OCR
RECOGNISING ACHIEVEMENT

**ADVANCED GCE**

**MATHEMATICS (MEI)** **4754A**

Applications of Advanced Mathematics (C4) Paper A

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

**Other Materials Required:**
None

**Tuesday 13 January 2009**
**Morning**

**Duration:** 1 hour 30 minutes

*4754A*

**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

**NOTE**

- This paper will be followed by **Paper B: Comprehension**.



**Turn over**

**Section A** (36 marks)

**1** Express $\dfrac{3x+2}{x(x^2+1)}$ in partial fractions. **[6]**

**2** Show that $(1+2x)^{\frac{1}{3}} = 1 + \frac{2}{3}x - \frac{4}{9}x^2 + \ldots$, and find the next term in the expansion.

State the set of values of $x$ for which the expansion is valid. **[6]**

**3** Vectors $\mathbf{a}$ and $\mathbf{b}$ are given by $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

Find constants $\lambda$ and $\mu$ such that $\lambda\mathbf{a} + \mu\mathbf{b} = 4\mathbf{j} - 3\mathbf{k}$. **[5]**

**4** Prove that $\cot\beta - \cot\alpha = \dfrac{\sin(\alpha-\beta)}{\sin\alpha\sin\beta}$. **[3]**

**5** **(i)** Write down normal vectors to the planes $2x - y + z = 2$ and $x - z = 1$.

Hence find the acute angle between the planes. **[4]**

**(ii)** Write down a vector equation of the line through $(2, 0, 1)$ perpendicular to the plane $2x - y + z = 2$. Find the point of intersection of this line with the plane. **[4]**

**6** **(i)** Express $\cos\theta + \sqrt{3}\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $\alpha$ is acute, expressing $\alpha$ in terms of $\pi$. **[4]**

**(ii)** Write down the derivative of $\tan\theta$.

Hence show that $\displaystyle\int_0^{\frac{1}{3}\pi} \frac{1}{(\cos\theta + \sqrt{3}\sin\theta)^2}\,d\theta = \frac{\sqrt{3}}{4}$. **[4]**

**Section B** (36 marks)

**7** Scientists can estimate the time elapsed since an animal died by measuring its body temperature.

**(i)** Assuming the temperature goes down at a constant rate of 1.5 degrees Fahrenheit per hour, estimate how long it will take for the temperature to drop

(*A*) from 98 °F to 89 °F,

(*B*) from 98 °F to 80 °F. **[2]**

In practice, rate of temperature loss is not likely to be constant. A better model is provided by Newton's law of cooling, which states that the temperature $\theta$ in degrees Fahrenheit $t$ hours after death is given by the differential equation

$$\frac{d\theta}{dt} = -k(\theta - \theta_0),$$

where $\theta_0$ °F is the air temperature and $k$ is a constant.

**(ii)** Show by integration that the solution of this equation is $\theta = \theta_0 + Ae^{-kt}$, where $A$ is a constant. **[5]**

The value of $\theta_0$ is 50, and the initial value of $\theta$ is 98. The initial rate of temperature loss is 1.5 °F per hour.

**(iii)** Find $A$, and show that $k = 0.031\,25$. **[4]**

**(iv)** Use this model to calculate how long it will take for the temperature to drop

(*A*) from 98 °F to 89 °F,

(*B*) from 98 °F to 80 °F. **[5]**

**(v)** Comment on the results obtained in parts **(i)** and **(iv)**. **[1]**

**[Question 8 is printed overleaf.]**

**8** Fig. 8 illustrates a hot air balloon on its side. The balloon is modelled by the volume of revolution about the $x$-axis of the curve with parametric equations

$$x = 2 + 2\sin\theta, \quad y = 2\cos\theta + \sin 2\theta, \quad (0 \leqslant \theta \leqslant 2\pi).$$

The curve crosses the $x$-axis at the point A (4, 0). B and C are maximum and minimum points on the curve. Units on the axes are metres.

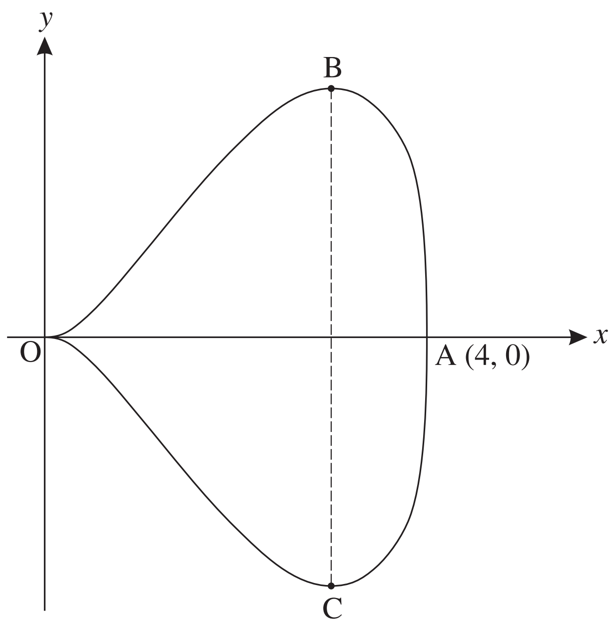


**Fig. 8**

**(i)** Find $\frac{dy}{dx}$ in terms of $\theta$. **[4]**

**(ii)** Verify that $\frac{dy}{dx} = 0$ when $\theta = \frac{1}{6}\pi$, and find the exact coordinates of B.

Hence find the maximum width BC of the balloon. **[5]**

**(iii)** (*A*) Show that $y = x\cos\theta$.

(*B*) Find $\sin\theta$ in terms of $x$ and show that $\cos^2\theta = x - \frac{1}{4}x^2$.

(*C*) Hence show that the cartesian equation of the curve is $y^2 = x^3 - \frac{1}{4}x^4$. **[7]**

**(iv)** Find the volume of the balloon. **[3]**

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