OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

4773/01

MATHEMATICS (MEI)

Decision Mathematics Computation

MONDAY 18 JUNE 2007

Morning
Time: 2 hours 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the answer booklet.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is 72.

1 The number of branches on a tree in a particular year is modelled as the number of branches that were on the tree in the previous year plus new growth of p times the number that were on the tree the year before that, $0 < p < 1$.

(i) Let u_n be the number of branches on the tree in year n. Write down a recurrence relation for u_{n+2} in terms of u_{n+1}, u_n and p. [2]

(ii) The tree was bought $(n = 0)$ with 20 branches. It had 25 branches after one year $(n = 1)$. Given that $p = 0.11$, solve your recurrence relation. [8]

(iii) Construct a spreadsheet to model your recurrence relation, and use it to check your answer to part **(ii)**.

Add to your spreadsheet to show the number of branches each year as the nearest integer to that given by applying the recurrence relation to the previous two integers. Print out your spreadsheet for n from 0 to 20. Print out the formulae which you used. (Just one example of each formula will suffice.) [4]

To control the growth of the tree it is pruned each year, after the new growth has taken place. New growth is not pruned, but a proportion, r $(0 < r < 1)$, of old branches is removed. Let v_n be the number of branches on the tree in year n, after pruning.

(iv) Modify your answer to part **(i)** to produce a recurrence relation for v_{n+2} in terms of v_{n+1}, v_n, p and r. [2]

(v) Modify your spreadsheet to allow for pruning and find a value of r which will lead to about 100 branches after 20 years (using $v_0 = 20$, $v_1 = 25$ and $p = 0.11$). [2]

2 Six trees are to be planted, two pines, one eucalyptus, one mimosa, one jacaranda and one acacia. There are 7 locations available, at each of which one tree can be planted. The table shows which trees can be planted in each of the locations.

Location	1	2	3	4	5	6	7
Trees	pine eucalyptus	eucalyptus jacaranda	mimosa acacia	pine jacaranda	pine mimosa	mimosa acacia	acacia

(i) Draw a bipartite graph to represent this information. [2]

The gardener decides that he does not wish to use location number 2. He starts placing trees by locations prior to planting.

(ii) Show that there is no solution if location 2 is not used. [2]

The gardener now decides to reject location 7 instead of location 2. He starts by placing the eucalyptus by location 1, the jacaranda by location 2, the acacia by location 3, the first pine by location 4 and the mimosa by location 5. He then realises that he has nowhere to put the second pine.

(iii) Represent the gardener's incomplete matching on a second bipartite graph. [1]

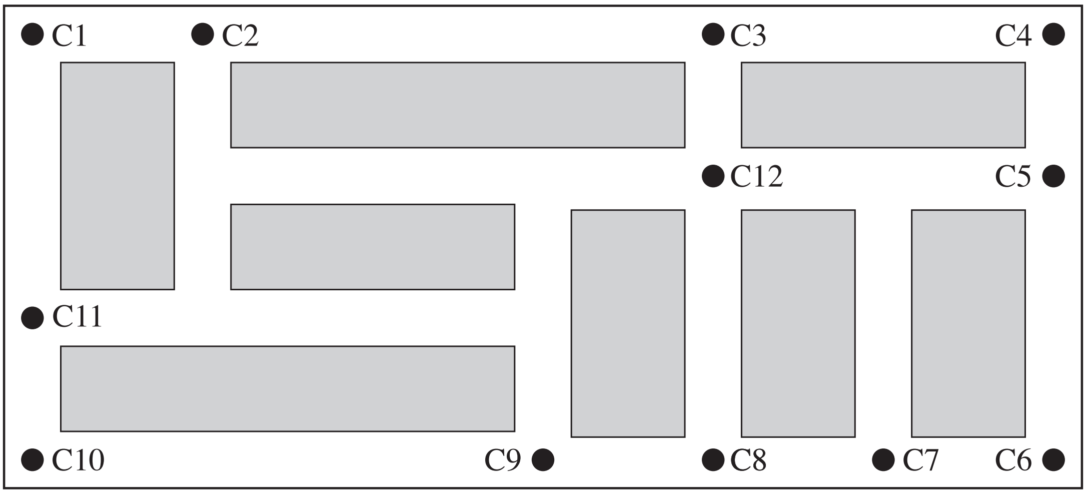
(iv) Give an alternating path, starting at the second pine and ending at location 6, of arcs taken alternately from the full bipartite graph and from the graph representing the incomplete matching. Hence give a complete matching. [3]

(v) The gardener would like to have an automatic procedure to solve similar tree-planting problems in the future. Produce an LP to solve the problem of finding a maximal matching from the information given in the original table.

Produce a print-out of your LP. Run it and produce a print-out of your results.

Interpret your results. [10]

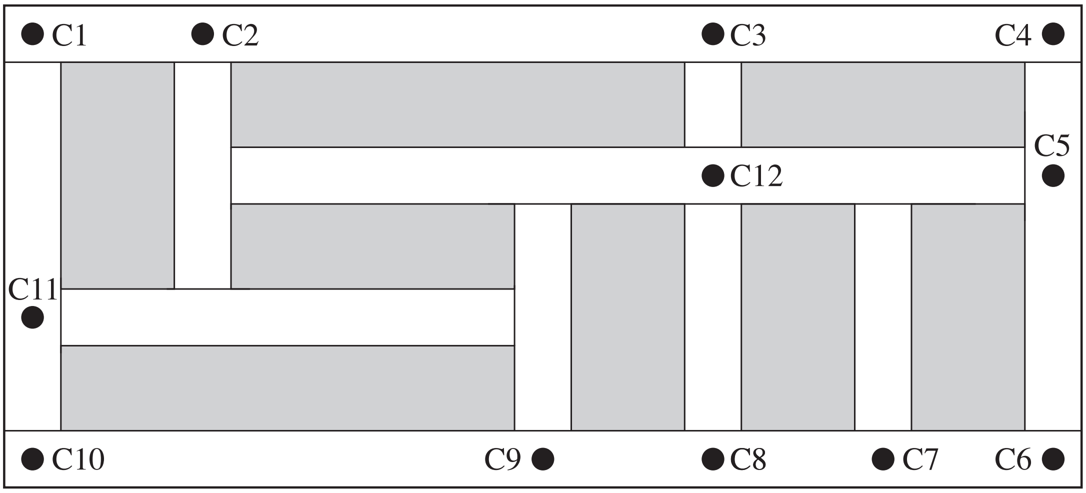
3 The builders of a shopping precinct have to decide where to place CCTV cameras. The diagram shows buildings (which are shaded), pavements, and 12 possible locations for cameras.



Cameras can be rotated to view along different directions, and all pavements must be in sight of at least one camera.

(i) By inspection select a set of 6 locations from which cameras can scan all pavements. [2]

The diagram below shows one way of splitting the pavements into rectangles.



(ii) Formulate an LP to select a minimum set of locations from which cameras can scan all of the rectangles. Produce a print-out of your formulation. [8]

(iii) Run your LP, produce a print-out of your output, and interpret the results. [3]

The costs of installing a camera depends on the location. They are listed below.

Location	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
Cost (£000)	5	2	3	5	4	1.5	2	2	5	3	4	7

(iv) Modify your LP to find the cheapest way of achieving full coverage of all pavements. [2]

(v) Run your modified LP, produce a print-out of your output, and interpret the results. [3]

[Question 4 is printed overleaf.]

4 A component in a machine has a short lifespan. It fails either after 1, 2 or 3 days, with probabilities given in the table.

Time to failure (days)	1	2	3
Probability	0.1	0.3	0.6

When a component fails it is replaced at the end of the day.

(i) Construct a look-up table to simulate the failure time for a component. Print out the formulae which you use. [3]

(ii) Set up a spreadsheet to simulate failure times for a number of components so that you can accumulate the times to failure. Simulate enough components so that the accumulated failure times exceed 16 days.

Print out your spreadsheet formulae. [3]

(iii) From your simulation in part **(ii)** record

- whether or not there was a failure on day 14,
- whether or not there was a failure on day 15,
- whether or not there was a failure on day 16,
- the total number of failures up to and including day 14.

Repeat your simulation 9 more times (10 times in total), recording information as before. Hence estimate the probability of a failure on day 14, the probability of a failure on day 15, the probability of a failure on day 16, and the expected number of failures up to and including day 14. [5]

Replacing a part when it fails costs £50, plus the cost of the component, which is £25. An alternative policy is to replace a component if it fails on its first day, and otherwise to replace it anyway, failed or not, at the end of its second day. Such a scheduled replacement costs £30 plus the cost of the component.

(iv) Simulate the operation of this scheduled replacement policy over a period of 14 days. Repeat your simulation 10 times and use your results from part **(iii)** to see whether or not this policy is cost effective. [6]

(v) How could you improve the reliability of your results? [1]

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