OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4764/01**

MATHEMATICS (MEI)

Mechanics 4

FRIDAY 22 JUNE 2007

Morning
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

[Turn over

Section A (24 marks)

1 A light elastic string has one end fixed to a vertical pole at A. The string passes round a smooth horizontal peg, P, at a distance a from the pole and has a smooth ring of mass m attached at its other end B. The ring is threaded onto the pole below A. The ring is at a distance y below the horizontal level of the peg. This situation is shown in Fig. 1.

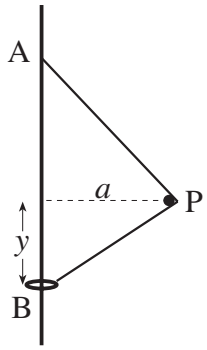

Fig. 1

The string has stiffness k and natural length equal to the distance AP.

(i) Express the extension of the string in terms of y and a. Hence find the potential energy of the system relative to the level of P. [5]

(ii) Use the potential energy to find the equilibrium position of the system, and show that it is stable. [5]

(iii) Calculate the normal reaction exerted by the pole on the ring in the equilibrium position. [2]

2 A railway truck of mass m_0 travels along a horizontal track. There is no driving force and the resistances to motion are negligible. The truck is being filled with coal which falls vertically into it at a mass rate k. The process starts as the truck passes a point O with speed u. After time t, the truck has velocity v and the displacement from O is x.

(i) Show that $v = \dfrac{m_0 u}{m_0 + kt}$ and find x in terms of m_0, u, k and t. [9]

(ii) Find the distance that the truck has travelled when its speed has been halved. [3]

Section B (48 marks)

3 **(i)** Show, by integration, that the moment of inertia of a uniform rod of mass m and length $2a$ about an axis through its centre and perpendicular to the rod is $\frac{1}{3}ma^2$. [6]

A pendulum of length 1 m is made by attaching a uniform sphere of mass 2 kg and radius 0.1 m to the end of a uniform rod AB of mass 1.2 kg and length 0.8 m, as shown in Fig. 3. The centre of the sphere is collinear with A and B.

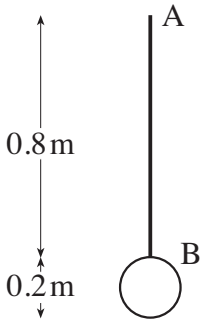

Fig. 3

(ii) Find the moment of inertia of the pendulum about an axis through A perpendicular to the rod. [7]

The pendulum can swing freely in a vertical plane about a fixed horizontal axis through A.

(iii) The pendulum is held with AB at an angle α to the downward vertical and released from rest. At time t, AB is at an angle θ to the vertical. Find an expression for $\dot{\theta}^2$ in terms of θ and α. [6]

(iv) Hence, or otherwise, show that, provided that α is small, the pendulum performs simple harmonic motion. Calculate the period. [5]

4 A particle of mass 2 kg starts from rest at a point O and moves in a horizontal line with velocity v m s^{-1} under the action of a force F N, where $F = 2 - 8v^2$. The displacement of the particle from O at time t seconds is x m.

(i) Formulate and solve a differential equation to show that $v^2 = \frac{1}{4}(1 - e^{-8x})$. [7]

(ii) Hence express F in terms of x and find, by integration, the work done in the first 2 m of the motion. [6]

(iii) Formulate and solve a differential equation to show that $v = \frac{1}{2}\left(\frac{1-e^{-4t}}{1+e^{-4t}}\right)$. [7]

(iv) Calculate v when $t = 1$ and when $t = 2$, giving your answers to four significant figures. Hence find the impulse of the force F over the interval $1 \leqslant t \leqslant 2$. [4]