OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4755

Further Concepts for Advanced Mathematics (FP1)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Thursday 15 January 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 **(i)** Find the roots of the quadratic equation $z^2 - 6z + 10 = 0$ in the form $a + b\text{j}$. **[2]**

(ii) Express these roots in modulus-argument form. **[3]**

2 Find the values of A, B and C in the identity $2x^2 - 13x + 25 \equiv A(x-3)^2 - B(x-2) + C$. **[4]**

3 Fig. 3 shows the unit square, OABC, and its image, OA′B′C′, after undergoing a transformation.

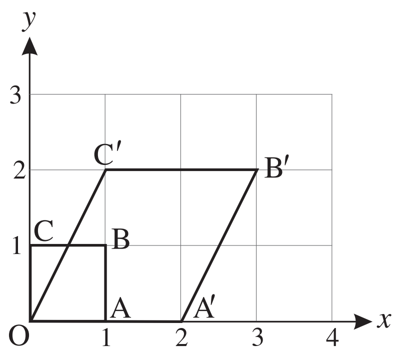

Fig. 3

(i) Write down the matrix **P** representing this transformation. **[1]**

(ii) The parallelogram OA′B′C′ is transformed by the matrix $\mathbf{Q} = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$. Find the coordinates of the vertices of its image, OA″B″C″, following this transformation. **[2]**

(iii) Describe fully the transformation represented by **QP**. **[2]**

4 Write down the equation of the locus represented in the Argand diagram shown in Fig. 4. **[3]**

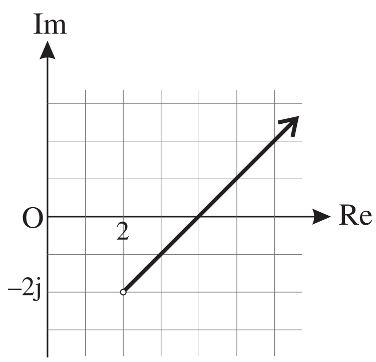

Fig. 4

5 The cubic equation $x^3 - 5x^2 + px + q = 0$ has roots α, -3α and $\alpha + 3$. Find the values of α, p and q. **[6]**

6 Using the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ show that

$$\sum_{r=1}^{n} r(r^2 - 3) = \tfrac{1}{4}n(n+1)(n+3)(n-2).$$ **[6]**

7 Prove by induction that $12 + 36 + 108 + \ldots + 4 \times 3^n = 6(3^n - 1)$ for all positive integers n. **[7]**

Section B (36 marks)

8 Fig. 8 shows part of the graph of $y = \dfrac{x^2 - 3}{(x-4)(x+2)}$. Two sections of the graph have been omitted.

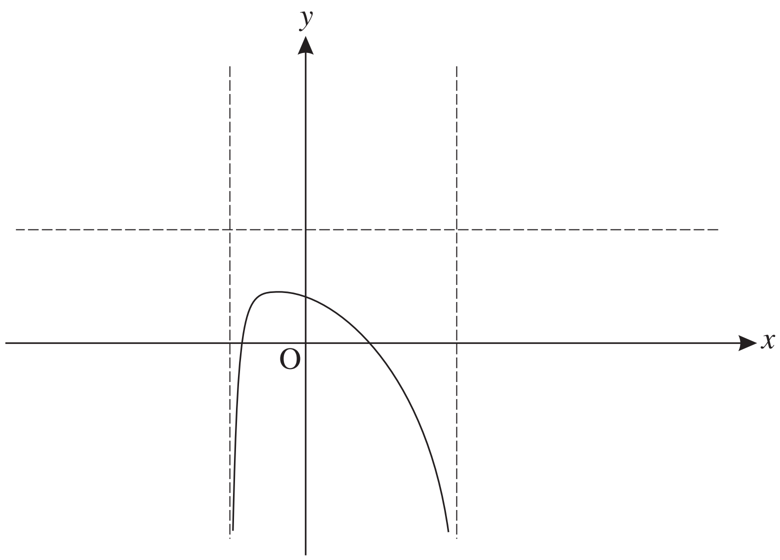

Fig. 8

(i) Write down the coordinates of the points where the curve crosses the axes. **[2]**

(ii) Write down the equations of the two vertical asymptotes and the one horizontal asymptote. **[3]**

(iii) Copy Fig. 8 and draw in the two missing sections. **[4]**

(iv) Solve the inequality $\dfrac{x^2 - 3}{(x-4)(x+2)} \leqslant 0$. **[3]**

[Questions 9 and 10 are printed overleaf.]

9 Two complex numbers, α and β, are given by $\alpha = 1 + \mathrm{j}$ and $\beta = 2 - \mathrm{j}$.

(i) Express $\alpha + \beta$, $\alpha\alpha^*$ and $\dfrac{\alpha + \beta}{\alpha}$ in the form $a + b\mathrm{j}$. **[5]**

(ii) Find a quadratic equation with roots α and α^*. **[2]**

(iii) α and β are roots of a quartic equation with real coefficients. Write down the two other roots and find this quartic equation in the form $z^4 + Az^3 + Bz^2 + Cz + D = 0$. **[5]**

10 You are given that $\mathbf{A} = \begin{pmatrix} 3 & 4 & -1 \\ 1 & -1 & k \\ -2 & 7 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 11 & -5 & -7 \\ 1 & 11 & 5+k \\ -5 & 29 & 7 \end{pmatrix}$ and that $\mathbf{AB}$ is of the form $\mathbf{AB} = \begin{pmatrix} 42 & \alpha & 4k-8 \\ 10-5k & -16+29k & -12+6k \\ 0 & 0 & \beta \end{pmatrix}$.

(i) Show that $\alpha = 0$ and $\beta = 28 + 7k$. **[3]**

(ii) Find $\mathbf{AB}$ when $k = 2$. **[2]**

(iii) For the case when $k = 2$ write down the matrix $\mathbf{A}^{-1}$. **[3]**

(iv) Use the result from part **(iii)** to solve the following simultaneous equations. **[4]**

$$\begin{aligned} 3x + 4y - z &= 1 \\ x - y + 2z &= -9 \\ -2x + 7y - 3z &= 26 \end{aligned}$$