



**ADVANCED GCE UNIT
MATHEMATICS (MEI)**

Further Methods for Advanced Mathematics (FP2)

THURSDAY 7 JUNE 2007

4756/01

Morning
Time: 1 hour 30 minutes

Additional materials:

Answer booklet (8 pages)

Graph paper

MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions in Section A and **one** question from Section B.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

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Section A (54 marks)

Answer all the questions

- 1 (a) A curve has polar equation $r = a(1 - \cos \theta)$, where a is a positive constant.
- (i) Sketch the curve. [2]
- (ii) Find the area of the region enclosed by the section of the curve for which $0 \leq \theta \leq \frac{1}{2}\pi$ and the line $\theta = \frac{1}{2}\pi$. [6]
- (b) Use a trigonometric substitution to show that $\int_0^1 \frac{1}{(4-x^2)^{\frac{3}{2}}} dx = \frac{1}{4\sqrt{3}}$. [4]
- (c) In this part of the question, $f(x) = \arccos(2x)$.
- (i) Find $f'(x)$. [2]
- (ii) Use a standard series to expand $f'(x)$, and hence find the series for $f(x)$ in ascending powers of x , up to the term in x^5 . [4]
- 2 (a) Use de Moivre's theorem to show that $\sin 5\theta = 5 \sin \theta - 20 \sin^3 \theta + 16 \sin^5 \theta$. [5]
- (b) (i) Find the cube roots of $-2 + 2j$ in the form $re^{j\theta}$ where $r > 0$ and $-\pi < \theta \leq \pi$. [6]
- These cube roots are represented by points A, B and C in the Argand diagram, with A in the first quadrant and ABC going anticlockwise. The midpoint of AB is M, and M represents the complex number w .
- (ii) Draw an Argand diagram, showing the points A, B, C and M. [2]
- (iii) Find the modulus and argument of w . [2]
- (iv) Find w^6 in the form $a + bj$. [3]

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3 Let $\mathbf{M} = \begin{pmatrix} 3 & 5 & 2 \\ 5 & 3 & -2 \\ 2 & -2 & -4 \end{pmatrix}$.

(i) Show that the characteristic equation for \mathbf{M} is $\lambda^3 - 2\lambda^2 - 48\lambda = 0$. [4]

You are given that $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ is an eigenvector of \mathbf{M} corresponding to the eigenvalue 0.

(ii) Find the other two eigenvalues of \mathbf{M} , and corresponding eigenvectors. [8]

(iii) Write down a matrix \mathbf{P} , and a diagonal matrix \mathbf{D} , such that $\mathbf{P}^{-1}\mathbf{M}^2\mathbf{P} = \mathbf{D}$. [3]

(iv) Use the Cayley-Hamilton theorem to find integers a and b such that $\mathbf{M}^4 = a\mathbf{M}^2 + b\mathbf{M}$. [3]

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 (a) Find $\int_0^1 \frac{1}{\sqrt{9x^2 + 16}} dx$, giving your answer in an exact logarithmic form. [5]

(b) (i) Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that

$$\sinh 2x = 2 \sinh x \cosh x. \quad [2]$$

(ii) Show that one of the stationary points on the curve

$$y = 20 \cosh x - 3 \cosh 2x$$

has coordinates $(\ln 3, \frac{59}{3})$, and find the coordinates of the other two stationary points. [7]

(iii) Show that $\int_{-\ln 3}^{\ln 3} (20 \cosh x - 3 \cosh 2x) dx = 40$. [4]

[Question 5 is printed overleaf.]

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 The curve with equation $y = \frac{x^2 - kx + 2k}{x + k}$ is to be investigated for different values of k .

(i) Use your graphical calculator to obtain rough sketches of the curve in the cases $k = -2$, $k = -0.5$ and $k = 1$. [6]

(ii) Show that the equation of the curve may be written as $y = x - 2k + \frac{2k(k+1)}{x+k}$.

Hence find the two values of k for which the curve is a straight line. [4]

(iii) When the curve is not a straight line, it is a conic.

(A) Name the type of conic. [1]

(B) Write down the equations of the asymptotes. [2]

(iv) Draw a sketch to show the shape of the curve when $1 < k < 8$. This sketch should show where the curve crosses the axes and how it approaches its asymptotes. Indicate the points A and B on the curve where $x = 1$ and $x = k$ respectively. [5]