OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI)

4776

Numerical Methods

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Tuesday 13 January 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 **(i)** Show by means of a difference table that a quadratic function fits the following data points.

x	–3	–1	1	3
y	–16	–2	4	2

[3]

(ii) Obtain the equation of the quadratic function, expressing your answer in its simplest form. **[5]**

2 **(i)** Use the formula for the difference of two squares to show that

$$\left(\sqrt{x+1}-\sqrt{x}\right)\left(\sqrt{x+1}+\sqrt{x}\right)=1. \qquad (*)$$

[2]

(ii) A spreadsheet shows $\sqrt{50001}$ as 223.6090 and $\sqrt{50000}$ as 223.6068.

Use the spreadsheet figures to obtain values of $\sqrt{50001}-\sqrt{50000}$

(*A*) by subtraction,

(*B*) by using (*)

Comment on your results. **[5]**

3 **(i)** For the integral

$$I=\int_0^{0.8}\sqrt{1-x^5}\,\mathrm{d}x$$

find the trapezium rule and mid-point rule estimates with $h = 0.8$ in each case. Use these estimates to obtain a Simpson's rule estimate. **[4]**

(ii) Given that the mid-point rule estimate with $h = 0.4$ is 0.784069 to 6 significant figures, obtain a second Simpson's rule estimate. Without doing any further calculations, give a value for I to the accuracy that is justified. **[4]**

4 **(i)** An approximation to $\cos x$, where x is small and in radians, is given by

$$\cos x \approx 1 - 0.5x^2.$$

Find the absolute and relative errors in this approximation when $x = 0.3$. **[4]**

(ii) The formula

$$\cos x \approx 1 - 0.5x^2 + kx^4$$

gives a better approximation if k is suitably chosen. By considering $x = 0.3$ again, estimate k. **[2]**

5 A student is investigating the iteration

$$x_{r+1} = x_r^2 - 3x_r + 3$$

for different starting values x_0.

Determine the values of x_1 and x_2 in each of the cases $x_0 = 3, x_0 = 2.99, x_0 = 3.01$.

Evaluate the derivative of $x^2 - 3x + 3$ at $x = 3$.

Comment on your results. **[7]**

Section B (36 marks)

6 **(i)** Show that the equation

$$\sqrt{\sin x} + \sqrt{\cos x} = 1.5, \qquad (*)$$

where x is in radians, has a root in the interval $(0.2, 0.3)$.

Perform two iterations of the bisection method and give the interval within which the root lies, the best estimate of the root, and the maximum possible error in that estimate. **[6]**

(ii) Now perform two iterations of the secant method, starting with $x_0 = 0.2$ and $x_1 = 0.3$. Give an estimate of the root to an appropriate number of significant figures.

Comment on the relative rate of convergence of the bisection method and the secant method. **[6]**

(iii) You are given that equation (*) also has a root α which is 1.298 504 to 6 decimal places. An iteration to find this root produces the following sequence of values.

r	0	1	2	3	4
x_r	1.4	1.314 351	1.298 887	1.298 504	1.298 504

By considering the values of $x_r - \alpha$, show that this iteration displays second order convergence making it clear what that means. **[6]**

[Question 7 is printed overleaf.]

7 A function f(x) has values, correct to 6 significant figures, as given in the table.

x	–0.4	–0.2	–0.1	0	0.1	0.2	0.4
f(x)	0.601 201	0.711 982	0.765 298	0.816 603	0.865 314	0.911 308	0.994 506

(i) Obtain three estimates of f′(0) using the forward difference method with h equal to 0.4, 0.2, 0.1. Show that the differences between these estimates are approximately halved as h is halved. **[4]**

(ii) Obtain three estimates of f′(0) using the central difference method. Show, by considering the differences between these estimates, that the central difference method converges more rapidly than the forward difference method. **[4]**

(iii) D_1 and D_2 are two estimates of a quantity d.

(A) Suppose that the error in D_2 is approximately half of the error in D_1. Write down expressions for the errors in D_1 and D_2 and hence show that $d \approx 2D_2 - D_1$.

(B) Now suppose that the error in D_2 is approximately a quarter of the error in D_1. Show that $d \approx \dfrac{4D_2 - D_1}{3}$. **[5]**

(iv) Use the results in part **(iii)**(A) and part **(iii)**(B) to obtain two further estimates of f′(0). Give an estimate of f′(0) to the accuracy that you consider justified. **[5]**