saying $a\theta = a\theta - b\sin\theta$.
In the final part few realised that b needed to be small in comparison to a.
The last two parts were often omitted.

5 Few candidates tried to work with the ratios for wavelength and height. Attempts using the ratio for troughs and crests were common. Those who attempted the correct ratio often made numerical errors - often height = 0.4. When the correct final ratio was obtained it was often compared with 1:100 rather than the required 1:7.

4755: Further Concepts for Advanced Mathematics (FP1)

General Comments

This paper was of an appropriate standard, with some questions that almost all candidates could make a good attempt at and others that provided a challenge for the most able. It may, however, have been a little long; some strong candidates seemed to have run out of time.

The paper appeared to be slightly easier than recent papers for weaker candidates but a little harder for strong ones.

By far the majority of candidates were clearly well prepared for the examination.

Comments on Individual Questions

1 **Properties of a matrix**

In this question candidates were asked to find the inverse of the given 2×2 matrix, and the effect of the matrix on the area of a figure which was transformed by the matrix. Almost all candidates answered the first part correctly but a significant minority were unable to do the second part.

2 **Locus on argand diagram**

While there were many correct answers to this question from strong candidates, there were also plenty of mistakes. The commonest of these were sign errors, missing or incorrectly used modulus brackets, using $\leq$ instead of =, and attempting to give the cartesian equation of the circle.

3 **Identity**

By far the majority of candidates got this question right. The errors that did occur mostly resulted from careless mistakes involving signs.

4 **Complex numbers**

Many candidates scored full marks on this question. There were, however, many poorly labelled Argand diagrams in part (i), careless mistakes in multiplying out $\alpha\beta$ in part (ii) and in dividing one complex number by another in part (iii). All but the weakest candidates knew what to do, but a significant proportion made simple slips.

5 **Roots of an equation**

Almost all candidates knew how to do this question and there were many correct answers. Most of the mistakes that occurred were careless errors in the manipulation, often involving signs. Both of the alternative methods were commonly used, but the substitution method was more efficient and resulted in fewer errors, though some failed to multiply the constant by 27.

6 **Method of differences**

This question was generally well answered. In part (i) a few candidates invalidated their establishment of the given result with sign errors when removing brackets. $\frac{r+3-r+2}{(r+2)(r+3)}=\frac{1}{(r+2)(r+3)}$ was seen quite often. The commonest mistake in part (ii) was to leave the answer as $\frac{1}{3}-\frac{1}{n+3}$, failing to substitute $n=50$.

7 **Proof by induction**

This was by far the least well answered question in Section A. Most candidates knew what they were trying to achieve but many failed to show that $3^k-1+2\times3^k=3^{k+1}-1$. There seemed to be a widespread belief that $2\times3^k=6^k$.

8 **Curve sketching**

This question was well answered. Many candidates scored full marks on it, or very nearly so.

(i) Candidates were asked for the points where the curve cuts the axes. Most scored full marks.

(ii) Candidates were asked for the vertical and horizontal asymptotes. The most common error was failing to recognise $y = 0$ as the horizontal asymptote.

(iii) Candidates were asked about approaches to the horizontal asymptote. A significant proportion failed to show any workings.

(iv) Candidates were asked to sketch the curve. Common errors were failing to label intercepts and asymptotes and incorrect approaches to the asymptotes.

9 **Cubic equation with two complex roots**

This question was often done well, but parts (ii) and (iii) differentiated well.

(i) Almost all candidates knew that if 1 + 2j was a root of the cubic, then 1 - 2j must also be a root.

(ii) Candidates were asked to explain why the third root must be real; they were expected to say that complex roots come in conjugate pairs and that because a cubic has three roots, the third must therefore be real. Many candidates omitted the word “conjugate”. Many produced entirely spurious arguments and a significant proportion failed to attempt an answer. A few gave alternative valid arguments, which were given full credit.

(iii) Part (iii) was an unstructured question for 9 marks to find the real root and the values of missing coefficients. Some candidates found efficient approaches to the work and took only a few lines to obtain the right answers. Others, by contrast, submitted several pages of work; in some cases there was nothing mathematically wrong but it was just not going anywhere useful. However, many candidates compounded a poor strategy with sign errors. The most efficient method was to consider the sums and products of the roots, but the majority used the factor theorem, which was more complicated and so prone to error. $2\mathrm{j} \times 2\mathrm{j} = 2\mathrm{j}^2$ was seen too often.

10 **Inverse of a 3×3 matrix**

This question was rather low scoring. Time pressure may have been a factor for some.

Many did not see the structure of the question, failing to see the connection between parts (i) and (ii), and (ii) and (iii).

(i) Candidates were asked to find the value of a constant when multiplying two 3×3 matrices. Most candidates did this correctly but a common mistake was to write $n = -21$ instead of $n = 21$.

(ii) Candidates were asked to write down the inverse of a 3×3 matrix and state the condition on a constant for this inverse to exist. This all followed from (i). A mark of zero on this part of the question was quite common. Some candidates did, however, see what was happening and obtained the correct answer. The very best did seem to simply right it down, which was possible if they could see the connection with part (i). Many started again from scratch. A few earned some of the marks, but most were not successful.

(iii) Candidates were asked to solve a system of 3 simultaneous linear equations. Candidates were given the option of following the logic of the question, using the inverse 3×3 matrix to solve the equations, or of using another method. Many candidates chose Gaussian elimination and right answers obtained by this method were common. There were also some right answers using the matrix method, which followed easily from (ii). Incorrect inverse matrices were followed through from (ii), so most candidates who got this far earned at least some of the marks.

4756: Further Methods for Advanced Mathematics (FP2)

General Comments

On the whole, the candidates seemed to find this paper quite hard; and the full range of marks, from 0 to 72, was obtained. There were many excellent scripts, with about 15% of candidates scoring 60 marks or more, as well as some who were clearly not ready for an examination at this level; about 20% of candidates scored less than 30 marks. Quite a few candidates appeared to be short of time, especially when they had used inefficient methods such as repeated differentiation in Q1(c)(ii) or multiplying out brackets (instead of using the binomial theorem) in Q2(a). In Section A, the question on matrices (Q3) was much the best answered; and in Section B the overwhelming majority of candidates chose the hyperbolic functions option.

Comments on Individual Questions

1 This question, on polar coordinates and calculus, had an average mark of about 10 (out of 18).

(a)(i) The sketch was usually drawn well. Most showed the cusp at $\theta = 0$ clearly, but some also had a sharp point at $\theta = \pi$, instead of an infinite gradient.

(a)(ii) Most candidates knew how to find the area, and the integration was very often carried out accurately. Sometimes the factors ½ and a^2 were missing, and some candidates were unable to integrate $\cos^2\theta$.

(b) About half the candidates used the correct substitution $x = 2\sin\theta$ (or $x = 2\cos\theta$) to obtain $\int \frac{1}{4\cos^2\theta}\,d\theta$ (or $\int \frac{-1}{4\sin^2\theta}\,d\theta$), but many could not proceed beyond this point. Other candidates tried inappropriate substitutions such as $x = \sin\theta$, $x = 2\tan\theta$, $u = 4 - x^2$, and achieved nothing useful.

(c)(i) The differentiation of $\arccos 2x$ was often done correctly, although $\frac{-1}{\sqrt{1-4x^2}}$ was a very common error. The denominator was sometimes given as $\sqrt{1-x^2}$.

(c)(ii) This part was poorly answered. Few candidates realised that all they needed to do was to find the first three terms of the binomial expansion of $-2(1-4x^2)^{-\frac{1}{2}}$ and then integrate; and those that did, often forgot about the constant term after integrating. There were very many attempts to differentiate $\arccos 2x$ five times, and hardly any of these were successful.

2 This question, on complex numbers, had an average mark of about 10.

(a) For many candidates this seemed to be a familiar piece of work which was carried out confidently and accurately. Quite a few candidates confused this with the process of expressing powers in terms of multiple angles and considered $(z - 1/z)^5$, without any success.

(b)(i) The process of finding the cube roots of a complex number was quite well understood, and there were very many correct answers. The argument of one of the roots was sometimes given as $\frac{19}{12}\pi$, outside the required range, instead of $-\frac{5}{12}\pi$; but the most common error was to start by giving the argument of $-2+2\mathrm{j}$ as $-\frac{1}{4}\pi$ instead of $\frac{3}{4}\pi$.

(b)(ii) The Argand diagram was usually drawn correctly with ABC forming an equilateral triangle, although the midpoint M was sometimes placed on the imaginary axis instead of in the second quadrant.

(b)(iii) Relatively few candidates were able to use their Argand diagram to find the modulus and argument of *w* correctly. In particular, the modulus of *w* was often thought to be the same as that of the cube roots.

(b)(iv) Most candidates knew how to use the modulus and argument of *w* to find the modulus and argument of w^6, and hence evaluate w^6; although errors in previous parts very often caused the final answer to be incorrect.

3 This question, on matrices, was the best answered question, with half the candidates scoring 14 marks or more. Candidates appeared to tackle it with confidence, and it was quite often the first question answered.

(i) Almost all candidates knew how to find the characteristic equation, although there were many algebraic, and especially sign, errors in the working.

(ii) The methods for finding eigenvalues and eigenvectors were very well known, and loss of marks was usually caused only by careless slips. However, some candidates did not appear to know that an eigenvector must be non-zero.

(iii) Almost all candidates correctly found P as the matrix with the eigenvectors as its columns (but this earned no marks if the resulting matrix was obviously singular, for example having a column of zeros or two identical columns). The diagonal matrix D very often contained the eigenvalues instead of their squares.

(iv) Most candidates knew the Cayley-Hamilton theorem, but many did not see how to obtain $\mathbf{M}^4$. Some attempts began by cancelling a factor M from the cubic equation, which is invalid since M is a singular matrix.

4 This question, on hyperbolic functions, was quite well answered, and the average mark was about 11. By far the most common reason for loss of marks was the failure to show sufficient working when the answer was given on the question paper.

(a) Most candidates realised that the integral involved arsinh, or went straight to the logarithmic form, although the factor $\frac{1}{3}$ was often omitted.

(b)(i) This was usually adequately, if not always elegantly, demonstrated.

(b)(ii) Most candidates differentiated correctly, although $\frac{\mathrm{d}}{\mathrm{d}x}(\cosh x) = -\sinh x$ and $\frac{\mathrm{d}}{\mathrm{d}x}(\cosh 2x) = \frac{1}{2}\sinh x$ were fairly common errors. However, it was rare to see full marks in this part. Having obtained $\cosh x = \frac{5}{3}$ it was not sufficient to write 'so $x = \ln 3$, $y = \frac{59}{3}$' since this point is given on the question paper. To earn the marks a candidate was expected to write (at least)

$$x = \ln\left(\tfrac{5}{3} + \sqrt{(\tfrac{5}{3})^2 - 1}\right) = \ln 3,\quad y = 10(3 + \tfrac{1}{3}) - \tfrac{3}{2}(9 + \tfrac{1}{9}) = \tfrac{59}{3}$$

or an equivalent exact calculation using $\cosh x = \frac{5}{3}$ and $\cosh 2x = 2(\frac{5}{3})^2 - 1$.

Candidates frequently failed to find one or other of the remaining stationary points $(0,\ 17)$ and $(-\ln 3,\ \frac{59}{3})$.

(b)(iii) The integration was usually done correctly (more often than the differentiation in part (b)(ii)), but very many candidates lost marks for not showing sufficient detail in the evaluation. Since the answer is given, it is not sufficient to state, for example $\sinh(2\ln 3) = \frac{40}{9}$; the calculation $\sinh(2\ln 3) = \frac{1}{2}(9 - \frac{1}{9})$ should be shown.

5 This question, on the investigation of curves, was more popular than in the past; and in a few centres all the candidates answered this question. Even so it was only attempted by about 4% of the candidates. There were a few very good complete solutions, but most attempts were fragmentary; the average mark was about 9.

(i) In the cases $k = -2$ and $k = 1$ it was quite common for one branch of the curve to be missing, presumably caused by poor choice of scales for the axes on the calculator.

(ii) Several candidates were unable to do this simple algebra.

(iii) Every type of conic was offered by at least one candidate, even though it is known to have asymptotes.

(iv) Few sketches were drawn carefully enough to show clearly all the details requested.

4757: Further Applications of Advanced Mathematics (FP3)

General Comments

There were some excellent scripts, with about 15% of candidates scoring more than 60 marks (out of 72). However, a lot of able candidates clearly found answering three long themed questions to be a difficult task, and overall the marks were somewhat disappointing. When things go astray part-way through a question, it is important to carry on with the later parts, but not all candidates have the confidence to do this.

Some candidates indicated that they were short of time; and indeed only a very few answered more than the three questions required.
The five questions seemed to offer roughly comparable challenges to the candidates; the average marks (out of 24) ranged from about 13 for Q3 to about 17 for Q2 and Q4. The most popular question was Q1 (attempted by about 85% of the candidates) and the least popular was Q3 (attempted by about 40% of the candidates). The most common combinations of questions seemed to be Q1 Q2 Q4 or Q1 Q4 Q5 or Q1 Q2 Q3.

Comments on Individual Questions

1 **Vectors**

(i) Most candidates realised that they should start by finding the directions of, and points lying on, the lines *K* and *L*. Showing that the lines are parallel was usually done correctly, but finding the distance between them caused problems, and was sometimes not even attempted. When it was recognised as the distance from a point to a line it was often found efficiently and accurately.
Quite a number of candidates took *L* to be the line of intersection of *Q* and *R* (instead of *P* and *R*); fortunately this misread did not significantly alter the work to be done throughout the question.

(ii) Surprisingly, this part was quite often omitted, presumably because it was not recognised as the simple problem of finding the distance from a point to a plane.

(iii) The correct point of intersection was very often found, but many candidates did not check properly that the lines do intersect.

(iv) The method for finding the shortest distance between skew lines was well understood, and usually applied correctly.

2 **Multi-variable calculus**

(i) Almost every candidate found the partial derivatives correctly.

(ii) The method for finding stationary points was well known, and was very often carried out completely correctly. The case $x = 0$ was sometimes overlooked; and sometimes, having obtained $x = 0$ *or* $y = 2x$, it was assumed that $y = 0$ when $x = 0$.

(iii) The section $x = 2$ was usually sketched correctly; but on the section $y = 4$ most candidates showed $(2, 4, 8)$ as a point of inflection instead of a maximum.

(iv) This was reasonably well done; although quite a large proportion put $\partial z / \partial x$ equal to +36 instead of −36. As this also leads to exactly two points, candidates were not alerted to their error.

3 **Differential geometry**

(i) The arc length was often found correctly. However, many candidates were unable to simplify $\mathrm{d}s / \mathrm{d}x$ and this prevented success in this part and in part (ii).

(ii) Most candidates began correctly with $\int 2\pi x \, \mathrm{d}s$ for the surface area, but many could not proceed beyond this, even when part (i) had been answered correctly.

(iii) Most candidates obtained a correct expression for the radius of curvature, but a large number failed to earn the final mark for simplifying it to the required form.

(iv) This was quite well answered, with many candidates finding the centre of curvature correctly.

(v) Most of the candidates who attempted this part knew what was required, and often found the envelope correctly.

4 **Groups**

(i) This was quite well answered, although some candidates simply stated that all groups of order 2 or 5 are cyclic, without giving a reason (for example, that 2 and 5 are prime numbers).

(ii) This was also well done, with most candidates selecting a generating element and calculating all its powers.

(iii) The correct subgroups were often found, and it was quite rare for ‘extra’ ones to be given; the subgroup of order 5 was sometimes omitted. There was sometimes an unnecessarily large amount of working, such as finding the subgroup generated by each of the 10 elements.

(iv) E was almost always stated to be the identity; and the reflections were very often correctly found by considering the elements of order 2.

(v) Most candidates stated that the groups were not isomorphic; when a reason was given it was usually based on the orders of the elements (for example, *P* has more elements of order 2, or *P* has no element of order 10). Strangely, very few candidates referred to the commutativity of *M* and the non-commutativity of *P*.

(vi) The orders of the elements were usually found correctly.

(vii) Most candidates used their answer to part (vi) appropriately to write down the required subgroups.

5 **Markov chains**

(i) The transition matrix was almost always given correctly.

(ii) Calculators were accurately used, and most candidates obeyed the instruction to give the elements to 4 decimal places.

(iii) The probability was usually found correctly. Just a few candidates used $\mathbf{P}^8$ instead of $\mathbf{P}^7$.

(iv) Almost all candidates wrongly assumed that the 8th and 12th letters were independent when finding the probability that they were the same. Only a handful of candidates used the diagonal elements of $\mathbf{P}^4$ as the required conditional probabilities.

(v) This part was very well answered.

(vi) The great majority wrote down the new transition matrix correctly.

(vii) The wording of the question was intended to encourage finding the limiting matrix $\mathbf{Q}^n$ when *n* is large, and hence writing down the equilibrium probabilities from that matrix, and when it was used this method was usually successful. Nevertheless, a large number of candidates preferred to find the equilibrium probabilities by solving simultaneous equations, and this method was much more prone to error.

(viii) Most candidates calculated this probability as ${p_D}^3$ instead of $p_D \times 0.1 \times 0.1$.

4758: Differential Equations (DE)

General Comments

The standard of work was generally very good, demonstrating a clear understanding of the techniques required. Candidates commonly answered questions 1 and 4. Question 3 was the least popular choice. Candidates often produced accurate work, but errors in integration were relatively common. Also there were again many candidates omitting or not dealing properly with the constant of integration when solving first order differential equations. It is vital for candidates to realise that the constant must always be included.

Comments on Individual Questions

1 Second order differential equations

(i) This was often completely correct. The commonest error was with the coefficients of the particular integral.
(ii) The particular solution was often correct, but errors with one of the constants were common. Most candidates were unable to calculate the amplitude, either omitting this calculation or just using one of the coefficients.
(iii) The calculations of displacement and velocity were often correct.
(iv) Most candidates did not seem aware of the connection between the two parts of the motion. Most stated a general solution using the same constants as in part (i). Many then proceeded to say that the motion was negligible because of the factor $e^{-10\pi}$, in contradiction to their answers to part (iii).

2 First order differential equations

(i) Many candidates completed this correctly, but many made errors in the integrating factor, commonly getting x^2 rather than x^{-2}. Some omitted the constant of integration.
(ii) Many candidates were able to identify the limit of their solution, but few were able to use the differential equation to deduce the limit.
(iii) Most candidates used the condition to find the particular solution, but some found it only for the case $n = 1$. Sketches often did not show the known information about the solution, such as the given conditions.
(iv) This was often done well, but some candidates used their previous solution rather than solving the differential equation. Some used the condition in part (iii) rather than the new condition.

3 Modelling a water tank emptying by separating variables, tangent field and numerically.

(i) The separation of variables was often started well, but errors in integration and omission of the constant caused problems for many.
(ii) The calculations were often hampered by errors in the solution for *y*.
(iii) The solution curve was almost always done well.
(iv) The numerical solution was often done well, but some candidates did not show sufficient working for a given answer.
(v) Many candidates made some progress with this part, but answers often were unclear or incomplete.

4 Simultaneous differential equations

(i) The elimination of *y* was often done well, although a few differentiated the first equation with respect to *x* rather than *t*.

(ii) The solution for *x* was often done very well.

(iii) When finding *y*, many candidates correctly used the first equation. Pleasingly, fewer candidates than in past examinations attempted to set up and solve a new differential equation.

(iv) The particular solutions were often done well. When calculating the initial gradients, many differentiated their solutions, rather than the simpler approach of using the differential equations. The sketches were often done well, but candidates were expected to show the initial conditions and the initial gradient should have been consistent with their values.

4761: Mechanics 1

General Comments

There were many high scores and few candidates who were unable to make a reasonable start to most of the questions. Very many candidates obtained high marks on questions 2, 3 and 7. Success with the other questions was not uniform and it seemed that some of the candidates were either not familiar with the ideas being tested or were not familiar with the form of the question. Q6 was the question that caused most problems. A good number of candidates tackled it with confidence for full or nearly full marks but more were unable to deal with the vector forms and couldn't make any progress beyond the first part.

As always, there were many very good, well presented scripts and some where the untidiness had clearly handicapped the candidate. Many candidates lost marks because they did not make their reasoning clear when asked to *show* something. Many candidates produced some good, clear diagrams but many did not and few produced as many as would have been helpful to them.

Comments on Individual Questions

1 **Resolution of forces and equilibrium**

Many candidates knew exactly what to do and did it efficiently but quite a few failed to write down clear equations for the equilibrium or had no plan and struggled.

(i) Most of the candidates realised that resolution was required but many of them either did not resolve in the right direction or failed to resolve accurately. Some simply ignored *Q* and thought that $P = \sqrt{40^2 + 120^2}$.

(ii) Fewer candidates managed this part than part (i), mostly because they failed to resolve or omitted a force.

2 **Sketching and using a speed – time graph**

This was the best answered question on the paper with most candidates obtaining a good score.

(i) Most candidates scored full marks on this part. The marks lost were usually because curves were used in place of straight line segments or there was a mistake with one of the values used.

(ii) Most candidates knew that they should find the area under the graph. Of these more split the region up into triangles and rectangles than two trapezia and one rectangle. Many candidates did not indicate *how* they had split up the region and quite a few omitted one part (often a 10 by 45 rectangle). Candidates who tried to apply the constant acceleration formulae were generally less successful.

(iii) This was the least well done part. Some worked with the 1700 m instead of 1700 less the answer to part (ii); others equated the extra distance to 40*T* – effectively arguing that the car is brought to rest at a constant speed!

3 **Equilibrium involving a pulley**

Despite the unusual situation described, there were many correct answers.

(i) Most candidates knew that the pulley had to be smooth for the tension to be the same throughout the string but some thought that the system being in equilibrium was enough.

(ii) The presentation of the argument was not always very good but this did not prevent many candidates from obtaining the correct value for the force and correctly naming it as a thrust. The most common errors came from candidates who first set up a pair of equations as if the situation were dynamic with the rod not present.

4 **The use of Newton's second law and the force in a tow – bar**

There were very many correct answers to parts (i) and (ii) and most candidates used the right principles in both parts. There were rather more errors seen in the attempts at part (iii) and many candidates would have done better if they had drawn a diagram and set out their work in a conventional way.

(i) A few used $F = mga$ or $F - mg = ma$ but the most common error was to omit one (or both) of the resistances. Quite a few candidates went from $P - 800 = 4000$ to $P = 3200$.

(ii) This part was generally done correctly if part (i) was correct. The chief error was to omit one or both of the original resistances.

(iii) A few candidates used the wrong mass or acceleration but the chief error was to omit one or more resistances or to apply the extra resistance of 2000 N to the wrong truck. There were a few candidates who based their method on wrong principles that did not include the use of Newton's second law.

5 **A block in equilibrium on a rough slope**

This question was done very well by some candidates but others made little progress with it. Very many candidates did not attempt to draw a diagram and these were, of course, more likely to omit forces or fail to resolve properly or at all.

For candidates who resolved up (or down) the plane, the chief errors were to omit a force, resolve only one of the forces or confuse sine with cosine. Candidates who tried to resolve in another direction usually omitted the normal reaction as well as making one or more of the errors listed above. Candidates who did not have a diagram showing the frictional force and did not specify the direction of the frictional force were not awarded the final mark.

It is worth repeating that many candidates, often whole centres, tackled this problem efficiently and accurately, obtaining full marks in just two or three lines.

6 **Newton's second law and constant acceleration situation in vector form; the modulus and direction of a vector**

This question was found much harder than expected. Seemingly, candidates were not able to work out what was required of them because of the use of vector form and the term 'position vector'. It was for most candidates the question with the lowest percentage score. Many candidates scored only 1, 2 or 3 marks.

(i) Most candidates knew they should add the vectors and use Newton's second law. Some of them lost a mark because they did not show enough working to establish the *given* answer.

(ii) This was done well by few candidates. Many used **s** = *t* **u** and many others worked out **v**(4) instead of **s**(4); many of those who attempted **s**(4) wrongly used $\mathbf{s} = t\mathbf{v} - \frac{1}{2}t^2\mathbf{a}$. Those who used integration usually integrated **u** once instead of **a** twice and many integrated the i, j and k they used in place of **i**, **j** and **k**.

(iii) Any answer from part (ii) was followed but many candidates gave the distance OA as a vector, in many cases the correct value of **r** not found for part (ii).

(iv) Those candidates who had obtained 3**i** + 4**k** usually obtained the correct angle. Some of those with a **j** component knew what to do but many just used the **i** and **k** components.

7 **Kinematics using calculus**

This question was well understood by most of the candidates. Many used differentiation and integration appropriately and accurately and scored good marks. Very few candidates made little progress at all.

(i) Usually done correctly

(ii) Usually done correctly. More candidates used substitution than factorised.

(iii) Most candidates did this well but a few made slips and a fair number omitted the final coordinates.

(iv) Most candidates knew that they should integrate and obtained the correct indefinite integral with few falsely using the constant acceleration formulae. A major source of error was the incorrect evaluation of the definite integral either because of an error in the calculation of the value with one of the limits or because the minus signs led the candidates to add instead of subtract these values. Quite a few candidates gave the answer as a displacement of – 18 m instead of a distance of 18 m.

(v) Some candidates realised that they could use symmetry and double their answer to part (iv) but many worked out the value by starting again with new limits.

(vi) Some knew how to deal with regions above and below the axis but many just integrated from *t* = 1 to *t* = 5 and so found the displacement.

8 **Projectile motion**

Many candidates managed parts of this question but only a few obtained full marks. The answers to parts (iii) and (iv) (A) and (C) showed that many candidates have some misconceptions about projectile motion.

(i) Most candidates knew broadly what to do but many failed to give enough working to *show* a given answer. Quite a few candidates wasted time working out the value of the angle of projection so that they could then take the sine and cosine to obtain (sometimes only approximately) the values from which they had deduced the angle in the first place.

(ii) Most candidates obtained the correct answer, some by applying $v^2 = u^2 + 2as$ to the vertical motion and some by going via the time taken to reach the highest point. Most of the errors were with signs.

(iii) Many candidates set up the correct quadratic equation but quite a few made slips when evaluating the roots, either by misremembering the formula or by calculation errors. A surprisingly large number of candidates found the horizontal distances at their two times but did not then subtract them to find the *horizontal distance travelled between these times*.
The most common error was falsely to assume that the vertical height was a linear function of time and so half the greatest height was reached in half the time to reach the greatest height.

(iv) (A) Although many candidates got this right a large number did not and for many different reasons. Some correctly tried to find $v_y(1.25)$ but wrote $v_y = 7 - 9.8 \times 1.25 = 19.25$, others used the right formula but took the initial speed as 25 or even 24. A common mistake was to find the vertical *height* when *t* = 1.25 and quote this as a velocity.

(B) This part was done well by many candidates. Some interpreted the sign of their $v_y(1.25)$ and others correctly argued that 1.25 s is after the time taken to reach the highest point.

(C) This was poorly done by many candidates. Some thought they were required to find the vertical component of the speed of the ball, some just gave the modulus of their answer to (A) and many who had found a height in (A) now correctly calculated $v_y(1.25)$ but incorrectly gave it as the answer to this part. Quite a lot of candidates who were using Pythagoras' Theorem to find the speed took the horizontal component of speed to be 24×1.25 instead of 24.

(v) This part was done well by many candidates, including many who had not scored very well on the rest of the question or, indeed, on the rest of the paper.

Most knew how to eliminate *t* and did so properly (albeit with some poor notation on the way). Quite a few established the final result.

Many candidates knew how to find the horizontal range. Most calculations were correct with some candidates setting *y* = 0 in the trajectory equation and others finding the total time of flight and using this to find the range.

4762: Mechanics 2

General Comments

Many excellent scripts were seen for this component with the vast majority of candidates able to make some progress worthy of credit on every question. Question 2 was, perhaps, less well answered than the other questions but even so most candidates obtained some credit for their work. The poor quality of diagrams hampered the progress of a substantial number of candidates on all of the questions. Presentation was satisfactory on the whole but in a few cases poor presentation led to arithmetic errors and a lack of coherence in the solutions. As has happened in previous sessions, some candidates did not appreciate the detail that was required in order to *'show'* a given answer and omitted relevant steps in the working or relevant comment in the explanation. Some candidates penalised themselves by premature approximation of answers leading to errors in accuracy on following parts.

Comments on Individual Questions

1 **Impulse and Momentum**

Many candidates gained significant credit on this question. Those that drew clear diagrams were usually more successful than those who did not.

(a)(i) This part was well done by almost all of the candidates.

(ii) Many numerically correct solutions were seen to this part but a significant minority of candidates did not produce a diagram and then usually failed to *show* that the direction of motion after impact was the same as the original direction of motion of A.

(iii) Many candidates failed to realise that impulse is a vector quantity and omitted its direction in their answer.

(iv) (A) Many of the diagrams were poor; labels were omitted; directions not indicated.

(B) This part posed few problems to the majority of candidates.

(C) Many correct solutions were seen to this part. Incorrect solutions usually arose because of sign errors in the application of Newton's experimental law.

(b) A large proportion of the candidates found this part of the question difficult. Many of them did not appreciate that the ball would move as a projectile with an initial velocity in the horizontal direction only and considered the initial motion to be at 8 m s^{-1} at 45° to the horizontal. Others failed to realise that the component of the velocity in the horizontal direction would be unchanged by the impact.

2 **Centres of Mass**

This question caused slightly more problems to candidates than other questions on the paper.

(i) Many candidates failed to give enough detail in their explanations to *show* the given answer. Some candidates introduced spurious negative signs. A small number of candidates did not appear to understand the relevance of the use of radians and offered $\frac{8\sin 90}{90} = \frac{16}{\pi}$ as an answer.

(ii) This part was well done on the whole with any errors usually arithmetic in nature.

(iii) Poor diagrams were seen on a large number of scripts. Diagrams showing the centre of mass directly below the point of suspension were in a minority and this led to errors in finding the lengths required to calculate angle α.

(iv) This part caused fewer problems than the previous part with many candidates able to obtain at least some credit for their work.

3 **Moments and Resolving**

It was pleasing to see some excellent answers to this question with many candidates gaining a substantial amount of credit.

(i) Many good responses to this part were seen with the majority of candidates able to obtain full credit.

(ii) The candidates seemed to have few problems with this part although a small minority failed to draw the diagram requested in the question.

(iii) While many good solutions to this part were seen, a sizeable number of candidates did not appreciate that the normal reaction was at right angles to the plank and drew the reactions acting vertically.

(iv) This part was not well answered. Many of the candidates failed to realise that for μ to be a minimum, the friction had to be acting at the place where the normal reaction was largest. Some candidates merely calculated a value for μ at one point and then failed to show or give a reason as to why this was the minimum value. Some complex calculations were offered by some candidates to find the frictional force with few of them appreciating that, for limiting friction, this had to be equal and opposite to the component of the weight down the plane.

4 **Work- Energy**.

As in previous sessions, those candidates who used work energy methods were on the whole more successful than those who attempted to use Newton's second law and the constant acceleration equations. It was encouraging to see a significant number of completely correct responses.

(i) This part was well done by most of the candidates.

(ii) The majority of candidates could obtain some credit for this part but a few failed to realise that the pail being raised at a steady speed implied that the force required was equal and opposite to the weight of the pail. Some utilised the force they had obtained in the previous part without realising that the conditions had changed.

(iii) Many of the candidates scored highly on this part. However, some attempted to solve the problem by applying Newton's second law and the constant acceleration equations in a vertical direction without giving any justification of their method.

(iv) A large number of completely correct responses were seen to this part. Errors, on the whole, usually arose from the omission of one term in the work energy equation or from a sign error.

4763: Mechanics 3

General Comments

The standard of work on this paper was very high. Most candidates found it to be a straightforward test of the topics, and there were many excellent scripts; about half the candidates scored 60 marks or more (out of 72). There were some weaker candidates, but very few scored less than 30 marks, and almost all appeared to have sufficient time to complete all that they could do. Circular motion was the only topic which seemed to cause significant difficulties for a large number of candidates.

Comments on Individual Questions

1 This question, on **dimensional analysis and simple harmonic motion**, was the best answered question, with an average mark of about 16 (out of 18).

(a)(i) Almost every candidate gave the dimensions correctly.

(a)(ii) The great majority of candidates found the dimensions of each term correctly, although some did not explain how their working had demonstrated the required result. A simple statement that the equation is consistent was sufficient to earn the final mark, but ideally some reference to all terms having the same dimensions should be made.

(b)(i) The graph showing the variation of the depth was usually drawn correctly, although there was sometimes no indication of scale on the time axis.

(b)(ii) The equation for *h* was found correctly by about half the candidates. The value of ω was usually calculated correctly from the period, but the amplitude was sometimes 0.6 instead of 0.3, and the central value 1.9 was quite often omitted.

(b)(iii) Most candidates used $\omega^2 x$ to find the magnitude of the acceleration, but *x* was very often taken to be 1.7 instead of 0.2. Other methods, such as differentiating the equation for *h* from part (b)(ii), were even more prone to errors. The direction was often given wrongly, and sometimes omitted.

2 This question, on **circular motion**, was the worst answered question, with an average mark of about 13; but even so about 30% of the candidates did score full marks. The horizontal circle (parts (i) and (ii)) caused more problems than the vertical circle.

(i) Although this was answered correctly by most candidates, a very common error was resolving parallel to PO ($R = mg\cos 60$) instead of vertically ($R\cos 60 = mg$).

(ii) Most attempted to use v^2/r for the acceleration, but a very common error was to take the radius to be 2.7 instead of $2.7\sin 60$, and some did not consider the horizontal component of the normal reaction.

(iii) This was quite well answered, although some candidates did not realise that they only needed to use conservation of energy.

(iv) Most candidates were able to set up the radial equation of motion and obtain the given result. A fairly common error was omission of the component of the weight.

(v) Most candidates understood that P leaves the surface when $R = 0$, although some were unable to follow this with a correct calculation of the speed.

3 This question, on **elasticity**, was generally well understood, and the average mark was about 15.

(i) Almost every candidate found the tension and the energy correctly.

(ii) The great majority of candidates realised that they should resolve vertically to find the mass of the ring.

(iii) Almost all candidates realised that they should consider energy, but very many failed to take sufficient care over the details. Common errors included the miscalculation (or omission) of the change in gravitational potential energy, forgetting about the initial elastic energy in the string, and sign errors in the energy equation.

(iv) Some assumed that the ring moves with simple harmonic motion, but the majority of candidates who attempted this part continued to consider energy, either from the lowest point or from the initial position. The most successful, and simplest, approach was to show that the ring has positive kinetic energy when it reaches the level of A; those who tried to find the highest point reached usually forgot that the string would become stretched again.

4 The methods required in this question, on **centres of mass**, were very well understood, apart from the last part. The average mark was about 14.

(a) Most candidates found the centre of mass of the lamina correctly.

(b)(i) Most candidates obtained the centre of mass of the solid of revolution correctly.

(b)(ii) Although most candidates realised that they should take moments, there were not many completely correct solutions to this part. Common errors were miscalculating the height of the solid, and especially taking the moment of the weight about the point of contact to be $mg \times 1.35$ instead of $mg \times 0.35$.

4764: Mechanics 4

General Comments

The standard of work was very high with most candidates demonstrating a good grasp of the mechanics and sound algebraic skills.

Comments on Individual Questions

1 **Stability**

(i) The calculation of potential energy was almost always done correctly.

(ii) The position of equilibrium was also usually calculated correctly.

(iii) When calculating the reaction, some candidates omitted to resolve the tension in the string. Some candidates made very lengthy calculations, often finding difficulty with the angle.

2 **Variable mass**

(i) Although many candidates were able to find the velocity, many did not show sufficient working for a given answer, in particular the derivation of the expression for mass. Some candidates integrated the Newton's second law equation directly to get mv = constant, others appealed to the conservation of momentum. These methods were perfectly acceptable, but some unnecessarily expanded $\frac{\mathrm{d}}{\mathrm{d}t}(mv)$ and set up and solved a differential equation. Finding the displacement from the given velocity was usually done correctly.

(ii) This was usually done correctly.

3 **Rotation**

(i) This was often done well, but some derivations lacked clarity. Some candidates set up an integral which bore no relation to the mechanics but just happened to give the correct answer.

(ii) This was also often done well, but some did not take account of the position of the axis for the rod or for the sphere.

(iii) Candidates who used energy often gave good solutions to this part. However errors in the potential energy were common. A clear diagram generally was helpful to candidates. Candidates who attempted this via a non-energy method were rarely successful.

(iv) Some candidates successfully used the rotational equation of motion, others successfully differentiated their expression from the previous part. However sign errors were common. Some candidates did not seem to know what was required here. When showing SHM, candidates are expected to make a conclusion once they have derived the relevant equation (simply stating "hence SHM" would be enough).

4 **Variable acceleration**

(i) Most candidates successfully found the required expression, but some omitted the constant of integration when solving the differential equation. It is vital for candidates to realise that the constant must always be included.

(ii) This was usually completed correctly.

(iii) The integration required in this part caused some problems. Some candidates used the standard result from the formula book, as intended. Others used partial fractions and others simply gave up. The constant of integration was sometimes omitted.

(iv) This was often completed correctly, but some candidates did not realise that they could use impulse equals change in momentum.

4766: Statistics 1

General Comments

The paper attracted a fairly wide range of responses, although there were relatively few scripts with very low scores. There was no evidence to suggest that candidates had insufficient time to attempt all questions. As in recent sessions, answers were often well presented but once again many candidates did not appear to appreciate the implications of using rounded answers in subsequent calculations.

Good answers were seen from many candidates in questions 1, 2, 3(i),(ii), 4(i),(i)i, 5(i), 6, 7(i)-(iii) and 8(i),(i)i. Candidates' work on Venn diagrams was much better than in recent papers, although in this paper candidates had to use a given diagram, rather than complete their own and perhaps this assisted them to perform well.

Candidates' responses to Q3(iii) suggest that more attention should be given to finding mean and standard deviation of transformed data. Calculation and interpretation of conditional probability as in Q7 continues to cause difficulties. In hypothesis testing, the work generally continues to improve; the use of point probabilities rather than tail probabilities seems to be declining, although many candidates are still not meeting the requirement to define *p* in words. There were a number of centres where candidates who scored well on the rest of the paper appeared to have minimal knowledge of hypothesis testing, possibly suggesting that this topic has only been covered superficially.

Comments on Individual Questions

Section A

1 **Album tracks; combinations and arrangements**

(i) Many totally correct answers were seen although candidates occasionally evaluated $^8P_4$.

(ii) Again very many correct answers were seen with the most frequent error being an answer of 16, often from 4^2.

2 **Customer spending; frequency table and total from histogram.**

(i) Most candidates correctly stated the group limits, although occasionally boundaries such as 19 or 21 instead of 20 were seen. Answers to the frequencies were less successful with a significant number of candidates giving the frequency density in place of frequency or doubling or halving each frequency.

(ii) Most candidates realised the necessity for finding the sum of the frequencies multiplied by the interval mid-point, although a few simply gave the sum of the frequencies as their answer. Others multiplied the mid-points by the frequency density. A few decided that the question required an estimation of the mean amount of money spent.

3 **Exam marks; mean, standard deviation, outliers, linear transformation.**

(i) Virtually all candidates obtained the mean correctly although some were less successful with the standard deviation. Errors here included use of an incorrect formula for S_{xx} but only occasionally division by n rather than (n-1).

(ii) There were many fully correct answers although there was occasionally use of 1.5s rather than 2s.

(iii) Many candidates were totally successful with the mean and standard deviation of the scaled data. The most frequent error was to calculate $s_y = 1.2s_x - 10$ instead of $s_y = 1.2s_x$. Some candidates decided to calculate the transformed summary statistics and then use these to find the new mean and standard deviation. Quite often this did lead to a correct new mean but almost without exception they were unable to adapt this approach to find the new standard deviation. The fact that only 2 marks were available should have alerted candidates that this did not warrant a further 2 pages of calculations.

4 **Recycling; Venn diagram, conditional probability.**

(i) Most candidates answered both parts entirely correctly, demonstrating their abilities to correctly read and interpret a Venn diagram.

(ii) A pleasing number of correct answers were seen to a question on a topic which candidates often struggle with. The idea was to use the Venn diagram to write down the probability without any calculation, but some chose to use the conditional probability formula which was of course equally acceptable. There was nonetheless a variety of errors leading to answers such as 13/50, 11/50 and 24/50, effectively missing the conditional nature of the question.

(iii) Correct answers to this part were conspicuous by their absence. Invariably answers such as 2 × 18/50 × 32/50 or 18/50 × 32/50 were given, with candidates not realizing that the second selection was from 49. Indeed sight of a second fraction with a denominator of 49 was a rarity, even from very high scoring candidates. This type of decreasing probability question has been set many times in the past and candidates should ask themselves a simple question – are the events independent or dependent?

5 **Rainfall and global warming, median and interquartile range, discussion.**

(i) A considerable proportion of candidates stated that the 11th value was the median rather than the average of the 11th and 12th. They were more successful with the interquartile range although the use of (7+1)/2 for the lower quartile was not unusual. A very few candidates treated the data as continuous and constructed a cumulative frequency curve, gaining no credit.

(ii) Full marks in this part were very rare. Many candidates, even those who overall scored highly, answered this as a question about summer rainfall, ignoring all reference to global warming being the cause. Such candidates thought that the conclusion was valid based on the median falling by 1 day and the IQR staying the same. This gained no credit.

6) **Telephone competition; probability, calculation of E(*X*) and Var(*X*).**

(i) Most candidates answered correctly, either by using a probability argument or by considering combinations. A few tried to justify the given value by using the other probabilities given in the table.

(ii) Most candidates calculated both expectation and variance correctly, although some inaccuracy was seen when candidates used decimal probabilities. Some candidates correctly found $E(X^2)$ thus scoring some credit, but then omitted the subtraction of $[E(X)]^2$ or used [E(*X*)] only in calculating Var(*X*). There are still some candidates who insist in dividing either E(*X*) or Var(*X*) or both by divisors *n* or (*n*-1). Such actions are penalised. Overall this question was a rich source of marks for many candidates.

Section B

7 **Screening test; tree diagram, probability, conditional probability, interpretation.**

(i) Almost all candidates gained all 4 marks here.

(ii) Again the vast majority of candidates were successful here.

(iii) Most candidates were again successful although a few multiplied instead of added the relevant products.

(iv) Many candidates were successful here although some candidates were unable to find this conditional probability. Common errors included answers of 0.0091, 0.0436/0.91, 0.0436/0.0091 and (0.0436 × 0.0091)/0.0436.

(v) The attempts at commenting on the answer to part iv) were very mixed with some candidates thinking that the larger the value of their answer, the more effective the test. A significant number of answers referred to a proportion of negative results rather than a proportion of those with the disease.

(vi) There were a few excellent answers but, without a complete tree diagram to assist them, most candidates failed to identify all the required possibilities. Common errors included partially correct answers such as 0.91 + 0.06 × 0.9 = 0.964, as well as entirely incorrect answers such as 0.91 × 0.99 + 0.06 × 0.9 = 0.9549.

8 **Job applications; binomial distribution, expected frequency, highest probability, hypothesis test, critical region.**

(i) Relatively few candidates were able to find this relatively straightforward upper tail probability correctly. Most failed to realise what was required by "at least". Answers of $P(X = 4) = 0.2093$, $P(X \geq 4) = 0.5489$ or 0.7582, $P(X \geq 4) = 1 – 0.2093$ or $= 1 – 0.7582$ appeared with regularity.

(ii) Most answers to part ii) were correct although few candidates resisted the urge to round their answer of 3.4 to an integer. Others insisted erroneously that $E(X) = 3$ or that $E(X) = 17 \times 0.4511$ (or their probability in part (i))

(iii) Answers to this part were disappointing, with many candidates stating that 3 was the most likely number of applicants as that value was closest to the expectation. Although the value with highest probability in the binomial distribution is close to the expectation, it is necessary to calculate probabilities both sides of the expectation to confirm the maximum. With 3 marks available, candidates should realise that more than this is required. Full credit could only be given when candidates had found <u>both</u> $P(X = 3)$ <u>and</u> $P(X = 4)$, (and also preferably $P(X=2)$) but some were content to make their judgement based on $P(X=3)$ alone. Those who did not calculate any probabilities earned no marks at all. Again this type of question has been set in the past and the required methodology has been commented on in previous reports.

(iv) Many candidates correctly stated their hypotheses in symbolic form. However, many incorrect notations were also seen. The required notation is clearly given in the mark scheme and candidates should be trained to use this, leading to a straightforward two marks. As in previous papers, still very few candidates realise the need to define the parameter '*p*' and thus most lose a third mark, even if they have stated their hypotheses correctly. Previous reports have referred to the importance of this. However the reason for the form of the alternative hypothesis was explained well by many candidates

(v) There was also an improvement here on earlier papers, with fewer candidates using point probabilities. However, a common error was to evaluate lower tail probabilities, despite having the correct upper tail hypothesis. Amongst candidates who did find an upper tail probability, a very common error was to state correctly that $P(X \geq 6) = 0.1057 > 5\%$ and $P(X \geq 7) = 0.0377 < 5\%$ before giving a wrong critical region of $X \geq 6$. Other answers obviously along the right lines failed to include any probabilities as justification, for example $P(X \geq k) < 0.05$, $P(X \leq k\text{-}1) > 0.95$, $k - 1 = 6$, $k = 7$, critical region is 7 and above. Candidates are expected to give numerical probabilistic justification for their answers. A further frequent omission was the failure to provide an explicit numerical comparison of the tail probabilities with the significance level of 5%, which again is always a requirement in hypothesis tests.

(vi) This was usually answered correctly by those candidates who had already shown an understanding of hypothesis testing in part (v).

4767: Statistics 2

General Comments

As with previous years, the majority of candidates were well prepared for this examination. Candidates are improving in their ability to carry out hypothesis tests, using correct notation and suitably thorough explanation. Most demonstrate good understanding of the Normal distribution; very few candidates use incorrect tail-probabilities in probability calculations compared with previous years. Marks for explanation and interpretation continue to be elusive to even the most able candidates.

Comments on Individual Questions

Section A

1 (i) Well answered. Many candidates lost marks through inappropriate use of continuity corrections. Most managed to calculate a probability using the correct tail of the Normal distribution.

(ii) Well answered. A few candidates omitted the binomial coefficient. Some found three eighths of their previous answer. Otherwise, most gained full marks.

(iii) The majority of candidates gained at least 3 of the 4 marks available. Many lost a single mark through inaccurate use of Normal tables, failure to use a continuity correction or using the continuity correction, 50.5. A small number attempted to use a Poisson approximation, gaining no credit.

(iv) Most candidates obtained two marks for providing correct hypotheses in terms of μ. The mark for defining μ proved harder to obtain. Many made no attempt to define μ at all; some of those who did, seemed unable to relate μ to the "new hairdresser". As with previous years, this mark still proves to be rarely given.

(v) Well answered. A variety of approaches were seen; the most common being as outlined in the mark scheme. A small number of students were penalised heavily for treating the sample mean as a single observation, thus avoiding use of the standard error $3/\sqrt{25}$. Most candidates obtained at least 4 of the 5 available marks. A few lost the final mark through failing to answer in context. In such questions, the concluding statement should always refer to the context in which the question is set.

2 (a)(i) Well answered. Most achieved full marks. Some candidates made mistakes with ranking or with calculating d^2, thus losing at least one mark. A number of candidates omitted the 6 from their calculation of r_s. Those failing to use ranks scored no marks on this part of the question.

(ii) Most candidates are now describing their hypotheses in tests for association, as outlined in the specification. Many failed to give their hypotheses in context, as required; in this particular question, "between *x* and *y*" was sufficient. Several lost a mark for omitting the word "positive" from their alternative hypothesis; a further mark was lost if "positive" was omitted from their conclusion. In the remainder of the question, most scored full marks, but marks were lost for failing to provide a conclusion in context.

(iii) Well answered, with most candidates scoring full marks.

(iv) Poorly done. Many answers merely repeated the wording given in the question without actually explaining why the pmcc test is preferable. Many candidates appeared not to realise that two explanations were required in this part of the question. For the second explanation, very few managed to refer to a critical value; most answers simply compared the values of the correlation coefficients with each other.

3 (i) *A* Most candidates scored full marks. A small number misinterpreted the question, finding P(X = 5) instead of P(X = 1).

(i) *B* Most candidates scored full marks. A small number used 1 - P($X \le 6$), losing both marks.

(ii) *A* Most candidates scored full marks.

(ii) *B* Well answered. Some candidates misinterpreted the question and found P(X = 1), using B(5, 0,3375)

(iii) Well answered. Common mistakes involved incorrect, or omitted, continuity corrections. Most candidates worked to an acceptable degree of accuracy.

(iv) *A & B* In answering questions such as this one, candidates should aim to provide a decision together with a reason to support it. Many candidates provided indecisive comments. Other candidates merely stated that calls would (or would not) arrive independently and at a uniform average rate, making no attempt to interpret what this meant. It is clear that most candidates have a poor understanding of what is meant by uniform average rate.

4 (i) Well answered. In stating hypotheses, some candidates lost a mark for failing to provide context. Calculations of expected frequencies were handled accurately, on the whole, leading to full marks for the test statistic; however, some candidates lost an accuracy mark through premature approximation. Most candidates had little trouble picking up the final 4 marks, although a significant number thought they should carry out a two-tailed test and were, consequently, penalised. A small number mentioned correlation in their conclusions.

(ii) It proved difficult for candidates to obtain full marks for this part of the question. Better attempts saw candidates comparing observed and expected frequencies. Those who referred to the contributions to the test statistic tended to write nonsense unless they demonstrated an appreciation of the difference between positive and negative contributions.

(iii) Well answered, with most gaining full marks. The Poisson approximation proved more popular and successful than the Normal approximation. Of those using the Normal approximation, several applied incorrect continuity corrections.

4768: Statistics 3

General Comments

Once again the overall standard of the scripts seen was pleasing: many candidates appeared well prepared for the paper.

As reported previously, it was noticeable that candidates' empathy with the use of correct mathematical notation was often poor. For example: integrals were often written without the terminator "d*x*" and the symbols "=" and "⇒" were treated as synonymous. Also, despite a comment in last June's report, many candidates continue to show a lack of appreciation of the level of detail of arithmetic required to convince the examiner that an answer printed in the question has been obtained genuinely.

Invariably all four questions were attempted, and attempted well, on the whole. Questions 2 and 4 were found to be particularly high scoring. There was no evidence to suggest that candidates found themselves short of time at the end.

Comments on Individual Questions

1 **Continuous random variables; Central Limit Theorem; duration of fireworks.**

(i) Almost all candidates got off to a good start here, experiencing no difficulty with the fairly straightforward integral that was involved.

(ii) The mode was found correctly, though most candidates were seen to disregard the root $t = 0$ without comment.

(iii) The value of E(T) was found easily. For Var(T) the layout and organisation of work was untidy at times, and all too often candidates were insufficiently careful about showing the printed answer convincingly.

(iv) Most candidates were able to write down the correct distribution here, based on the Central Limit Theorem.

(v) This was generally well answered. When candidates got it wrong it was usually because they constructed the interval either using a t value instead of a Normal value or using an incorrect alternative to the sample standard deviation. Most spotted that the mean of the model lay outside the interval, thus calling the model into question.

2 **Combinations of Normal distributions; motorway toll charges.**

(i) This part was found to be very straightforward.

(ii) This part, too, was well answered. It was pleasing to note that fewer candidates slipped up with the inequality of the requirement than in the past.

(iii) Usually the mean of the total takings was correct, but the variance was correct less often. Typically the error came about through a lack of proper understanding of the difference between Var(4X) (= 4^2Var(X)) and Var(X_1 + … + X_4) (=Var(X_1) + … + Var(X_4)). In several cases the former was used when it should have been the latter. Furthermore the notation of the former was often seen when the subsequent working seemed to indicate that the latter was intended. That the weeks should be independent of each other was not as well known as would have been liked.

(iv) There were many correct solutions to this part. As one might expect the errors that were seen all related to the calculation of the required variance.

3 **The *t* distribution: paired test for the population mean difference; confidence interval for a population mean; absenteeism in the workplace.**

(a)(i) For candidates at this level too many seemed unable to set down clearly and concisely the hypotheses for this paired *t* test. It is reasonable to expect them to be familiar with the conventional notation "μ" for the population mean (difference, in this case) and to define it as such. For the necessary assumption, "Normality" on its own was not enough; candidates were expected to be explicit in naming the population of differences.

(ii) The *t* test itself was usually carried out successfully. Candidates seemed well versed in what they had to do. There is still the issue of encouraging candidates to express their final conclusion in suitable language.

(b) Most candidates answered this part well, apart from the required assumption. This time it was the Normality of the "days lost after" that was needed, and again candidates were expected to be explicit in identifying that population. The majority of candidates were able to provide the required interpretation of their interval in relation to the target. However, some carried over the mean and standard deviation from part (a), a consequence of which was that their confidence interval included a negative part which would be difficult to interpret in context. A few candidates thought that, since their interval was higher than the target value, then the target had been surpassed.

4 **Chi-squared test of goodness of fit; Wilcoxon single sample test for a population median; distance between flaws in lengths of plastic strip.**

(i) This was well answered. Many candidates seemed to be making good use of their calculators, obtaining a correct value of the test statistic with little fuss. There were only occasional errors over the number of degrees of freedom and hence the critical value. As in Question 3 the language of the conclusion sometimes left room for improvement, and there were some who thought they were fitting data to a model rather then the other way round.

(ii) Apart from a handful of candidates who ill advisedly attempted a *t* test, this part of the question was well answered. The work submitted was well organised and easy to follow.

4769: Statistics 4

General Comments

This is the second time that the new-specification Statistics 4 module has been sat. Although the entry is small, it is pleasing that the opportunity to proceed to high levels in the applied mathematics strands is still available.

There was some extremely good work, and only a little very poor work.

The paper consists of four questions, each within a defined "option" area of the specification. The rubric requires that three be attempted. All four questions received many attempts – another encouraging feature, as it indicates that centres and candidates are spreading their work over all the options.

Sadly there were again cases of "faking" of answers that were given within the questions. This was discussed at some length in last year's report. This year, I will merely reiterate that it is *entirely unacceptable*.

Comments on Individual Questions

1 This was on the "estimation" option. It consisted of comparison of two estimators of θ for the uniform distribution on (0, θ).

The first of those estimators was $2\bar{X}$. Most candidates showed quickly enough that it was unbiased, but surprisingly many did not spot that, with the sample data given, its value was 0.92 even though we *knew* that θ must be at least 1, thus making it a fairly useless estimator! Candidates tended instead to struggle in making unconvincing comments about its variance. The question then moved on to a new estimator whose mean square error was to be found; this was usually done fairly successfully, some candidates being much more efficient in their work than others, and some not really being able to cope at all. Candidates who had spotted the key disadvantage of the first estimator were usually able to see that the new estimator could not possibly suffer from it, but others struggled to find anything sensible to say.

2 This was on the "generating functions" option. It led candidates through the steps of proving that the limiting distribution of the B(n, p) random variable as $n \to \infty$ is N(np, npq).

Most candidates proceeded thoroughly and carefully through the technical mathematical work, much of which should have been standard bookwork. However, surprisingly many could not simply write down 0 and 1 for the mean and variance in part (iii). In part (iv), several candidates were rescued, with greater or less legitimacy, by the provision in the question of the answer. In part (v), some candidates did not realise that the first step towards the limiting result was to expand the exponential terms from part (iv). Most, however, did this quite well, sometimes not being entirely convincing in their use of the result given in the question (simply averring that their version of the f(n) in that result was actually *equal to* zero rather missed the point).

3 This question was on the "inference" option. It was based on an unpaired Normal test, proceeding to consideration of Type II error.

Mostly the test and confidence interval (parts (i) and (ii)) were well done, though some of the usual errors did appear from time to time. Part (iii) met with mixed success; it was done very well by some candidates, whereas others fell by the wayside en route. Surprisingly many failed to consider both "tails" in finding the last probability; though one of them turns out to be negligible in the extreme, this cannot be known until there has been some investigation of it! A variety of suggestions in favour of and against the Wilcoxon alternative came forward in part (iv).

4 This was on the "design and analysis of experiments" option.

It opened with some important considerations of experimental design. Some candidates showed good appreciation of the points here; others did not. In part (iii), the required design was randomised blocks (correctly oriented with respect to the fertility gradient); some credit was allowed for suggestions of Latin squares, though that design is not really appropriate here as it is too complicated for the situation.

The analysis in the last part was usually done well. However, the point must *yet again* be made that many candidates were very inefficient in their calculations. This is definitely getting worse. What might be called the "s_b^2/s_w^2" method is *extremely* cumbersome for hand calculation. It is intricate, takes a great deal of time, and is liable to produce errors. It is poor practice. The "squared totals" method (as exhibited, somewhat in summary form, in the published mark scheme) is *very* much better for hand calculation. It is appreciated that the "s_b^2/s_w^2" method is that by which the analysis of variance is first approached in the MEI textbook that supports this module, but the book does go on to mention the "squared totals" method. Candidates should be sure to understand the "squared totals" method and to use it routinely when carrying out these calculations by hand.

Finally, it was encouraging that many candidates were able to state the assumptions about the distribution of the experimental error correctly.

4771: Decision Mathematics 1

General Comments

Candidates were able to cope well with this examination. Most had been well prepared and were able to collect a high proportion of the marks allocated for the more fundamental parts of the questions. There were a small number of "extension" marks which very few candidates were able to score.

This was the second delivery in which candidates were provided with a printed answer book. Again, it worked well. There were still one or two centres which had all of their candidates append 8 page answer booklets to their printed answer books. As before these were a nuisance, since they seldom contained anything more than a few notes, whilst requiring examiners to check and annotate each page. If candidates run out of space for a question they should write on the blank page(s) at the end of the printed answer book. Only if this proves insufficient should a 4 page answer booklet be issued.

Comments on Individual Questions

1 **Graphs**

Almost all candidates could draw the graphs in parts (i) and (iii). A few forfeited all marks in parts (ii) and/or (iv) by not realising that the first graph was not simple and the second not a tree. Many who did get the correct "no" answers were unable to mount adequate justifications. There was much confusion between loops and cycles.

2 **Algorithms**

(i) Most were successful with first fit.

(ii) Most were successful, but several implemented first fit increasing.

(iii) Most were able to give the required packing, but only about 50% offered weight as an alternative criterion for packing. Some thought it iniquitous that one hiker would have to carry 3 items and the other only 2! They were denied the mark. Others went on flights of fancy concerning the possible natures of the items being packed. These also failed to gain the mark.

3 **LP**

There was a range of responses to this question. Most were able to make a decent attempt at drawing the graph, although candidates' shadings were sometimes difficult to discern. Fewer showed evidence of trying to compute or read off the coordinates of the relevant points. A similar proportion failed to show evidence of how the best point was selected. There were few instances of candidates losing marks through inaccurate drawing.

It should be noted that it is **not** necessary to use (0,21) to draw a graph of $3x + y = 21$. Plenty of other points are available, e.g. (4,9), (5,6), (6,3) etc. Scaling axes so as to use (0,21) leads to a very small feasible region.

4 **CPA**

(i) Most candidates were successful in listing immediate predecessors. Of those that failed rather more showed a complete lack of understanding than those who gave complete list of predecessors.

(ii) Whilst most were able successfully to complete forward and backward passes, a substantial minority failed, particularly with the backward pass. With some weak candidates examiners had to be vigilant to see which box was being used for which pass.

(iii) These parts were all concerned with scheduling. Part (iii) was straightforward,
(iv) but many candidates either found parts (iv) and (v) less easy, or were unable
(v) adequately to communicate their schedules.

5 **Networks**

(i) This was very easy. Nevertheless, it was pleasing that so few candidates failed to score full marks on it.

(ii) Examiners had to struggle through the usual crop of answers in which it was not clear whether or not Dijkstra had been used. It is particularly important for candidates to show their order of labelling, and for them only to write in new working values when the new **do** replace the old.

(iii) Surprisingly few candidates scored the marks for this part of the question. Most who attempted it tried to find a maximum weight route (which would not be well defined), as against the required maximin route.

(iv) Many candidates were able to score the second and third marks here. Only one or two were able to define the potential new working value for the first mark.

6 **Simulation**

(i) Most candidates gave correct "dry" and "wet" routes. As always, a proportion failed when it was necessary to reject some random numbers.

(ii) The pitfalls of examining! We were ready and waiting for those candidates who failed to reject the 98 and the 99, having the consequential weather string ready for "follow-through" marks. We were several candidates into the marking procedure before it was realised that candidates who had the correct rules, but who only ever applied the "dry" rule, were producing the same string! Thankfully it was possible to determine what **was** going on, and to allocate marks accordingly.

(iii) Not all candidates gave a probability in answering this, the worst being those who gave 3:7 instead of 3/7. (3:4 was never seen!)

(iv) Nearly all candidates gave "repetition", though many took quite a bit more than a word, or even a sentence, to say it. Only one or two made reference to the initial condition.

(v) Many made reference to the concept of classifying a day as "dry", etc. They were given a mark even if they failed to get to grips with the issue. They were given it only once, even if they referred to many separate aspects of this issue, as many did.

Some criticised the assumption that the transition probabilities remain constant, although the examiners often had to be generous in their interpretations of what had been written to award a mark – e.g. "global warming"? Those that stated that the probabilities would be different in different seasons were not given credit – they had not read the question.

The most difficult issue for the examiners concerned those who questioned the Markov nature of the model. Anyone who said, for instance, that the weather today might also depend on the weather two days ago would clearly have been worth a mark – such a candidate would have in mind, at least implicitly, a second order recurrence relation. On the other hand a candidate with the same thoughts might well have written down "The model only takes account of yesterday's weather". That was thought not to be worth credit, being a statement of fact.

Some thought that having sets of probabilities with different denominators was worthy of criticism.

4772: Decision Mathematics 2

General Comments

Again, candidates were mostly able and well-prepared, and gave good performances.

Comments on Individual Questions

1 **Logic**

(a) Most candidates were able to answer part (i) correctly, but many were floored by part (ii). Not all of those who gave the correct answer could justify it, and there were even some who produced correct arguments having given an incorrect answer.

(b) The truth table work was well done. Most attempting it had 8 rows to their tables. Many had their entries completely correct and some just made the odd slip.

(c) Few candidates were able to see through what was required here. This was the case even in instances where a thorough and correct line of reasoning had been supplied in part (a)(i).

2 **Decision Analysis**

Most candidates were able to score heavily on this question, but a substantial minority failed to produce a correct tree at the beginning of part (i). Many of those had an initial bifurcation for "first race/second race", with attendant confusion over the node type. Such candidates were struggling for marks after a misunderstanding of that magnitude.

The utility analysis in part (ii) required that candidates both applied the utility function to a payoff and multiplied by a probability so as to give an expected utility.

3 **Networks**

(i) Almost all candidates scored these marks.

(ii) Most were able to give both answers and explanations. Some gave the route as **1-2-3**, presumably thinking that the matrix represents the first vertex en route.

(iii) An easy mark, scored by most.

(iv) Also relatively easy, and high scoring. A few fell at the interpretation.

(v) Many candidates came adrift here by incorrectly applying the technique to the original network, instead of to the complete network of shortest distances. The resulting "lower bound" is 14, which is bigger than the upper bound of 12 found in part (iv). Candidates who found themselves in this situation seemed to be unconcerned or oblivious to the problem, and went on to make strange comments in part (vi).

(vi) Surprisingly few candidates were able to make the correct deduction that the answer is 11 or 12.

4 **LP**

Candidates were very competent in the basic techniques of simplex, as tested in part (i).

In part (ii) they were instructed to initiate an extended simplex application from their solution to part (i). This requires exactly the same skills as the setting up of a two-stage or big-M tableau, and many were able to do it. The instruction was intended as a help, since with the correctly deduced tableau, one iteration leads to optimality. There were many candidates who were not able to follow the instructions, and who set up a tableau "ab initio". For these candidates full marks were still available, but more iterations were needed, and many succeeded thus.

The structure of the final part was similar, but it would have been much more difficult to solve ab initio, and no-one did so.

4773: Decision Mathematics Computation

General Comments

There were fewer problems this year involving missing printouts. However, many candidates could usefully spend a few moments helping the examiner, and themselves, by arranging their printouts in the correct order and orientation, and checking that they are correctly labelled.

Comments on Individual Questions

1 **Recurrence relations**

It was very surprising to see so many candidates failing to produce the Excel output that was needed. It was expected that there would be many who failed correctly to "integerise" as specified, but many, even among those who succeeded with the recurrence relation algebra, failed to answer the first part of (iii) adequately.

2 **Networks**

This question was entirely on matchings.
Some candidates worked through the question without introducing vertices P1 and P2 for the two pine trees. This was possible, but was not always carried through successfully.
Not all candidates were able to mount a convincing argument in part (ii). However, in addition to the logically argued solutions, a few candidates produced an appropriate LP which showed a complete matching was not possible.
It was very noticeable that the computing work (part (v)) was done better more often than was the theoretical work. In particular, there were many candidates who were unable to tackle the alternating path in part (iv).

3 **LP modelling**

Many candidates were able to cope well with this question, and good solutions were often seen. Some candidates thought that C3 could not see all the way to C8, but this erroneous assumption was often then incorrectly implemented. A few candidates were unclear about the use of inequalities in their constraints. Marks were unnecessarily lost when candidates failed to make clear the interpretation of their LP output.

4 **Simulation**

Many candidates found this to be the most difficult of the questions. Parts (i), (ii) and (iii) were relatively easy, but were often not done efficiently. Furthermore candidates often made it **very** difficult for examiners to check what was being done.
In part (iv) few candidates made clear how they were modelling the changed distribution of failure times, probably because they were themselves confused about it.
Few correct cost comparison calculations were seen.
Most managed to pick up the final mark!

4776: Numerical Methods

General Comments

Though there were some very good scripts, a substantial number of candidates seemed ill prepared for this exam. Some very fundamental ideas – maximum possible error, relative error – seemed unfamiliar; some standard techniques – Newton-Raphson, Lagrange interpolation– were not accurately understood. There was a lot of poor algebra, and a significant minority of candidates worked their solutions with too small a number of significant figures, losing accuracy and losing marks. Over all, there was a sense that some candidates were insufficiently experienced on Numerical Methods papers.

Comments on Individual Questions

1 **Bisection**

The bisection method was well understood, but the layout of solutions was often poor. Here, as elsewhere in numerical work, a tabular layout is best. Many candidates did not understand maximum possible error. Two inappropriate approaches were seen: some took the value of the function as the maximum possible error; others simply iterated many times more than required in the hope that they would have gone far enough.

2 **Numerical integration**

The first half of the question was frequently done well, though some candidates still do not appreciate the relationship between the mid-point rule, the trapezium rule and Simpson's rule. The point here is that the weighted average of M and T with $h = 0.5$ gives S with $h = 0.25$. Extrapolation was often not done well, though some candidates showed confidence with one of the several approaches possible. Full marks were available for full extrapolation or for just finding the next term in the sequence of Simpson's rule estimates as the convergence is so rapid.

3 **Cosine rule, errors**

The basic elements of this question often let candidates down. Some got the cosine rule wrong even though it is GCSE work and in the formula book. Some had their calculators in radian mode even though the question is very clearly set in degrees. Some forgot to take the square root to find a. Some, not reading the question carefully, found the errors in the approximation for the cosine rather than the approximation for a.

4 **Relative errors, π**

Most candidates were unable to identify r in the formula $X = x(1 + r)$ as the relative error. The binomial expansion was beyond many; though with the result given most could then go on to gain the marks for part (iii). There were no marks for calculating the relative errors from first principles in this part.

5 **Lagrange interpolation**

As usual, there were candidates who confused the x and f(x) values in Lagrange's formula, and those who could not handle the algebra. Some whose working was otherwise correct multiplied the expression for f(x) by 20 in order to eliminate decimal fractions. This question was, nevertheless, often done well.

6 **Newton-Raphson method**

(i) In part (i), a good, detailed, clearly labelled sketch of the Newton-Raphson method was sufficient to get full marks, though candidates often helped their case if they offered a short written explanation. The major error here was to give another method altogether, such as fixed-point iteration.

(ii) In part (ii), some candidates sketched tan$x$ and 2x as separate graphs, making it impossible to gain any of the marks. Good solutions identified starting points that would give convergence to the root at zero, or difficulties starting at the turning point or beyond the asymptote.

(iii) In part (iii), there was some fudging of the derivative of f(x), some work in degrees, and some inaccurate use of calculators. However, there were also many correct solutions. Showing that the iteration is faster than first order defeated quite a few. Many showed that the ratios of differences are not constant. This demonstrates that the process is not first order. Observing that the ratios decrease shows that the process is *faster* than first order.

7 **Difference table, Newton's method**

(i) The difference table in part (i) was generally done well, though there were some sign errors. The function is not quadratic because the second differences are not equal. Candidates who answered that the function was almost quadratic because the second differences were almost equal were given the credit as they were judged to have understood the point. Those who said that the function was quadratic because the differences were approximately equal did not receive full credit. (Some candidates seemed to believe that the function would be quadratic if the second differences were within 10% of one another.)

(ii) In part (ii), the method was generally well understood but the algebra defeated some.

(iii) The final part was, for most, a routine application of the approximating quadratic just found. Those who had made a substantial error in part (ii) frequently had quadratics that gave absurdly large errors. This should have been a clear sign of an earlier mistake.

4777: Numerical Computation

General Comments

The candidature for this paper was, once again, small and so generalisations are difficult. Several candidates scored high marks, but the rest scored poorly, showing little knowledge of the necessary theory and little familiarity with the techniques. The poorest candidates seemed not fully at home in the use of a spreadsheet.

Comments on Individual Questions

1 **Solution of an equation; acceleration**

In part (i) the algebra was sometimes unconvincing. Parts (ii) and (iii) were done better, though some candidates did not appreciate that, when the acceleration formula is used to produce an improved estimate, that value should be used to re-start the process.

2 **Gaussian 4-point rule**

There was just one good attempt at this question, with the candidate scoring highly. The other attempts were poor, with candidates unable to make much progress in the theoretical or practical parts of the question.

3 **Predictor-corrector method**

This was the least popular question, and also the least well done with no candidate achieving more than half marks. Euler's method was known, but the modified Euler and the predictor-corrector extensions were beyond all candidates.

4 **Gaussian elimination**

Those who talked this question did well, with no candidate scoring less than half marks. The fundamental ideas of Gaussian elimination to solve equations, find inverses and find determinants were well understood and successfully implemented on a spreadsheet.

Coursework report
Summer, 2007
4753/02, 4758/02, 4776/02

Moderators were pleased to receive the MS1 and the sample of work from the vast majority of centres in good time, but there were a few centres where inconvenience was caused by the late arrival of the work.

There have been a number of cases where incorrect work is ticked and given credit (in C3 and, in particular, NM). We do ask that if the work is not checked then assessors do not tick it, and preferably write "not checked".

The majority of centres also included the Centre Authentication Form, CCS160, but again there was a degree of inconvenience caused by a few centres where this form had to be requested, in a few cases more than once. Centres are reminded that this is now a requirement and failure to submit the form results in all marks from the centre in the component being set to 0.

As always, teachers will find that most of what is stated below has been said before. We feel we need to repeat what we have said before because we continue to experience the same difficulties with marking.
These documents are therefore crucial to centres who are engaged in the process of assessment and we would encourage Heads of Departments and Examination Officers to ensure that all those involved in the assessment have a copy of the report to inform them for future sessions.
We wish to stress that the vast amount of work we have seen displays a high level of commitment by candidates and assessors with appropriate marks being awarded. In a few cases, however, this is unfortunately not the case.

Methods for Advanced Mathematics (C3); Numerical solution of equations (4753/02)

A small number of centres continue to assess the work using an incorrect cover sheet. This incorrect sheet was originally published with the specification but was amended within weeks of publication. Subsequently, centres have been sent the correct sheets and asked to destroy the old versions. Some centres even used both the correct and incorrect ones within their assessment and even within a single group. The old versions are now over three years old! This will have been noted on individual reports to centres, and if you receive such a comment please will you ensure that all incorrect sheets are destroyed.

Change of sign
There is still a tendency for candidates to launch into theoretical considerations at the commencement of the coursework. This is not worthy of credit – marks are to be allocated to work which is meeting the criteria for the specific equation being solved. This includes graphical work where the graph of the function needs to be annotated to demonstrate the method working in this specific case.
The failure of the method is sometimes carried out inappropriately. In particular, this includes situations where the search of a sign change actually locates the root or locates the discontinuity. Equations such as $\frac{1}{x-1}=0$ or $(x-2)^2=0$ are therefore deemed as trivial and should not be used.

Newton-Raphson method

The second mark in this domain is for finding all remaining roots, the first mark being for the first root. If there is only one root then this mark should not be awarded. The requirements of the task include the need for the equation to be used in this domain to have at least two roots (see specification book, page 62). Assessors are still interpreting this criterion as automatically having been satisfied for an equation with only one root once the first mark is awarded. This is not so. Error bounds need to be established. It is not enough to note that successive iterates agree to *n* decimal places and that therefore the root is found to *n* decimal places. Error bounds are typically established by a change of sign calculation.

Some candidates illustrate a "failure" of the method by simply moving the first value of *x* away from the root. The criterion states that they should be demonstrating a failure to locate the expected root "despite a starting value close to it". Merely choosing an unrealistic value for x_0 does not satisfy this criterion. We would expect a candidate to choose a starting value at one of the end points of the integer range within which the root lies.

Particularly in this domain, candidates who use computer resources to do the work for them should give some indication that they understand the method by doing some of the work themselves, either using a spreadsheet or calculator.

Rearrangement method

The main problem in this domain continues to be the description of why convergence or not was achieved. It is expected that candidates will make some reference to the fact that the gradient of the line $y = x$ is 1 and that convergence will therefore only be achieved if $|g'(x)| < 1$. Merely stating that $g'(x) < 1$ with no explanation does not fulfil the criterion.

Comparison

Candidates should comment on the resources (hardware and software) they have used and how effective it has been in aiding the coursework. There is no "right" answer to this - candidates who have used spreadsheets and/or "Autograph" may well come to an entirely different conclusion from candidates who have only had a scientific calculator at their disposal.

Written communication

Candidates will continue to confuse equations with functions and even expressions.
A candidate who writes "I am going to solve the equation $y = x^3 + x - 7$" or even "I am going to solve the equation $x^3 + x - 7$" should not be credited with having written correct notation and terminology. The moderators continually find that a large number of candidates are awarded this mark with a positive comment given, yet the work is full of the confusions described above.

Oral communication

As with the investigations in the other two units, it is a requirement that the assessor fulfils this criterion and writes a brief report on how it was done and the results. Assessors are reminded that it is not permissible to give credit for any of the other criteria as a result of this oral communication.

Differential Equations (4758/02)

The marks submitted by approximately twenty per cent of the centres were changed.
One of the most common problems was the very brief list of assumptions which were not discussed and related to the original model. This is, of course, fundamental to the concept of modelling. This comment is also relevant to the marking of Domain 5 when the assumptions are modified to produce a second model. For full marks in this domain one would expect a justification for the new model (other than trying something else since the original one was not satisfactory).

The differential equations that are used for modelling and their solution should reflect the work covered in the Differential Equations Specification. The use of a basic separable variable as a first model is only really acceptable, for full marks in Domain 2, provided a differential equation more appropriate to the specification is used in the revision phase. Similarly, the solution of equations using basic numerical methods when an analytical solution is feasible would not merit full marks in domain 6.

In the comparison domain, tables **and** graphs should be used whenever possible.

Finally care must be taken to avoid circular arguments; that is using the experimental data to produce predicted results of the same data. It is better to find the various parameters from one set of readings and then apply them to a second set of readings.

Numerical Methods (4776/02)

Most candidates tackled numerical integration; only a few candidates chose unsuitable topics. Generally speaking there was more work which was assessed in line with national standards. It was disappointing to observe, however, many instances of work that was (obviously) wrong being marked correct. The following comments represent some common themes:

Domain 1

A clear statement of the problem in correct mathematical notation (including limits) (not in excel code) is a minimum requirement for the first mark. Assessors should not have to trawl through spreadsheet output to work out what is being done.

We saw a good number of scripts where a problem was specified, but another problem was solved. Surprisingly these were often given full marks in both domains 1 and 3.

If a whole group (or section of a large group) tackle the same problem in the same way, it is hard to justify full marks for problem specification (and maybe for strategy) as clearly there has been little initiative on the part of the candidate. Yet full marks were often awarded in such circumstances.

Domain 2

Many candidates reproduced reams of bookwork describing the methods (for which there should be no credit) or tried the methods on a simple function such as $f(x) = x^2$ to justify the use of Simpson's rule. Those who deserved full marks *either* wrote a paragraph (for example) to the effect that Simpson's rule is a higher order method, and converges faster, hence its selection *or* demonstrated with the aid of a sketch that for their function "M" overestimates and "T" underestimates (or vice versa) and hence both algorithms would be used to provide bounds for the final answer. Many also went on to explain that Simpson's rule could be calculated directly from M and T, and that this would be an improved approximation. In a good number of cases, however, candidates incriminated themselves by writing rubbish – yet were still given full marks. Muddled explanations should be penalised!

Domain 3

Subdividing as far as 16 strips does not usually warrant a "substantial application" – we have intimated that 64 strips is reasonable. This was mostly well done and well assessed.

Domain 4

No marks are available for discussing the relative merits of different spreadsheet packages. What is required is a clear explanation of how the algorithms were implemented – either by detailed commentary in the text, or (preferably) by a well annotated printout of spreadsheet cell formulae. The second mark was often awarded on flimsy evidence.

Domain 5

The work in this domain (and the assessment of it) has improved. Most candidates found the ratio of differences and appropriately used it to extrapolate to an improved solution. This can gain full marks when it is well done. Problems arose when extrapolation was carried out too soon, before "*r*" had settled down, or when *r* was **not** converging to its theoretical value, but the theoretical value was used anyway. There are still some candidates who take their best answer as being "exact", and use it to show that, for example, the Midpoint rule is second order……even though they've just used $r = 0.25$ (because they have shown by the ratio of differences that the method is second order) to extrapolate!! A few still refer to the known value (eg of π, or of a value obtained from a graphics calculator) and use this to find relative error etc. No marks should be awarded for this analysis.

Domain 6

It is surprising how many candidates do not state their final solution, and yet are still given the marks! The assessor should not have to trawl through spreadsheet output to find it. 6 s.f. accuracy should be considered a minimum: some candidates were given the first two marks for 3 s.f. only. A number of candidates simply wrote down all the figures in their last approximation – often the quoted level of accuracy could not possibly be justified from the figures generated. The final two marks were often awarded for no good reason. (Usually for ramblings about excel only working to 9 decimal places (which is incorrect) or the possibility of human error). When the integral is well behaved there is less scope here, but the following points could be addressed.
Validity: Compare best "M" and best "T" – these should give bounds for "I". Compare best "S" (ie extrapolated value to ∞) with last iterated value – S_{64}? A discussion of the number of decimal places quoted in the final answer can then take place.
Limitations: Compare "*r*" with theoretical value – has it converged? (There is more scope here with functions that are not well-behaved). How much accuracy could have been lost? Are there any odd things about the function – eg x^x when $x = 0$?

Domain 7

A brief report is expected. This was not always given.

7895-8,3895-3898 AS and A2 MEI Mathematics
June 2007 Assessment Session

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
4751	Raw	72	54	46	38	31	24	0
4752	Raw	72	54	47	40	33	26	0
4753	Raw	72	60	52	45	38	30	0
4753/02	Raw	18	15	13	11	9	8	0
4754	Raw	90	65	57	49	41	34	0
4755	Raw	72	59	51	44	37	30	0
4756	Raw	72	52	45	38	32	26	0
4757	Raw	72	53	46	39	32	25	0
4758	Raw	72	55	47	40	33	25	0
4758/02	Raw	18	15	13	11	9	8	0
4761	Raw	72	59	51	43	36	29	0
4762	Raw	72	59	52	45	38	31	0
4763	Raw	72	61	53	45	37	30	0
4764	Raw	72	62	54	46	38	31	0
4766	Raw	72	55	48	41	35	29	0
4767	Raw	72	58	51	44	37	30	0
4768	Raw	72	62	53	45	37	29	0
4769	Raw	72	54	47	40	33	27	0
4771	Raw	72	59	53	47	41	35	0
4772	Raw	72	52	45	39	33	27	0
4773	Raw	72	59	51	43	36	29	0
4776	Raw	72	53	46	40	33	26	0
4776/02	Raw	18	13	11	9	8	7	0
4777	Raw	72	55	47	39	32	25	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
7895-7898	600	480	420	360	300	240	0
3895-3898	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
7895	43.5	64.3	80.2	90.9	97.5	100	9403
7896	57.9	78.6	90.1	96.2	98.6	100	1301
7897	88.2	97.1	100	100	100	100	34
7898	100	100	100	100	100	100	2
3895	27.4	42.6	57.3	70.9	82.9	100	12342
3896	55.4	73.4	85.1	92.1	97.1	100	1351
3897	75.2	87.2	97.3	99.1	100	100	109
3898	71.4	82.1	82.1	96.4	96.4	100	28

For a description of how UMS marks are calculated see;
http://www.ocr.org.uk/exam_system/understand_ums.html

Statistics are correct at the time of publication

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